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A NEW METHODOLOGY FOR A QUARTERLY MEASURE OF THE OUTPUT GAP

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A new methodology for a quarterly measure of the Output Gap[1]

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[1]This paper represents the authors personal opinions and does not reflect the view of the Italian Department of Treasury. Routines on the mixed frequency factor model are coded in Ox 3.3 by Doornik (2001) and are based on the programs realized by Tommaso Proietti for the Eurostat project on EuroMIND: the Monthly Indicator of Economic Activity in the Euro Area.

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Abstract

This paper presents a new mixed frequency methodology to estimate output gaps and potential output on a quarterly basis. The methodology strongly relies on the production function method commonly agreed at the European level (D'Auria et. al., 2010) but significantly improves it as allows to assess the impact of real time forecast for GDP and other underlying variables. This feature of the model is particularly welcome in the current Italian budgetary framework which has foreseen the introduction of the principle of a budget balance in cyclically adjusted and structural terms in the Constitution. By allowing to measure output gap with a quarterly span on the basis of recent developments indicators, the methodology provides interesting hints on the cyclical position of the economy in real time to be used for deriving cyclically-adjusted fiscal aggregates.

Keywords: output gap, potential output, mixed frequency models
J.E.L. Classification: E32, C32, C53

1 Introduction

The commonly agreed Production Function method developed by the Output Gap Working Group (OGWG) (D'Auria et al., 2010) of the European Commission (EC) to estimate potential growth and output gaps and government balance net of cyclical components has recently gained large relevance both at national and at the EU level. The new provisions of the Stability and Growth Pact and the Fiscal Compact foresee that countries cannot deviate from their own medium term objectives (MTO), that is a government deficit or surplus close to zero and expressed in structural terms that can allow the debt to rapidly converge to 60 per cent of GDP and assure the long term sustainability of public finances. Analogously, the Constitutional reform approved by the Italian Parliament in 2012 forces public finances to stick to the MTO, to quickly correct for any significant deviation from it and to take into account of the effects of the business cycle when planning medium term fiscal targets. By using annual macroeconomic series, the production function methodology allows to derive potential output estimates both in real time as well as over a short/medium term horizon (normally the one spanned by the EC services forecasts). In addition, such methodology extends the potential output components out-of-sample so as to minimize the end-point-bias problem linked to the filtering of underlying series and to project "past trends" over the medium term.

Such a framework, though convenient for multilateral surveillance, may be subject to some shortcomings. First of all, the adoption of annual data may result in an inappropriate use of available statistical information at higher frequency (quarterly or monthly) that may have some relevance in the derivation of potential growth and output gap in real time.

Secondly, as the out-of-sample extension is currently carried out through simple univariate autoregressive methodologies for a number of variables (such as hours worked, investments and participation rates) it is not possible to take into account the cross correlations and linkages among such important determinants of potential output growth over the medium term.

Finally, potential output estimates are, typically, carried out assuming exogenously the macroeconomic outlook of EC Spring of Autumn forecasts (or, alternatively, national authorities projections). This choice may result in large revisions of underlying figures due to unavoidable judgmental forecast errors as well as in a huge variability of the out-of-sample projections. Such real time variability may be extremely harmful for policymakers when assessing the achievement of their own MTOs in compliance with European and constitutional rules.

The relevance of all of these issues is well known and widely recognized. Accordingly, improving the reliability of figures in real time by exploiting higher frequency data as well as the macroeconomic linkages among potential output determinants appears as being crucial.

In the past, the EC tried to propose output gap measures based on higher frequency data (i.e quarterly figures) which, notably, are considered as more suited for estimating the business cycle. The adoption of yearly averages of quarterly (or monthly) business survey figures to estimate Total Factor Productivity is an example which goes in this direction. Moreover, the shift to the Bayesian Kalman filter approach to estimate Total Factor Productivity can be considered as a successful attempt to minimize the end-point-bias over the medium run. Finally, the EC often recognized the importance of exploiting the underlying links among macroeconomic variables for producing out-of-sample projections over the medium term. In this respect, lately, the EC presented at the OGWG a note in which the link between the dynamics of average hours worked and the evolution of the participation rate in the EU Member States was assessed in order to explore the implications of using this relationship as a basis for the 3-years medium-term extension of the average hours worked series.

In order to deal with all these issues we propose a new methodology based Production Function approach which uses a flexible Kalman Filter mixed frequency model (annual plus quarterly) to estimate each factor of production (Labour, Capital and TFP) for determining the level of potential output in real time. In addition, we also propose a multivariate model using mixed frequency State Space representation to extend out of sample in a multivariate framework the pattern of hours worked and participation rates.[1] The advantage of using a mixed frequency model rests on the fact that available and timely information may efficiently be used to provide more reliable real time estimates of potential output with respect to those obtained through low frequency annual data. In addition, our proposed model is flexible enough as it could be estimated by imposing external constraints, such as the convergence on annual values such as EC Forecast. Finally, the Kalman filter specification allows to derive a common factor model that may be crucial for extrapolating labour supply variables over longer out-of-sample horizons.

Recently we have observed an increasing interest on the methodologies based on mixed frequency models. They are particularly useful to extract the information content from high frequency indicators that are used as proxy for target variables observed at lower frequency and eventually with a time lag. In addition, these models are particularly suited as a time series disaggregation tool, given their multivariate nature and given that the target variable

[1] Works are also in progress to extend NAIRU out of sample (t+2-t+5) by exploiting results of a multivariate model of Labour supply.

is estimated at a higher frequency.

The mixed frequency literature has initially been developed using state space factor models, estimated via the Kalman filter. Most of the applications exploit monthly series, like industrial production or confidence surveys to predict the quarterly GDP. This approach has been followed by Mariano and Murasawa (2003), Mittnik and Zadrozny (2004), Aruoba et al. (2009), Camacho and Perez Quiros (2009) and Frale et al. (2009). These models can also be used as a multivariate tool for time series disaggregation, as done in Frale et al. (2008), Harvey and Chung (2000), Moauro and Savio (2005).

Relying on these contributions, the note is organised as follows. Section 2 presents the mixed frequency methodology focusing in particular on the specification of the factor model. Section 3 describes the application to Italy presenting the methodology adopted and the respective results for the estimates of Potential Labour, Capital and TFP. The reliability of our results in real time is also assessed by comparing the variability of potential growth with respect to that obtained by the EC through different forecast vintages. Section 4 presents some concluding remarks.

2 Methodology

Although the currently agreed methodology at the European level for the estimation of the potential output is comprehensive and well established, two possible directions for improvement deserve to be explored: first, the use of quarterly data, and, second, the adoption of a multivariate factor model for estimating potential labour.

The use of disaggregated information allows to exploit timeliest and more updated information as yearly data are released with substantial delay and only once a given year is ended. For instance, yearly data on GDP, let's say for the year 2011, are published only in March 2012, whereas the first information about GDP for the first quarter of 2011 is already available in May 2011. This means that the information contained in partial quarterly figures could be efficiently used for updating the yearly projections, at least, 10 months in advance. This is particularly relevant in periods of high variability of business cycle such as recessions or quick expansions, when the macroeconomic situation could quickly deteriorate or improve. Moreover, it is well known in the literature that business cycle features are better captured by high frequency series, quarterly or monthly, which are more sensitive to changes in the business economic activity. By contrast, annual data fail to take into account such underlying variability. This is the reason why, to date the cycle either monthly series (such as the industrial production index) or quarterly data (e.g. GDP) are generally used instead of yearly figures.

As far as the second innovation is concerned, we consider fundamental to use a multivariate model in order to properly forecast potential labour. It is granted that participation rate, employment, active population and hours worked are correlated at least because of definition links [2]. As a consequence, making independent univariate forecasts of those series could generate incoherent patterns from the economic point of view and increases the probabilities of measurement and forecast errors.

In this respect, the use of mixed frequency models allows to solve, simultaneously, both of the issues identified above that is: using the most recent and updated information to estimate potential output in real time (quarterly) and estimate labour supply relations using a multivariate framework. In addition, the mixed frequency approach and, in particular, the Kalman filter are enough flexible to allow the introduction of some constraints so as to be consistent with pre-determined yearly aggregates (such as for example EuroPOP 2010 demographic projection, or EC forecast).

2.1 The factor model with mixed frequency

There are many possibilities for linking a set of indicators available at high frequency to the target variable observed at lower time interval.

In particular, there has been recently a large interest in the literature for mixed frequency dynamic factor models where a vector of N time series, $\mathbf{y}_t$, available at different frequencies (e.g. quarterly and yearly) is decomposed into one (or more) common nonstationary component, f_t, and some idiosyncratics,γ_t, specific to each series. Both the common factor and the idiosyncratics follow autoregressive standard processes as shown by the following representation:

$$\begin{array}{lcll} \mathbf{y}_t & = & \boldsymbol{\vartheta}_0 f_t + \boldsymbol{\vartheta}_1 f_{t-1} + \boldsymbol{\gamma}_t + \mathbf{S}_t\boldsymbol{\beta}, & t = 1, ..., n, \\ \phi(L)\Delta f_t & = & \eta_t, & \eta_t \sim \mathrm{NID}(0, \sigma^2_\eta), \\ \mathbf{D}(L)\Delta\boldsymbol{\gamma}_t & = & \boldsymbol{\delta} + \boldsymbol{\eta}^*_t, & \boldsymbol{\eta}^*_t \sim \mathrm{NID}(\mathbf{0}, \boldsymbol{\Sigma}_{\eta^*}), \end{array} \tag{1}$$

where $\phi(L)$ is an autoregressive polynomial of order p with stationary roots and $\mathbf{D}(L)$ is a diagonal matrix containing autoregressive polynomials of order p_i (i=1 to N) . The vector δ contains the drifts of the idyosincratics. The regression matrix $\mathbf{S}_t$ contains the values of exogenous variables that are used to incorporate possible calendar effects and intervention

[2]The correlation exists also with wages growth. A further extention of the multivariate model considers the inclusion of wages.

variables (level shifts, additive outliers, etc.), as well as the elements of $\boldsymbol{\beta}$ that are used for initialisation and other fixed effects. The disturbances η_t and $\boldsymbol{\eta}_t^*$ are mutually uncorrelated at all leads and lags.

The model states that each series in differences, $\Delta\, y_{it}$, is obtained as the sum of a common autoregressive process of order p, $\phi(L)^{-1}\eta_{it}$ an individual $AR(p_i)$ process, $d_i(L)^{-1}\eta_{it}^*$ and a mean term δ_i. The error terms, η_{it} and η_{it}^* are difference stationary and independent.

The quarterly model is cast in a linear State Space Form (SSF) and, assuming that the disturbances have a Gaussian distribution, the unknown parameters are estimated by maximum likelihood, using the prediction error decomposition, performed by the Kalman filter.

The SSF can be suitably modified to take into account the mixed frequency nature of the series. Following Harvey (1989), the state vector is augmented by an ad hoc cumulator function which translates the problem of aggregation in time into a problem of missing values. The cumulator is defined as the observed aggregated series at the end of the season (e.g. last quarter of year), otherwise it contains the partial cumulative sum of the disaggregated values (e.g. quarters) making up the aggregation interval (e.g. year) up to and including the current one [3].

Given the multivariate nature of the model and the mixed frequency constraint, the maximum likelihood estimation can be numerically complex. Therefore, the univariate filter and smoother for multivariate models proposed by Koopman and Durbin (2000) is used as it provides a very flexible and convenient device for handling high dimension and missing values. The main idea is that the multivariate vectors $\mathbf{y}_t$, $t = 1, \ldots, n$, where some elements can be missing, are stacked one on top of the others to yield a univariate time series $\{y_{t,i}, i = 1, \ldots, N, t = 1, \ldots, n\}$, whose elements are processed sequentially.

In the Appendix we report the State Space form and the procedure for the time disaggregation procedure.

3 Application for Italy

The methodology presented in the paper has been applied to the Italian case in order to estimate the Potential GDP growth and the relative contributions of labour, capital and total factor productivity.

Given the annual data provided by the EC, we use quarterly series by Istat or Eurostat

[3]Therefore in each year we observe a sequence such as: "'.NaN,.NaN,.NaN, sum(q1,q2,q3,q4)=y"' where the last value is the yearly amount and each missing entry is the cumulative partial sum of quarters up to the current one. For stock variables the yearly amount corresponds to the average of quarterly values in the year.

so as to disaggregate (to the quarterly frequency) yearly values in sample and to produce quarterly forecast out of sample.

In section 3.1 we present the results of the disaggregation and forecast of potential GDP and we compare them to the EC's results (aggregating our quarterly results to yearly values).

Each key input of potential GDP, namely potential labour (LP) , capital (K) and (TFP) is estimated in sample at the quarterly frequency and forecasts are produced on different time horizons. The potential GDP is then computed through the classical formula:

$$\bar{Y} = LP^{0.65} \times K^{0.35} \times SRK \tag{2}$$

where SRK is the Kalman filtered Solow Residual.

It has to be stressed that an important feature of the model is the fact that it allows not only to constrain quarterly estimates to be consistent with annual historical data but also to impose out of sample constraints. In fact, different constraints can be imposed easily with the aim to bind the quarterly data to any projections along different time horizons, such as for example those of the EC.

Thus the model allows either to exactly replicate the EC forecast, or, alternatively, to constraint only historical data or to impose different constraints on different variables. For example, since the commonly agreed methodology by EC uses the AWG (Aging Working Group) projection to extrapolate the population of working age after the short term forecast horizon, that constraint can be easily included in the model.

Section 3.2 is devoted to present some sensitivity analysis on the stability of the estimates with respect to their revision by applying different input forecasts and between successive EC forecast vintages. The results allows to appreciate the strengths of the proposed methodology in terms of flexibility and robustness.

3.1 Estimation results

This section deals with the detailed presentation of the results of our estimates with respect to the EC forecast exercise of Spring 2012. The main methodological changes are for the Labour and Capital components whereas the TFP is computed with the standard EC model just recast in quarterly values.

In the following sections we show how to use timely quarterly data and how to build the multivariate mixed frequency models for Labour and Capital so as to exploit efficiently the cross correlation among data underlying series. For both components we present estimated factor loadings along with their standard errors and we plot disaggregated quarterly series as resulting from the model.

Finally we collect all results and compute the potential output and output gap with the standard procedure.

3.1.1 Potential Labour

The current methodology applied by the EC for the estimation of potential labour involves several steps. In each of them, a singular component of the total labour supply is estimated through a multivariate (NAIRU) and univariate approaches which foresee a mechanical or a simplistic extrapolation of projections out of sample.

We propose a multivariate dynamic factor model for labour series where different components of the labour supply are jointly estimated and forecast maintaining the coherence among them.

In particular we use yearly Employment, Unemployment rate, Active Population and Hours worked constrained to match EC series (those of Spring forecast 2012 exercise) up to 2013, unless for active population that is constrained up to 2018.[4]

On top of that, we use quarterly series of Hours worked and Participation rate so as to disaggregate in sample yearly data and to include more recent information. The quarterly series of Participation rate and hours worked are those published by ISTAT and the participation rate is consistent with the definition used by the EC and calculated as follow:

$$PARTS = \frac{Empl + Unempl}{POPW} \tag{3}$$

where $Empl$ is the total employment, $Unempl$ is the total unemployment and $POPW$ is the total population between 15 and 64 years.

The rationale beyond the use of a multivariate dynamic factor model is to extract a common factor representing the underlying pattern of the labour supply to which the different series are correlated accordingly to a specific factor loading. The results are shown in table 1 where the estimated factor loadings are presented together with their standard errors. Moreover figure 1 shows the disaggregated series of, respectively employment, unemployment rate, hours worked and active population in first differences together with the estimated common factor. The population has been included in the model in first difference so as to match the

[4] A further extension of the model considers the inclusion of also the series of wage growth. This allows to use the NAIRU Kalman Filter model to project NAIRU out of sample and replace the mechanical extrapolation procedure currently in place for the years (t+3)- (t+5).

cyclical characteristic of the other series which generally are more dynamic. The common factor has been assumed to follow an AR(1) process which is quite standard in the literature.[5]

Table 1: Labour market moldel- Estimated factor loadings with standard errors

	Loading	*SE*	*Student-t*
Employment	0.09	0.02	3.70
Unemployment rate	2.31	0.44	5.21
DPopulation	0.13	0.02	7.06
Hours worked	-0.02	0.01	3.58
Common factor: $(1-0.72L)\,\Delta\mu_t=\eta_t,\quad \eta_t\sim N(0,1)$			

The model produces directly quarterly values of hours worked, while quarterly participation rate is calculated through equation (3). Hence potential levels of both series (hours worked and participation rate) are extracted by applying HP filter. This is only a preliminary attempt and other filters such as Kalman or Band-pass can be applied to improve the quality of results. The NAIRU quarterly series is obtained by appropriately changing the parameters of the GAP program by the EC.[6] Finally the potential Labour is computed by applying the usual formula:

$$LP = (POPW \times PARTS \times (1 - NAIRU)) \times HOURST \tag{4}$$

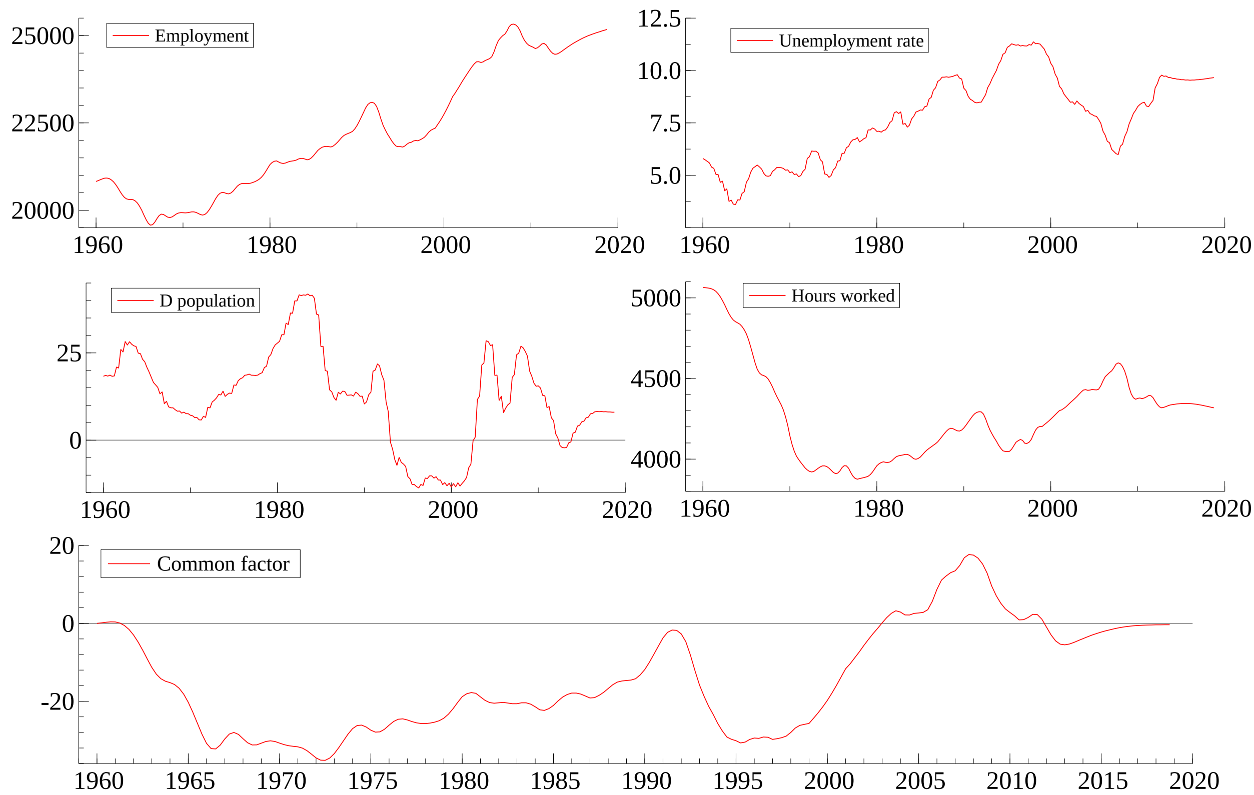
where $PARTS$ is the smoothed participation rate, and $HOURST$ is the trend of the average of hours worked.
To compare our results to those obtained by the EC, we aggregate the potential labour series by averaging the quarterly information over a yearly frequency. Figure 2 shows Italian potential labour over the period 1981-2016 as obtained by the EC compared with our estimates.

As expected the quarterly method seems to be more sensitive to business cycle swings and thus it produces slightly more volatile results. Moreover, the inclusion of updated quarterly values for the year 2012 (up to first quarter of 2012) allows to better capture the slowdown due to the recent economic recession.

[5]The model is flexible enough to allow for other specifications such as, for instance, AR(2), ARIMA, etc.

[6]However, we are currently working on a different specification of the multivariate model for labour supply allowing to forecast also the series of wage growth. On the basis of this specifications, the quarterly NAIRU can be projected out of sample also for the period (t+3) - (t+5).

Figure 1: Disaggregated series for the Labour market multivariate model



Note: Axis are shown in normalized scale for visibility reasons.

3.1.2 Capital

As far as the estimation of Capital is concerned, we rely on the EC model at the yearly level and we disaggregate the series at the quarterly frequency by using a multivariate model similar to that used for the Labour supply. In particular, we use quarterly data on Investments published by ISTAT to disaggregate the yearly series of Capital taking into account also yearly potential output as estimated in a first run of the procedure as to mimic the practice in the EC's approach. Table 2 shows estimated factor loadings and standard errors for Capital whereas figure 3 presents the disaggregated series of capital.

We would like to emphasize that Investments and Potential Output are not the focus of

Figure 2: Potential Labour

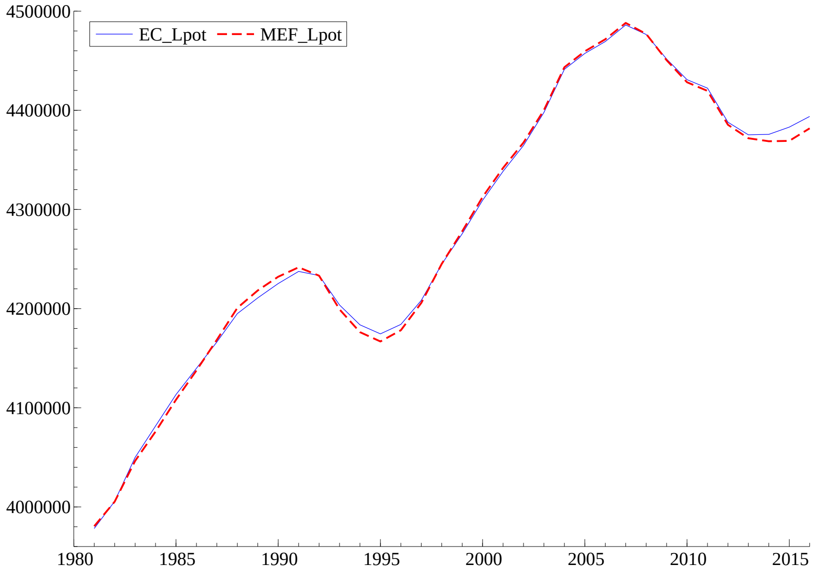


Table 2: Model for Capital- Estimated factor loadings with standard errors

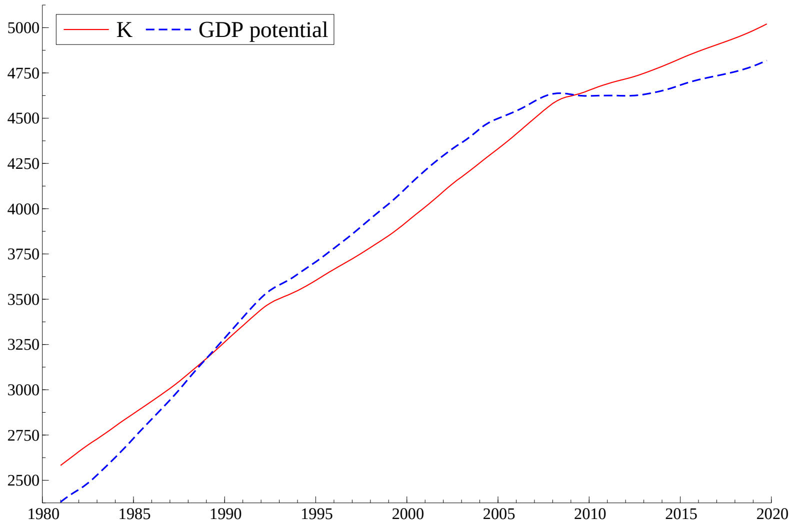
	Loading	*SE*	*Student-t*
Quarterly data			
Investments	0.17	0.04	4.57
Yearly data			
Capital	0.25	0.08	3.16
GDP potential	0.46	0.07	6.68
Common factor:	$(1 - 0.94L)\,\Delta\mu_t = \eta_t, \quad \eta_t \sim N\,(0, 1)$		

the model but only instruments to disaggregate yearly Capital at the quarterly frequency.

3.1.3 Total Factor Productivity

Once labour supply and capital stock are estimated, Solow residual and the corresponding estimate of the Total factor Productivity at quarterly frequencies can be computed. In order to do that, we use a quarterly version of the program GAP, where prior distribution at the quarterly frequency has been derived accordingly (see figure 8 and figure 9 for technical spec-

Figure 3: Disaggregated quarterly Capital



Note: Axis are shown in normalized scale for visibility reasons.

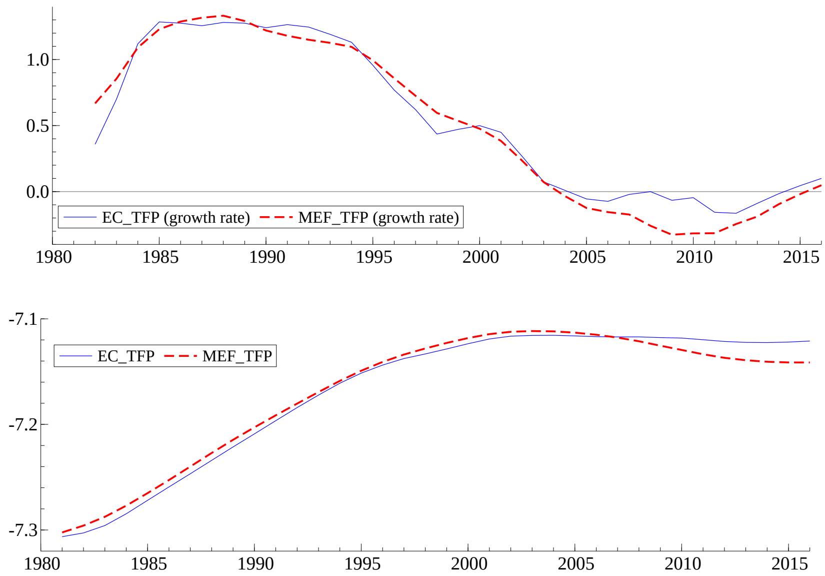
ifications) [7]. The Solow residual is calculated until the end of the short term forecast horizon by using quarterly forecast of GDP obtained by applying a multivariate model similar to that of Labour consistently with yearly EC's projections for the years 2012 and 2013.
The quarterly capacity index used as a proxy for the unobserved level of capacity utilization is the usual **CUBS** of the EC's procedure calculated at quarterly frequency by transforming the **ESI.SERV** and **ESI.BUIL** indicators from a monthly to a quarterly frequency [8]. Figure 4 shows our results compared with those of EC.
As before the quarterly frequency allows to better capture the cyclical swings and thus to produce a more dynamic TFP [9] .

[7]We would like to thank people from the JRC center of the EC for the useful help in deriving suitable priors for quarterly frequency.

[8]These indicators come from the Business and Tendency Surveys published by the EC. See for more details the web site http://ec.europa.eu/economy_finance/db_indicators/surveys/index_en.htm

[9] It has to be stressed that our methodology allows to forecast GDP values on a longer time horizon (until the medium-term forecast horizon), so it may be considered the possibility to employ a longer series of Solow residual to estimate the trend total factor productivity.

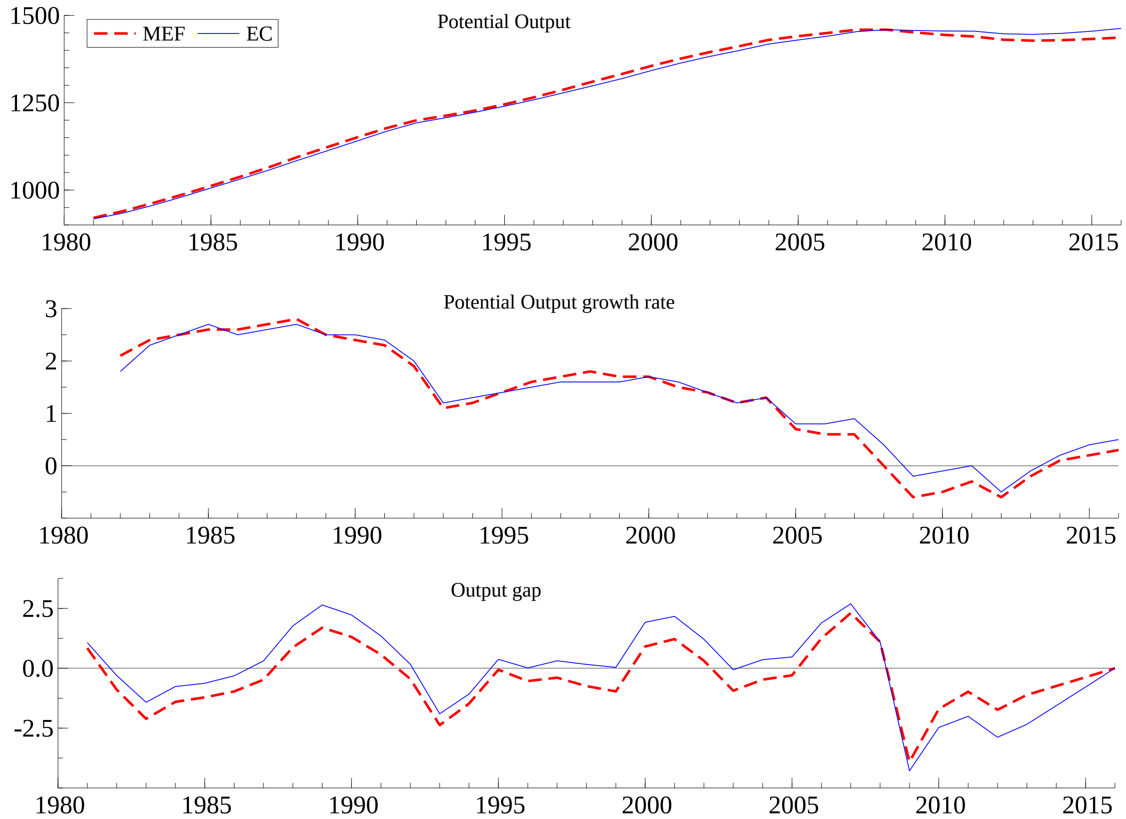
Figure 4: Quarterly trend total factor productivity



3.1.4 Potential Output and Output Gap

Combining the results obtained in previous paragraphs we compute the quarterly potential output by using the Cobb-Douglas production function (2). Results are shown in figure 5 in levels and growth rates along with the estimated value of the potential GDP by EC in Spring 2012. Our results shows a lower potential output growth after the crisis and consequently a smaller output gap.

Figure 5: Potential output and output gap: MEF vs EC Spring 2012 estimates



3.2 Analysis of revisions and sensitivity to forecasts

In this section, we present an insight on the stability of our estimates with respect to the revision of the variables and consistently with the updating of available data. Moreover, the impact of changes in short term forecasts of input series on long term growth prospects is also assessed.

Figure 6 presents the estimates of potential output resulting by applying different input forecasts. In particular, whereas the constraint to historical data is always maintained, we assume different ways to link the model to yearly EC forecast data for the period 2012-2016.

More in details:

- Case 1: The model is constrained to the 2012 Spring forecasts over the period 2012-2013 unless for Active Population that is linked to EUROPOP 2010 projection up to 2018. In such scenario, the only difference with respect to the commonly agreed Production Function methodology is due to the use of quarterly data in the multivariate model for Labour Supply and in both Capital and TFP components.

- Case 2: The model keeps the link to historical figures but it is set to freely produce forecasts for all the underlying series with the only exception of Active Population which is still constrained to EUROPOP projections up to 2018.

- Case 3: also the constraint on the active population is relaxed and the model produces the forecast for all variables from 2012 up to 2016.

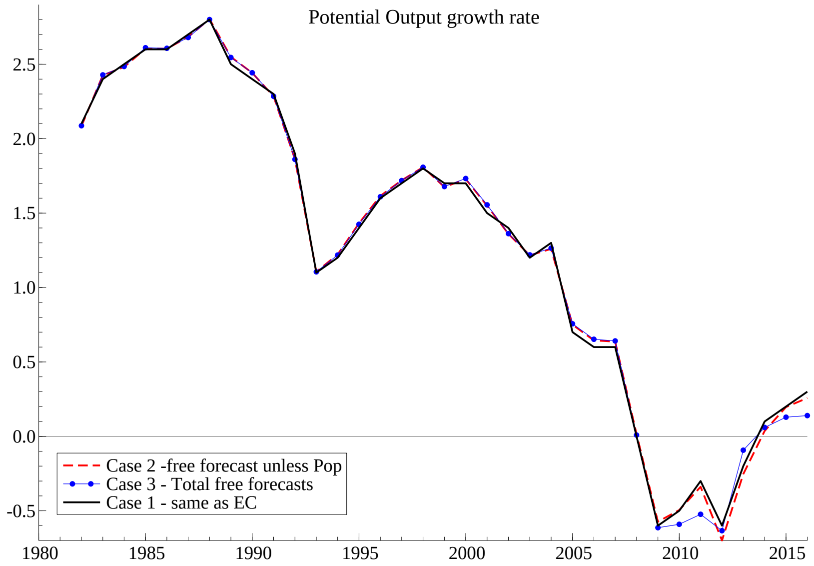
The inspection of figure 6 shows the relevance of Active population in defining the long term growth.

In fact estimates by Case 1 and Case 2 are quite similar and the only relevant change occurs once the constraint to the Active population is relaxed.

Moreover, the differences in sample with EC estimates are due by the use of quarterly values which allows to capture cyclical swings and thus it produces a more dynamic output growth.

A second experiment of sensitivity analysis is made in terms of revision of estimates between successive vintages of EC Forecasts. Figure 7 presents the estimates of Potential Output (in growth rates) in the last three consecutive vintages of forecast: Spring 2011, Autumn 2011 and Spring 2012. The same vintages specification is presented for our method. It is clearly visible that the proposed new methodology appears to be less influenced by revision of data especially on an historical basis. Whereas the EC methodology produces substantial revisions which extends backward until 2000, our model shows stable results for statistical historical figures. In addition, as our model is bounded to the results of EC forecasts, the revisions in the outer part of the sample mainly represent the forecast error underlying the projection exercise in each vintage. Moreover it has to be highlighted that

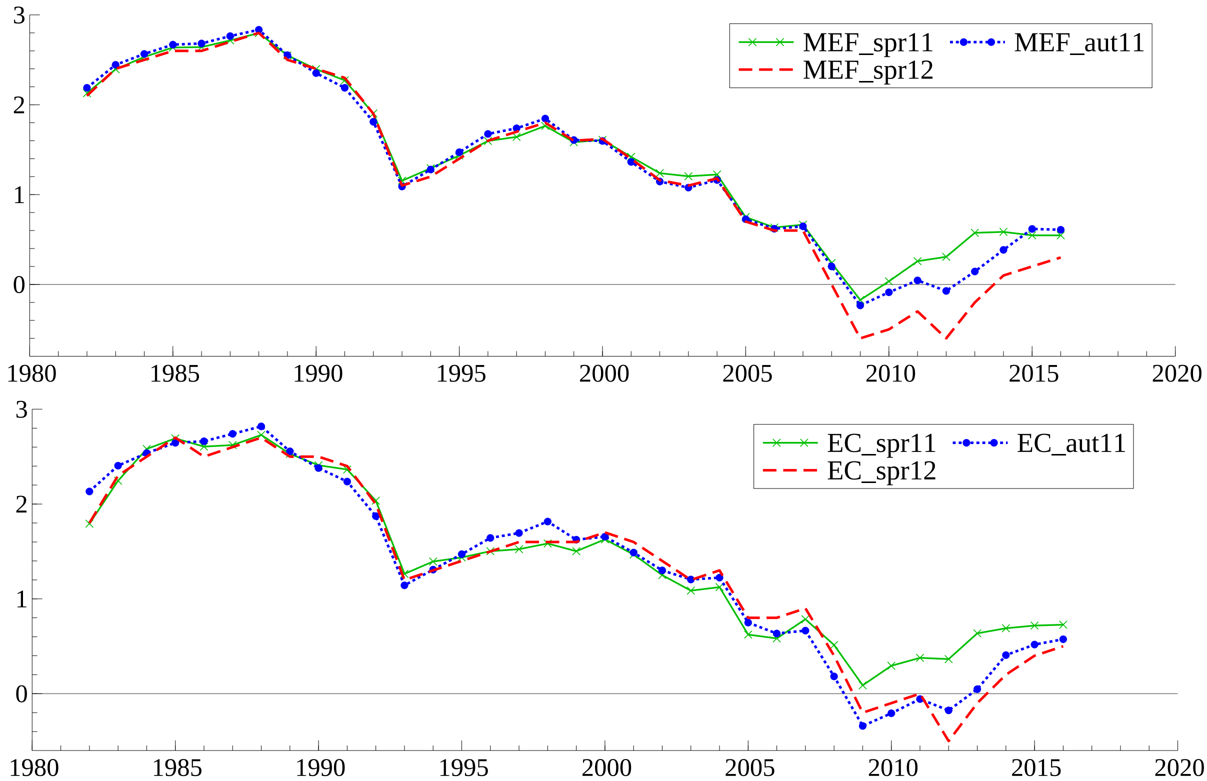
Figure 6: Sensitivity analysis: Impact of the forecasts



Note: Case 1: Historical data up to 2011 + Spring forecast 2012-2013+ Active Population a up to 2018, the difference is due only to the new methodology). Case 2: Historical data up to 2011 + forecasts by the model, unless Active Population constraint up to 2018. Case 3: Historical data up to 2011 + forecasts by the model for all variables including Active Population.

an other source of revision with respect to 2012 EC Spring forecast is represented by the introduction in our estimates of the latest figures on the first quarter of 2012. This produces a drop in potential growth for 2012 which is not reflected in the latest EC forecast.

Figure 7: Sensitivity analysis: Revision of estimates



4 Conclusion

This methodology presents a new way of estimating on a quarterly basis the single components of potential output (Labour, Capital and TFP).

For Capital and Labour a multivariate Kalman Filter model with mixed frequency has been adopted. Though computationally more demanding, this specification, through the use of quarterly and annual observations, is able to reproduce and timely update- more often than under the current OGWG framework - both the historical, real time and projected series of the EC forecasts.

As it is mostly based on higher frequency observations, our methodology allows also to capture business cycle features and the variability of economic fluctuations in a more efficient fashion than what would result by using annual data.

Besides all this, one of the most important innovations of our methodology is represented by the use of a multivariate State Space model for projecting out of sample all the components of labour supply and Capital Stock. The choice of a multivariate framework for projecting jointly out of sample (over the years $(t+3)$ - $(t+5)$) hours worked, participation rates, unemployment rates (and eventually wage growth) allows to exploit the underlying macroeconomic relations existing, respectively, among the components of Labour supply and Capital stock and to provide a sound alternative to the simple univariate procedures in use so far.

According to our estimates, results for Italy appear as more robust and stable than those obtained with the current methodology at least as far as past historical revisions of underlying figures in different forecast vintages are concerned. As shown by Cacciotti and Pradelli (2009), revisions of past values of unobserved variables are potentially large and may surely affect the results in real time. In such a context, the relative stability in potential output growth estimates for past observations is a desirable feature especially for its use in the current fiscal framework for determining the medium term average growth of the expenditure benchmarks.

In our opinion, this model offers some appealing features. In particular, it allows to assess on a quarterly basis the reliability of real time estimates of output gaps and potential growth based on underlying annual macroeconomic projections. Such a property appears as being essential in a fiscal framework where the compliance the MTO is crucial to assess whether particular correction mechanisms should be activated or not on the basis of real time variables and a specific macroeconomic medium term outlook. In addition, a quarterly framework based on mixed frequency variables allows to assess the revision in output gaps and structural deficits due to the updated on macroeconomic variables, providing the policy makers with an efficient tool to measure possible slippages from the MTO well in advance and giving to them the possibility to reshape fiscal policies in case of need.

Appendix

A.1-State space representation and Temporal aggregation[10]

In this section we cast model (1) in the state space form (SSF). We start from the single index, $\phi(L)\Delta\mu_t = \eta_t$, considering the SSF of the stationary AR(p) model for the $\Delta\mu_t$, for which:

$$\begin{aligned} \Delta\mu_t &= \mathbf{e}'_{1p}\mathbf{g}_t, \\ \mathbf{g}_t &= \mathbf{T}_{\Delta\mu}\mathbf{g}_{t-1} + \mathbf{e}_{1p}\eta_t, \end{aligned}$$

where $\mathbf{e}_{1p} = [1, 0, \dots, 0]'$ and

$$\mathbf{T}_{\Delta\mu} = \begin{bmatrix} \phi_1 & \\ \vdots & \mathbf{I}_{p-1} \\ \phi_{p-1} & \\ \phi_p & \mathbf{0}' \end{bmatrix}.$$

Hence, $\mu_t = \mu_{t-1} + \mathbf{e}'_{1p}\mathbf{g}_t = \mu_{t-1} + \mathbf{e}'_{1p}\mathbf{T}_{\Delta\mu}\mathbf{g}_{t-1} + \eta_t$, and defining

$$\boldsymbol{\alpha}_{\mu,t} = \begin{bmatrix} \mu_t \\ \mathbf{g}_t \end{bmatrix}, \quad \mathbf{T}_\mu = \begin{bmatrix} 1 & \mathbf{e}'_{1p}\mathbf{T}_{\Delta\mu} \\ 0 & \mathbf{T}_{\Delta\mu} \end{bmatrix},$$

the Markovian representation of the model for μ_t becomes

$$\mu_t = \mathbf{e}'_{1,p+1}\boldsymbol{\alpha}_{\mu,t}, \quad \boldsymbol{\alpha}_{\mu,t} = \mathbf{T}_\mu\boldsymbol{\alpha}_{\mu,t-1} + \mathbf{H}_\mu\eta_t,$$

where $\mathbf{H}_\mu = [1, \mathbf{e}'_{1,p}]'$.

A similar representation holds for each individual μ^*_{it}, with ϕ_j replaced by d_{ij}, so that, if we let p_i denote the order of the i-th lag polynomial $d_i(L)$, we can write:

$$\mu^*_{it} = \mathbf{e}'_{1,p_i+1}\boldsymbol{\alpha}_{\mu_i,t}, \quad \boldsymbol{\alpha}_{\mu_i,t} = \mathbf{T}_i\boldsymbol{\alpha}_{\mu_i,t-1} + \mathbf{c}_i + \mathbf{H}_i\eta^*_{it},$$

where $\mathbf{H}_i = [1, \mathbf{e}'_{1,p_i}]'$, $\mathbf{c}_i = \delta_i\mathbf{H}_i$ and δ_i is the drift of the $i-th$ idiosyncratic component, and thus of the series, since we have assumed a zero drift for the common factor.

Combining all the blocks, we obtain the SSF of the complete model by defining the state vector $\boldsymbol{\alpha}_t$, with dimension $\sum_i (p_i + 1) + p + 1$, as follows:

$$\boldsymbol{\alpha}_t = [\boldsymbol{\alpha}'_{\mu,t}, \boldsymbol{\alpha}'_{\mu_1,t}, \dots, \boldsymbol{\alpha}'_{\mu_N,t}]'. \tag{5}$$

[10]This section is mainly taken from Frale et al. (2011)

Consequently, the measurement and the transition equation of SW model in levels is:

$$\mathbf{y}_t = \mathbf{Z}\boldsymbol{\alpha}_t + \mathbf{X}_t\boldsymbol{\beta}, \quad \boldsymbol{\alpha}_t = \mathbf{T}\boldsymbol{\alpha}_{t-1} + \mathbf{W}\boldsymbol{\beta} + \mathbf{H}\boldsymbol{\epsilon}_t, \tag{6}$$

where $\boldsymbol{\epsilon}_t = [\eta_t, \eta^*_{1t}, \ldots, \eta^*_{Nt}]'$ and the system matrices are given below:

$$\begin{array}{ll} \mathbf{Z} = \left[\boldsymbol{\theta}_0, \vdots\, \boldsymbol{\theta}_1 \ \vdots\, \mathbf{0} \,\vdots\, \mathrm{diag}(\mathbf{e}'_{p_1}, \ldots, \mathbf{e}'_{p_N})\right], & \mathbf{T} = \mathrm{diag}(\mathbf{T}_\mu, \mathbf{T}_1, \ldots, \mathbf{T}_N), \\ \mathbf{H} = \mathrm{diag}(\mathbf{H}_\mu, \mathbf{H}_1, \ldots, \mathbf{H}_N). & \end{array} \tag{7}$$

The vector of initial values is written as

$$\boldsymbol{\alpha}_1 = \mathbf{W}_1\boldsymbol{\beta} + \mathbf{H}\boldsymbol{\epsilon}_1,$$

so that $\boldsymbol{\alpha}_1 \sim \mathrm{N}(\mathbf{0}, \mathbf{W}_1\mathbf{V}\mathbf{W}'_1 + \mathbf{H}\mathrm{Var}(\boldsymbol{\epsilon}_1)\mathbf{H}')$, $\mathrm{Var}(\boldsymbol{\epsilon}_1) = \mathrm{diag}(1, \sigma^2_1, \ldots, \sigma^2_N)$.

The first $2N$ elements of the vector $\boldsymbol{\beta}$ are the pairs $\{(\mu_{01}, \delta_i, i = 1, \ldots, N\}$, the starting values at time $t = 0$ of the idiosyncratic components and the constant drifts δ_i.

The regression matrix $\mathbf{X}_t = [\mathbf{0}, \quad \mathbf{X}^*_t]$ where $\mathbf{X}^*_t$ is a $N \times k$ matrix containing the values of exogenous variables that are used to incorporate calendar effects (trading day regressors, Easter, length of the month) and intervention variables (level shifts, additive outliers, etc.), and the zero block has dimension $N \times 2N$ and corresponds to the elements of $\boldsymbol{\beta}$ that are used for the initialisation and other fixed effects.

The $2N + k$ elements of $\boldsymbol{\beta}$ are taken as diffuse.

For $t = 2, \ldots, n$ the matrix $\mathbf{W}_t$ is time invariant and selects the drift δ_i for the appropriate state element:

$$\mathbf{W} = \left[\begin{array}{c} \mathbf{0} \\ \mathrm{diag}(\mathbf{C}_1, \ldots, \mathbf{C}_N) \end{array}\right], \mathbf{C}_i = [\mathbf{0}_{p_i+1,1} \vdots \mathbf{c}_i],$$

whereas $\mathbf{W}_1$

$$\mathbf{W}_1 = \left[\begin{array}{c} \mathbf{0} \\ \mathrm{diag}(\mathbf{C}^*_1, \ldots, \mathbf{C}^*_N) \end{array}\right], \mathbf{C}^*_i = \left[\mathbf{e}_{1,p_i+1} \vdots \mathbf{c}_i\right],$$

A.2-Temporal aggregation

Suppose that the set of coincident indicators, $\mathbf{y}_t$, can be partitioned into two groups, $\mathbf{y}_t = [\mathbf{y}'_{1t}, \mathbf{y}'_{2t}]'$, where the second block gathers the flows that are subject to temporal aggregation, so that

$$\mathbf{y}^*_{2\tau} = \sum_{i=0}^{\delta-1} \mathbf{y}_{2,\tau\delta-i}, \quad \tau = 1, 2, \ldots, [T/\delta],$$

where δ denote the aggregation interval: for instance, if the model is specified at the quarterly frequency and $\mathbf{y}^{\dagger}_{2t}$ is yearly, then $\delta = 4$.

The strategy proposed by Harvey (1989) consists of operating a suitable augmentation of the state vector (5) using an appropriately defined cumulator variable. In particular, the SSF (6)-(9) need to be augmented by the $N_2 \times 1$ vector $\mathbf{y}^c_{2t}$, generated as follows

$$\begin{aligned}\mathbf{y}^c_{2t} &= \psi_t \mathbf{y}^c_{2,t-1} + \mathbf{y}_{2t} \\ &= \psi_t \mathbf{y}^c_{2,t-1} + \mathbf{Z}_2\mathbf{T}\boldsymbol{\alpha}_{t-1} + [\mathbf{X}_{2t} + \mathbf{Z}_2\mathbf{W}_t]\boldsymbol{\beta} + \mathbf{Z}_2\mathbf{H}\boldsymbol{\epsilon}_t\end{aligned}$$

where ψ_t is the cumulator coefficient, defined as follows:

$$\psi_t = \begin{cases} 0 & t = \delta(\tau - 1) + 1, \quad \tau = 1, \dots, [n/\delta] \\ 1 & \text{otherwise} . \end{cases}$$

and $\mathbf{Z}_2$ is the $N_2 \times m$ block of the measurement matrix $\mathbf{Z}$ corresponding to the second set of variables, $\mathbf{Z} = [\mathbf{Z}_1', \;\; \mathbf{Z}_2']'$ and $\mathbf{y}_{2t} = \mathbf{Z}_2\boldsymbol{\alpha}_t + \mathbf{X}_2\boldsymbol{\beta}$, where we have partitioned $\mathbf{X}_t = [\mathbf{X}_1' \;\; \mathbf{X}_2']'$. Notice that at times $t = \delta\tau$ the cumulator coincides with the (observed) aggregated series, otherwise it contains the partial cumulative value of the aggregate in the seasons (e.g. quarters) making up the larger interval (e.g. year) up to and including the current one.

The augmented SSF is defined in terms of the new state and observation vectors:

$$\boldsymbol{\alpha}^*_t = \begin{bmatrix} \boldsymbol{\alpha}_t \\ \mathbf{y}^c_{2t} \end{bmatrix}, \quad \mathbf{y}^{\dagger}_t = \begin{bmatrix} \mathbf{y}_{1t} \\ \mathbf{y}^c_{2t} \end{bmatrix}$$

where the former has dimension $m^* = m + N_2$, and the unavailable second block of observations, $\mathbf{y}_{2t}$, is replaced by $\mathbf{y}^c_{2t}$, which is observed at times $t = \delta\tau, \tau = 1, 2, \dots, [n/\delta]$, and is missing at intermediate times. The measurement and transition equation are therefore:

$$\mathbf{y}^{\dagger}_t = \mathbf{Z}^*\boldsymbol{\alpha}^*_t + \mathbf{X}_t\boldsymbol{\beta}, \quad \boldsymbol{\alpha}^*_t = \mathbf{T}^*\boldsymbol{\alpha}^*_{t-1} + \mathbf{W}^*\boldsymbol{\beta} + \mathbf{H}^*\boldsymbol{\epsilon}_t, \tag{8}$$

with starting values $\boldsymbol{\alpha}^*_1 = \mathbf{W}^*_1\boldsymbol{\beta} + \mathbf{H}^*\boldsymbol{\epsilon}_1$, and system matrices:

$$\mathbf{Z}^* = \begin{bmatrix} \mathbf{Z}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_2} \end{bmatrix}, \quad \mathbf{T}^* = \begin{bmatrix} \mathbf{T} & \mathbf{0} \\ \mathbf{Z}_2\mathbf{T} & \psi_t\mathbf{I} \end{bmatrix}, \quad \mathbf{W}^* = \begin{bmatrix} \mathbf{W} \\ \mathbf{Z}_2\mathbf{W} + \mathbf{X}_2 \end{bmatrix}, \quad \mathbf{H}^* = \begin{bmatrix} \mathbf{I} \\ \mathbf{Z}_2 \end{bmatrix}\mathbf{H}. \tag{9}$$

The state space model (8)-(9) is linear and, assuming that the disturbances have a Gaussian distribution, the unknown parameters can be estimated by maximum likelihood, using the prediction error decomposition, performed by the Kalman filter; given the parameter values, the Kalman filter and smoother will provide the minimum mean square estimates of the states

$\boldsymbol{\alpha}_t^*$ (see Harvey, 1989, and Shumway and Stoffer, 2000) and thus of the missing observations on $\mathbf{y}_{2t}^c$ can be estimated, which need to be "decumulated", using $\mathbf{y}_{2t} = \mathbf{y}_{2t}^c - \psi_t \mathbf{y}_{2,t-1}^c$, so as to be converted into estimates of $\mathbf{y}_{2t}$. In order to provide the estimation standard error, however, the state vector must be augmented of $\mathbf{y}_{2t} = \mathbf{Z}_2\boldsymbol{\alpha}_t + \mathbf{X}_2\boldsymbol{\beta} = \mathbf{Z}_2\mathbf{T}\boldsymbol{\alpha}_{t-1} + [\mathbf{X}_2 + \mathbf{Z}_2\mathbf{W}]\boldsymbol{\beta} + \mathbf{H}\boldsymbol{\epsilon}_t$.

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Figure 8: Quarterly specification for GAP program

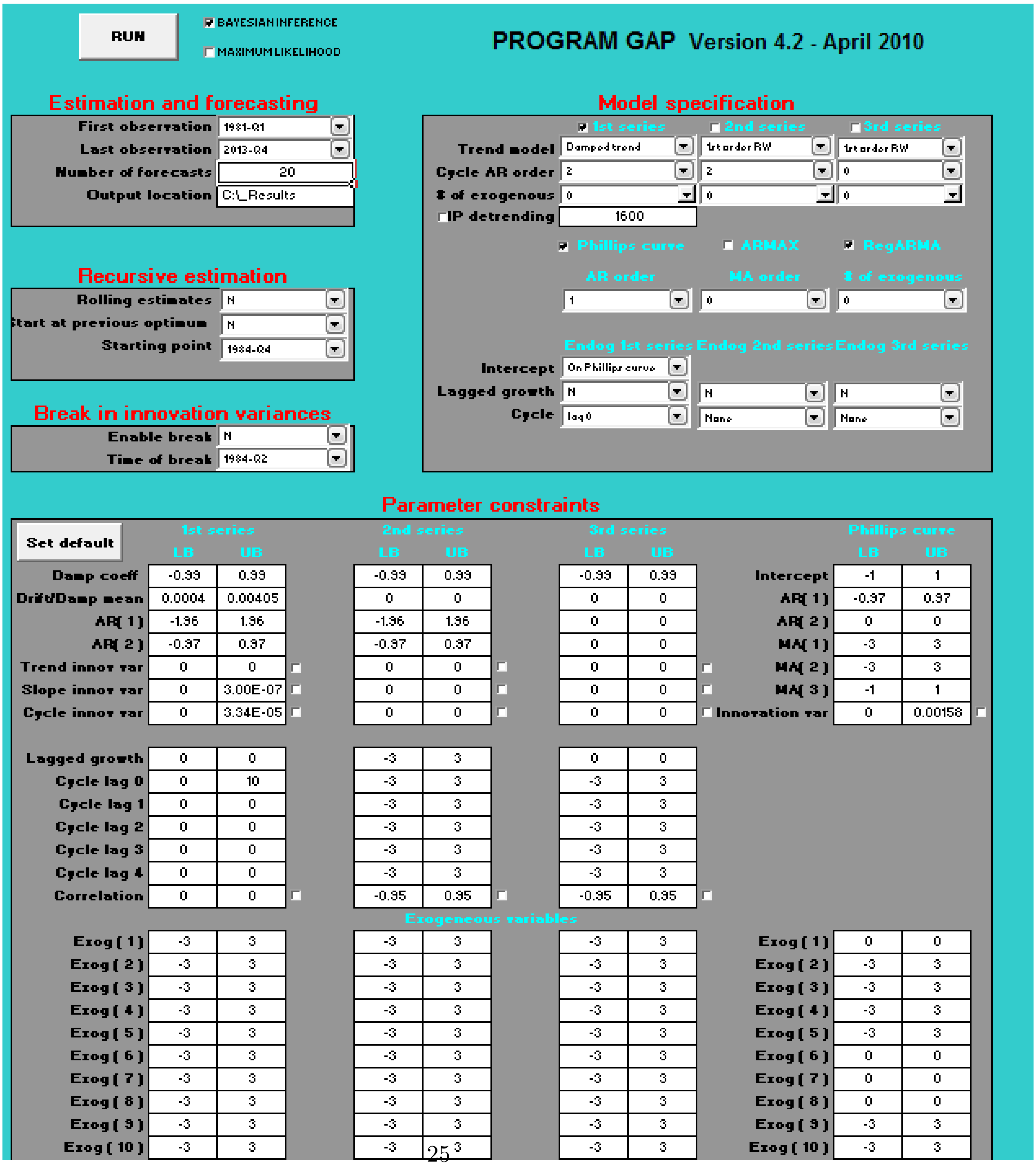

Figure 9: Priors in the quarterly TFP