When to Stop - A Cardinal Secretary Search Experiment

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Abstract

For the cardinal secretary search problem, whose candidates have identically and independently distributed values, the (risk neutral) benchmark solution reveals exponentially decreasing acceptance thresholds. As acceptance thresholds are value aspirations for any given number of remaining candidates, we can compare actual with optimal aspirations. Conditions vary the known number of candidates, the experimentally elicited choice data: "hot" collects play data, "warm" asks for an acceptance threshold only when needed, and "cold" for a complete profile of acceptance thresholds. In "cold" participants confront successively different known numbers of candidates in increasing or decreasing order. We analyze how actual search qualitatively and quantitatively differs from benchmark behavior and how these differences affect search length and average success.

Key words: **Optimal Stopping, Secretary Problem, Sequential Search Mechanism**

Date: September 7, 2017

Financing: This research project was financed by the Max Planck Institute for Research on Collective Goods

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1. Introduction

Optimality in dynamic decision tasks with finite horizon relies on backward induction in the form of dynamic programming (see Bellman, 2013). In our specific setup all candidates are ex-ante equal, the random values of all candidates are independently and identically determined, so that the remaining number of candidates is the only state variable. The optimal strategy of a risk neutral decision maker, a complete sequence of first more and later less ambitious acceptance thresholds, is derived in the appendix A[1]. Although one could have experimentally induced risk neutrality by implementing binary lottery incentives[2] (the value of the accepted candidate determines linearly the probability of earning a larger rather than a smaller monetary amount), we let participants successively confront several search tasks, each with possibly many chance moves, in order to reduce the variance of average earnings across all tasks.

One can hardly expect participants to engage in dynamic programming but only to intuitively reason in qualitatively and quantitatively similar ways. When describing and statistically analyzing actual search behavior we mainly focus on qualitative deviations from optimality, for instance, its invariance property that aspirations condition only on the number of remaining candidates. Behaviorally, however, past experiences like the values of so far rejected candidates or previous search tasks may matter. Since we are at best boundedly rational, we want to explore how we actually decide when to stop and how this differs from optimal search.

A participant accepts or rejects (except for the last and automatically accepted) the successively revealed value v_t of trial t directly after seeing it without recall (when rejecting v_t in trial t this value is lost and cannot be retrieved). Since all values v_t for $t = 1, \ldots, n$ (with $n (\geq 2)$ denoting the initial number of candidates) are randomly and independently generated according to the uniform density, concentrated on the internal (0,1), so far rejected values do not inform about future ones, and the number of remaining candidates $n - t$ is the only state variable[3]. In the experiment the values are discrete. We have shifted up and enlarged the interval by allowing only integer values v from 24 to 123, i.e. the equally probable possible discrete values are 24,25, ...,122,123. The number n of candidates is 5, 10, or 15. Figure 1 illustrates how the optimal aspirations or acceptance thresholds depend on the remaining number of trials, i.e. the number of remaining candidates after rejecting the present one minus 1 (due to the last candidate who has to be accepted).

[1] We gratefully acknowledge the support of Alessandro Arlotto and Marco Scarsini who supplied the recursive formula and its algorithm which they, furthermore, adjusted to the discrete distribution, used in this experiment.

[2] This requires only that larger monetary amount is better than smaller one and that probabilities are calculated properly.

[3] This would be different when regret, measured by the positive difference between the highest rejected value and the actually accepted value, would matter.

We vary the number n (> 1) of candidates within subjects (participants confront all three n-tasks) and between subjects only whether n increases or decreases. The other between subjects variation of choice elicitation is more psychologically grounded. In "cold" participants are asked for complete strategies, i.e. a complete pattern of acceptance thresholds. Data of this treatment are especially suited to assess how actual and optimal aspirations differ, for example, whether actual aspirations are less often adapted than optimal ones (see, on aspiration adaptation, Sauermann and Selten, 1962).

The choice data in "hot" provide sequences of so far rejected values and the finally accepted one. We predicted that, when confronting a rather high value, participants will tend to accept more willingly in "hot" than when just imagining such a value in "cold" , similarly to the impatience of small children in the "marshmallow delay" task[4]. This suggests, on average, earlier stopping in "hot".

The intermediate "warm" treatment asks for acceptance thresholds as "cold" but only when needed, i.e. one successively states acceptance thresholds in "warm" being aware of the lost candidates so far. We expected sharper declines of acceptance thresholds after many rejections due to such awareness in "warm". Behaviorally, post-decisional regret, possibly measurable by how much the highest value seen so far exceeds the present one, could affect the next stated acceptance threshold in "hot" whereas in "cold" such regret would have to be anticipated. Using (risk neutral) optimality (RN-optimality from now on) as a benchmark, our focus is on deviations from RN-optimality and whether and how the elicitation method, the sequence of n, the number of remaining candidates and past experiences shape behavior.

Our findings reveal that participants substantially deviate from RN-optimal search, especially when there are many candidates still to be seen. The deviations overwhelmingly let them stop too early although also the opposite behavior is significant. Not only the elicitation method, varied between subjects, matters but also whether the number n of candidates increases or decreases. This sequence effect suggests that participants view the six successive rounds holistically, as a single comprehensive task, which justifies assuming risk neutrality due to the many payoff relevant random moves of this comprehensive choice task.

Section 2 informs about the related literature. Section 3 describes the experimental protocols. The data and main findings are described and statistically validated in section 4 before the final discussion in section 5.

[4] When delaying consumption of a given marshmallow by 30 minutes a child can eat two marshmallows, what is claimed to be positively correlated with professional success in adulthood (see Mischel et al. 2010 for a thorough review)

2. On the literature

Reviews of the optimal stopping literature are Freeman (1983), Ferguson (1989), Samuels (1991), and Szajowski (2008). Wu and He (2016) provide a more recent informative overview. Additionally, in economics an influential standard text on search is Phelps (1968). Even when paying the decision maker (DM) according to the value of the accepted candidate, the decision maker DM decides to stop or not partly based on additional information, e.g. like whether or not the presently available candidate is the best so far, see Bearden (2006), and experimentally Bearden et al. (2006). A field case of such example could be a head hunter (institution) who may be able to rank candidates relatively but cannot assess the firm-specific usefulness of the candidate for the hiring firm. Altogether the theoretical and experimental literature is still influenced by the traditional incentive to hire the best, for example by rendering cardinal value payoff relevant only when hiring single best candidates so far (Ferenstein and Krasnosielska, 2009).

An early survey of secretary search experiments is provided by Stein et al. (2003). Todd and Miller (1999) report more generally on empirical search studies, not necessarily based on the stylized secretary search paradigm. Mostly the empirical literature explores satisficing by employing, in analogy to the revealed preference approach of empirical neoclassical economics, the revealed aspiration approach: one infers success aspirations from observed choice data presupposing - in analogy to optimality in revealed preference theory - satisficing behavior. Güth and Weiland (2011), like our study, directly ask participants to state aspirations first for sampling size and later for acceptance.

Bearden et al. (2006) pay cardinal incentives but let stopping condition on rank. Other studies include essential search costs (Kogut, 1990; Moriguti 1993; Seale, 1996; Seale and Rapoport, 1997 and 2000; Stein et al.,2003), random number of candidates (Presman and Sonin, 1972; Ferenstein and Krasnosielska, 2009) or investigate (too little or too much) consumer search (Zwick et al., 2003). Bayesian secretary search with known priors for monetary values, as in our own study, so far seems neglected. It may be that secretary search tasks have met so much interest due to capturing value ambiguity and suggesting incentives like wanting only to hire the single-best. This may be true but even then it is important to also consider the opposite border case, namely, the cardinal secretary search tasks.

3. The choice tasks and (risk neutral) optimality

A strategy in the search task is a complete profile of acceptance thresholds, one for each subgame (a subgame is defined by the sequence of so far rejected candidates). According to the risk neutral benchmark solution, RN-optimality, acceptance thresholds depend monotonically only on the remaining number of candidates and not on the value sequence of the so far seen candidates[5], see Figure 1. Participants in the experiment may choose other than RN-optimal strategies, for instance, by violating monotonicity[6] or conditioning on strategically irrelevant aspects like foregone rejected values. The experimental elicitation of behavior in "cold" and "warm" is restricted[7] to conditioning on the number of remaining candidates but does not impose monotonicity across trials (stated acceptance threshold may increase with fewer candidates left).

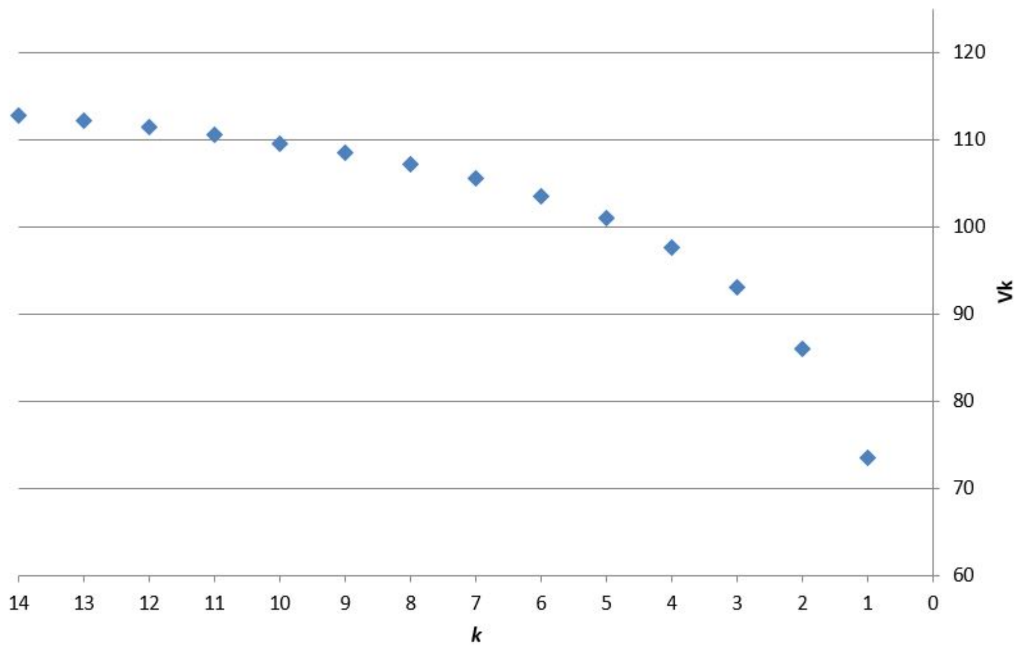

Figure 1 **RN-optimal acceptance thresholds for the numbers $k = 14 - k$, of remaining "secretaries" when rejecting the present one, k.**

"Warm" elicits acceptance thresholds trial by trial. This allows comparing "cold" and "warm" acceptance thresholds until acceptance. "Hot" implements the actual dynamics of sequential search. Participants confront the first value and may stop by accepting it or continue till a candidate is finally accepted where the last candidate must be accepted. To compare RN-optimality with "hot" play data one can focus on acceptance of non-RN-acceptable candidates and rejection of RN-acceptable ones and compare the average length of search and average search success. To gather more informative data "hot" relies only on the largest number of candidates ($n = 15$).

[5] Game theoretically this means to impose subgame consistency (subgames with the same number of remaining candidates are isomorphic and should have the same solution)

[6] In "hot" one may accept a lower value but reject a higher one. In "cold" and "warm" one may display non-monotonicity in the form of higher acceptance thresholds with fewer candidates.

[7] In view of the large multiplicity of sub-games applying an unrestricted strategy method in the search tasks at hand is practically impossible.

The three protocols (”cold”, “warm” and “hot”) require different instructions (see Appendix B for the instructions of “Cold-increasing” treatment). Participants obtained a show-up fee of 5 Euro. The four between-subject treatments are listed in Table 1, informing about the sequence of tasks in the six successive rounds of each treatment and the elicitation mode, which changes only in “hot”.

Table 1 Treatment design

	“Cold-Increasing”	“Cold-Decreasing”	“Warm”	“Hot”
Round 1	n=5	n=15	n=5	n=15-”hot”
Round 2	n=5	n=15	n=5	n=15-”hot”
Round 3	n=10	n=10	n=10	n=15-”hot”
Round 4	n=10	n=10	n=10	n=15-”hot”
Round 5	n=15	n=5	n=15	n=15-”warm”
Round 6	n=15	n=5	n=15	n=15-”cold”

Notes: Columns feature between subjects treatments with partly varying n across rounds

In all treatments (except hot) participants repeat the same task only once in order to distinguish possible pure learning that arisses when confronting the same number $n = 5, 10, 15$ from learning to cope with changing numbers n (see Table 1 for details). “Hot” is only run with $n = 15$ in order to acquire more informative play data; after four successive rounds of “hot” one round of “warm” and “cold” is played in rounds 5 and 6.

4. Data analysis

We compare between-subjects treatments, mainly relying on outcome data like stopping times, average payoffs, and standard deviations for $n = 15$ of “cold”, “warm”, and “hot”, where we distinguish whether $n = 15$ has been experienced first, respectively last (see Table 1). For $n = 15$ one can also compare across “cold-increasing”, “warm”, and “hot” the frequencies of accepted but RN-unacceptable and rejected but RN-acceptable values. We, however, focus on choice data of “cold” and “warm” for which one can compare acceptance thresholds. For these conditions we begin by analyzing monotonicity, i.e. whether earlier acceptance thresholds exceed later ones in the same search task.

4.1. Anti-monotonicity

We focus on in rounds 3 & 4 which are most suitable for this comparison due to $n = 10$ in rounds 3 & 4 in all treatments eliciting acceptance thresholds. Anti-monotonicity is defined by at least one violation of monotonicity in these rounds. According to Table 2 the total share depends strongly on the format of choice elicitation. The anti-monotonicity share is naturally lowest for “warm” which provides fewer opportunities to reveal anti-monotonicity, compared to "cold". In "cold" we distinguish between increasing and decreasing sequence of n. Of all participants in "cold-in(de)creasing" 27.08% (42.55%) reveal at least one anti-monotonicity in rounds 3 and 4, compared to 6.38% in "warm". which is significantly smaller (z=-2.681, p=0.004) using the one-tailed Wilcoxon Rank-sum Test (henceforth WRST). "Cold-decreasing" has the largest share of anti-monotonic participants, 42.55% which is significantly higher (z=-4.057, p=0.000, WRST) than in "warm". The difference in anti-monotonicity between "cold-increasing" and "cold-decreasing" treatments is just short of significant (z=1.575, p=0.115, two-tailed WRST).

We also compared anti-monotonicity shares across "cold" and "warm" after balancing their frequencies for observing monotonicity violations via simulating randomly shorter acceptance profiles for cold-increasing (with the lower anti-monotonicity share) according to search length in "warm". Here the anti-monotonicity rate, 14.5%, is still above "warm" with 6.38% (z=-1.295, p=0.097, WRST). This lets us conclude:

Result 1 "Cold", increasing and decreasing, both trigger significantly higher shares of anti-monotonicity in rounds 3 and 4 than in "warm".

Table 2 Percentage of anti-monotonic participants

	"Cold-increasing" n=10	"Cold-decreasing" n=10	"Warm" n=10
End-game Anti-monotonic (%)	6.25	17.02	0.00
Other Anti-monotonic (%)	20.83	25.53	6.38 9
Total (%)	27.08	42.55	6.38

One might have expected a lower share of monotonicity violation for "Cold" due to the obvious intuition that for fewer remaining candidates chance and aspirations should be worse. On the other hand, having to anticipate consequences in "cold" like a long unsuccessful search could trigger attempts to ultimately render an apparent failure into a success, e.g. by stating a final high aspiration which, when satisfied, render the long(est) search still a success.

Rather than on anti-monotonicity in general let us therefore consider the specific anti-monotonicity in "cold" treatments violating monotonicity by only a striking increase of acceptance thresholds when only one or two candidates are left (see top row of Table 2; the 0-share for "warm" is due to missing data). The phenomenon is, in our view, closely related to becoming (more) risk loving after losing (see Kahnemann and Tversky, 1984). A participant in "cold" when stating the last aspirations, seems to reason like: "if I have searched too long due to being unlucky in all earlier trials I should still aim at making a success".

Result 2: "Cold", especially "cold-decreasing" triggers many participants (17.02) to adapt their last aspiration upwards.

Although in view of optimality the format of choice elicitation should not matter, it crucially affects how many participants violate intuitive monotonicity. Many participants seem view a decrease of n as a loss which may explain the much larger anti-monotonicity shares in "cold-decreasing".

4.2. Acceptance Data

We compare the four between-subjects treatments of Table 1, "cold-increasing", "cold-decreasing", "warm" and "hot", by outcome data, separately for monotonic and anti-monotonic participants although this is more selective in "cold". According to Table 3 average search is longest for monotonic participants in "cold-increasing". The shorter search of "cold-decreasing" is mainly due to its $n = 15$ plays by (in round 1&2) inexperienced participants. The positive difference [8] between "cold-increasing" and "cold-decreasing" is significant, both for all and only monotonic participants (z=02.376, p=00.018; and z=01.707, p=00.088, two-tailed WRST's). Average length [9] of search in "cold-increasing" and in "warm" are closest (5.05 versus 5.01 for all participants, 5.51 versus 4.91 for only monotonic ones) suggesting that n-sequence affects search length more than the elicitation format.

Table 3 **Stopping Time - Average number of participants drawn (seen) before accepting when $n = 15$**

	Cold-Increasing		Cold-Decreasing		Warm		Hot	
	Mean	Sdt. Dev.	Mean	Sdt. Dev.	Mean	Sdt. Dev.	Mean	Sdt. Dev.
All (%)	5.052	3.985	3.936	3.704	5.011	3.947	4.051	3.391
Monotonic Only (%)	5.614	3.987	4.426	3.456	4.909	3.891	4.149	3.478

Result 3 *(i)* In "cold" participants search significantly longer when n increases. "Warm" relies on the same n-sequence and is similar in average search length as "cold-increasing".

[8] For "warm" such a comparison is problematic due to the fewer chances for anti-monotonic behavior.

[9] When comparing stopping of "warm" and "cold", especially "cold/increasing", one could reduce the random effects due to different random value sequences by assuming the observed short value sequences of "warm" to simulate the average stopping probability of all "cold/increasing" participants in the same trials.

(ii) "Hot" triggers significantly shorter plays than "cold-increasing" and "warm" (p=0.027 when comparing with "cold-increasing" and p=0.059 when comparing with "warm"; two-tailed WRST). "Cold-decreasing" with its high anti-monotonicity share seems strikingly different.

(iii) Across all conditions in Table 3 monotonic participants (in rounds 3 and 4) search longer than non-monotonic ones (z=3.417, p=0.000, two-tailed WRST). The longer search of monotonic participants, who are behaving more intuitively, confirms that monotonicity in aspiration formation goes hand in hand with more patience and possibly less emotional decision making.

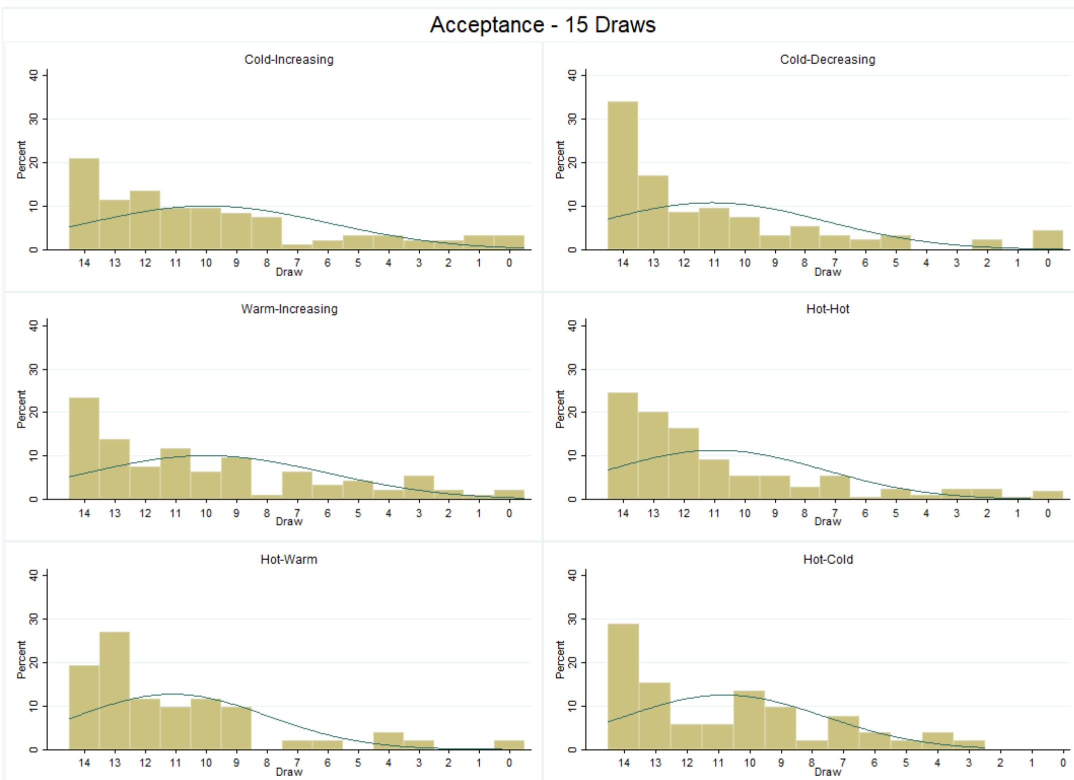

Figure 2 Stopping time - 15 Draws

Figure 2 visualizes the distributions of stopping trials for $n = 15$. Accepting the first value is most likely in the right-hand panels, i.e. decreasing n-sequence in "cold" renders first acceptance more likely. For n=10 and n=5 there exist only three comparisons each (see Figure 3). Differences across between-subject treatments become minor with fewer candidates (smaller, respectively with fewer candidates).

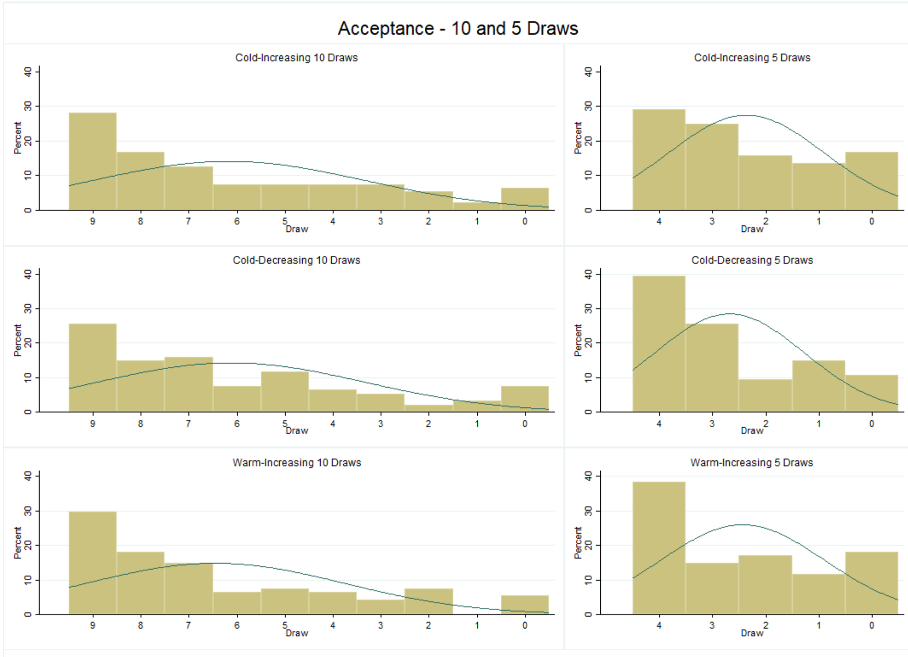

Figure 3 Stopping time for $n = 10$ and $n = 5$

4.3. Payoff Comparisons

Average payoffs (see the kernel densities in Figures 4 and 5 for monotonic participants only (for all participants in Appendix C). "Hot" payoffs are higher due to "hot" participants playing 6 rounds with n=15. To account for this difference, the left panel of Figure 5 presents the kernel densities of payoffs only for the first two rounds of "cold-decreasing" and "hot", and the right panel of Figure 5 for the last two rounds of "cold-increasing", "warm" and "cold", all relying on n=15. Table 4 lists for monotonic participants the average payoffs and their standard deviations, separately for n=15, 10, and 5. Table 5 includes all participants. The obvious payoff enhancement of more candidates n is missing in "cold-decreasing" (see first and second play in Tables 4 and 5), even when considering only monotonic participants.

Altogether repeating the same n-task hardly enchances payoffs, behaving monotonically does so partly as does a larger number n of candidates.

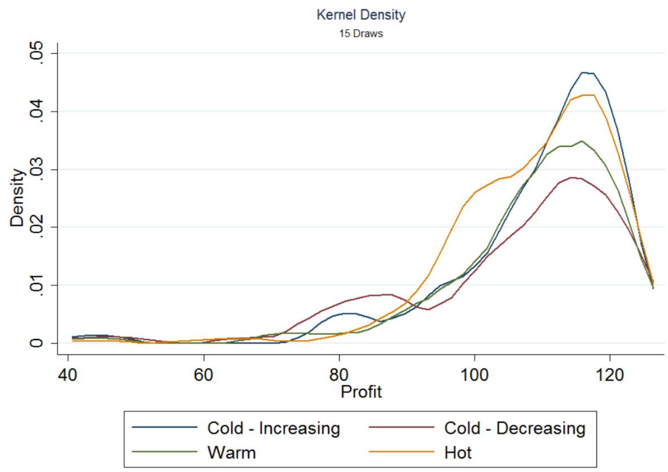

Figure 4 **Kernel density of profits ($n = 15$) - monotonic participants only**

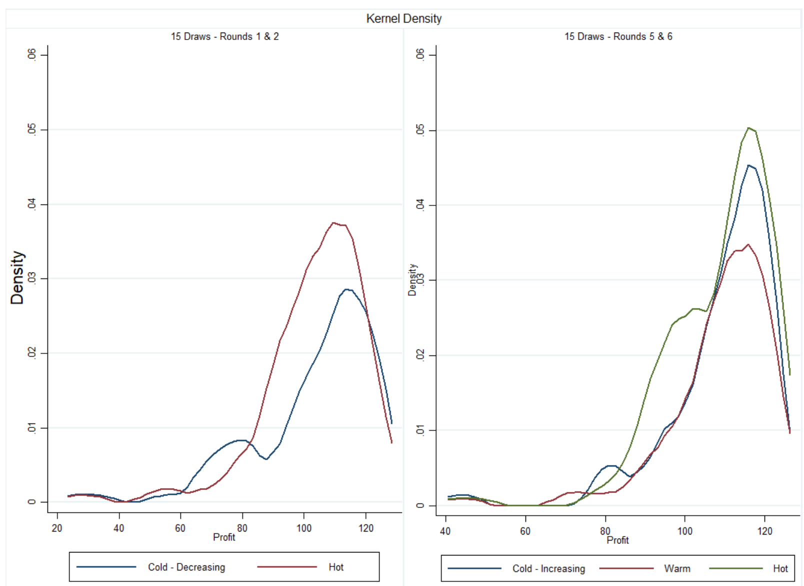

Figure 5 **Kernel density of profits in the first two and last two rounds (with $n = 15$) - monotonic participants only**

Table 4 **Payoff per round - monotonic participants only**

	Cold-Increasing			Cold-Decreasing			Warm			Hot ($n = 15$)		
Play	5	10	15	5	10	15	5	10	15	Hot	Warm	Cold
1st	96.91 (19.13)	102.14 (17.87)	108.49 (15.18)	102.00 (21.55)	111.11 (8.84)	106.96 (11.49)	92.36 (26.35)	107.11 (15.97)	105.55 (17.47)	106.45 (15.38)	112.13 (11.09)	113.66 (8.04)
2nd	99.34 (20.8)	104.51 (19.94)	110.06 (11.23)	105.19 (17.42)	101.93 (21.03)	108.67 (11.29)	92.89 (23.47)	105.73 (11.58)	108.14 (11.87)	108.72 (8.20)		
3rd										109.45 (8.99)		
4th										109.15 (8.25)		

Table 5 Payoff per round - all participants

	Cold-Increasing			Cold-Decreasing			Warm			Hot ($n = 15$)		
Play	5	10	15	5	10	15	5	10	15	Hot	Warm	Cold
1st	94.92	100.42	107.98	98.32	102.47	101.83	92.83	106.87	106.15	105.17	109.81	112.02
	(21.02)	(19.23)	(15.61)	(23.15)	(19.59)	(21.83)	(25.82)	(15.55)	(17.07)	(15.85)	(15.43)	(11.40)
2nd	99.42	103.50	106.04	100.04	98.53	106.60	93.91	105.66	107.91	108.72		
	(20.13)	(19.27)	(19.09)	(22.16)	(22.19)	(14.97)	(23.12)	(11.34)	(11.54)	(8.20)		
3rd										108.69		
										(9.71)		
4th										109.65		
										(8.06)		

<u>Result 4</u> *(i)* Repeating the same n-task in round t=2,4,6 after round t-1 with the same n, does not significantly increase payoffs across treatments (p=0.232 in "cold-increasing"; p=0.994 in "cold-increasing"; p=0.479 in "warm";and p=0.924 in the first 2 rounds of "hot", using two-tailed WRST).

(ii) Monotonicity significantly enhances payoffs in all treatments except "warm" due to its low share of anti-monotonic participants (p=0.088 in "cold-increasing", p=0.000 in "cold-decreasing", and p=0.003 in "hot" using a two-tailed WRST).

(iii) For monotonic participants larger n yields larger average payoffs (p=0.001 when comparing $n = 10$ with $n = 5$, and p=0.018 when comparing $n = 10$ with $n = 15$ using a two-tailed WRST).

Our results so far reveal that reacting more emotionally triggers anti-monotonicity and shorter search for which one pays on average payoffs, similar to the lower success in life of children who are behaving myopically in the marshmallow task.

4.4. Choice Data

We begin with graphical illustrations. Figure 6 plots for the number of remaining decisions (on the abssises) the average aspirations (acceptance thresholds) for $n = 15$ across the 14 trials of "hot" (round 6 only), "cold-increasing" and "cold-decreasing" and confronts them with RN-optimal ones (dotted). Playing "cold" after "hot" play triggers average aspirations closest to RN-optimality. Neglecting overshooting at the last trial, "cold-decreasing" differs most from RN-optimality with "cold-increasing" lying in between. The striking difference between "cold-increasing" and "cold-decreasing" (see Figures 6 and 8) illustrates once again that the decrease in n is perceived as a loss and triggers a emotional behavior which in turn implies shorter search and lower payoffs, on average.

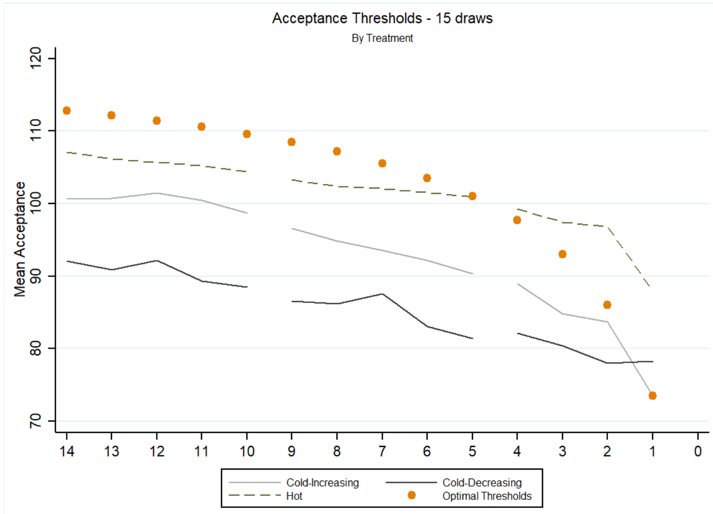

Figure 6 **Acceptance Thresholds for** $n = 15$

Result 5 *(i)* for $n = 15$ average "hot" aspirations are closest to RN-optimal ones, when neglecting their overshooting in the last four draws. "Cold-increasing" average aspirations for $n = 15$ are closer to RN-optimality than those for "Cold-decreasing" (Figure 6),

(ii) Comparing average choice behavior for $n = 15$ between "cold-increasing" and "cold-decreasing" reveals a striking and significant sequence effect (p=0.000 using a two-tailed WRST) whereas average aspirations for $n = 5$ and $n = 10$ do not differ significantly between the two cold treatments (see Figures 6, 7, and 8).

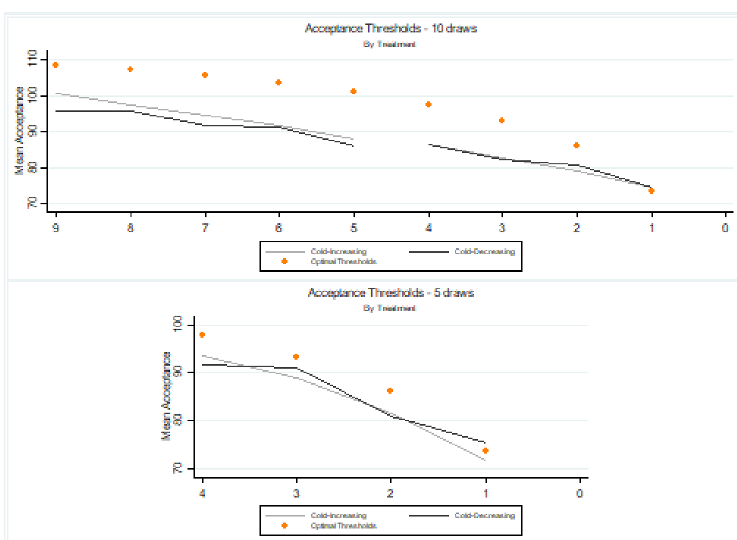

Figure 7 **Acceptance Thresholds for** $n = 10$ **and and** $n = 5$

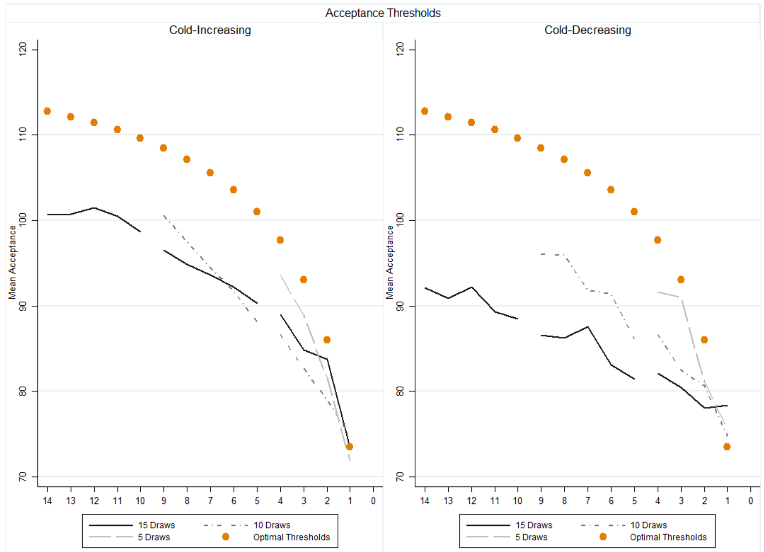

Figure 8 Acceptance Thresholds in the Cold-Increasing Treatment

In our view, "cold-decreasing" participants experience the more rewarding $n = 15$ tasks first, and since they are quite inexperienced, stop on average too early due to their low $n = 15$-acceptance thresholds. In "cold-increasing" the obviously more prosperous $n = 15$-tasks are confronted later and, based in more experience with the type of tasks, better exploited.

Table 6 Percentage of aspirations profiles whose three initial aspirations are below the optimal ones

	Cold-Increasing			Cold-Decreasing			Hot		
	Draw 1	Draw 2	Draw 3	Draw 1	Draw 2	Draw 3	Draw 1	Draw 2	Draw 3
n=5	49.74	43.75	30.21	39.36	35.11	25.53			
n=10	56.25	86.05	68.12	54.26	65.96	74.47			
n=15	68.75	72.92	73.96	68.09	76.60	77.66	71.15	76.92	75.00

Table 6 reports the share of participants with lower acceptance thresholds than RN-optimal ones during the first three draws. For "cold" treatments the first aspiration is more frequently below the more RN-optimal one when time-horizon n is larger (p=0.008 when comparing n=5 & n=10, p=0.009 when comparing n=10 & n=15, and p=0.000 when comparing n=5 & n=15, using a logit regression and pooling both treatments).

Result 6 Initial aspirations are below RN-optimal ones more frequently for n=15 than for n=10 and less often for n=5.

This indication of stopping too early (compared to RN-optimality) had to be expected. RN-optimality neglects all regret concerns which may guide human decision making. Having rejected a better option earlier only to accept a worse one later on is something most of us have experienced in life which makes it easy to anticipate the regret of such experiences via imagining that "I should have accepted the earlier candidate!" by adjusting search length. Even when having anticipated such regret it still seems to strongly affect our decision making.

5. Conclusions

The cardinal iid-secretary search task is special and may seem less interesting compared to the standard version. There is nothing to learn from the past: all what matters is the number of remaining candidates. But this renders it very suitable to confirm purely behavioral aspects of search behavior like path dependence and anticipated regret. We have implemented not only the cardinal iid-secretary search task by using the ("cold") monotonic strategy method allowing best to compare optimal and actual behavior but also compared this with other formats of choice elicitation. In view of bounded rational theory participants have to form aspirations and to engage in satisficing (see Simon, 1995). Thus it is is directly observable in "cold" and "warm" how acceptance aspirations are formed and adapted (see Sauermann and Selten, 1962). RN-optimal search behavior can be described as optimal satisficing (when accepting risk neutrality). Satisficing as such does not require optimality but only acceptance and qualifies as at least qualitatively more rational when guaranteeing monotonicity of aspirations. Other than in "hot", the usual elicitation method in "secretary" search experiments, one does not have to infer aspirations from acceptance choices.

There is striking heterogeneity in individual behavior: some participants start with more and others with less ambitious aspirations than RN-optimal ones implying, on average, longer, respectively shorter search. The dominant tendency is to begin less ambitiously and thus to stop earlier. Other surprising findings are the strong n-sequence effect and the widely differing degrees in violating monotonicity across treatments. One such noteworthy monotonicity violation applies to the last two aspirations which "stand out". We interpret this as an attempt to let a long search nevertheless end with a success, similar to risk seeking after experiencing a loss. If shrinking n is seen as a loss this also would account for the striking n-sequence effect in "cold".

In view of experimental methodology cardinal secretary search tasks are interesting as they directly reveal success aspirations and their dependence on the remaining number of candidates. But as shown, they are also suitable paradigms to shed new light on the debate among experimentalists whether to use the "cold" strategy method or to elicit only "hot" play data. One wonders why the debate so far concentrates on comparing elicitation methods for social and strategic interaction experiments and neglects isolated decision making. For the latter we convincingly confirm that elicitation method and task sequencing matters.

Regarding the field relevance, one of our restriction is the known number n of (remaining) attempts which may be due to idiosyncratic characteristics of the decision maker like the limited time of search. In the animal kingdom an already starving predator has fewer attempts to hunt like somebody urgently searching for an apartment. Financial markets with stationary random-walk assets and traders, having to invest immediately, would also resemble such situations.Of course, we also neglect competition in search inducing, on average, earlier stopping (see, for instance, Güth and Weiland, 2011). Hence, our interest in the cardinal secretary search task is both due to theoretical and methodological reasons as well as due to its field relevance.

In future research one might want to employ binary lottery incentives in order to experimentally induce risk neutrality by paying only for one randomly selected round rather than all (six) rounds. One could also elicit choice behavior via employing a “hybrid choice elicitation” to which one might refer as the CWH-method: participants as in “cold” choose a strategy and additionally, trial after trial, as in “warm”, an acceptance threshold, possibly differing from the one chosen n in “cold”. Then as in “hot” they would decide whether to accept the known value at the given trial or not.

References

[1] Bearden, J. N. (2006). A new secretary problem with rank-based selection and cardinal payoffs. *Journal of Mathematical Psychology*, 50(1), 58-59.

[2] Bearden, J. N., Rapoport, A., & Murphy, R. O. (2006). Experimental studies of sequential selection and assignment with relative ranks. *Journal of behavioral decision making*, 19(3), 229-250. Chicago

[3] Bellman, R. (2013). Dynamic programming. *Courier Corporation.*

[4] Ferenstein, E. Z., & Krasnosielska, A. (2009). A version of the Elfving optimal stopping time problem with random horizon. *Game theory and applications*, 14, 40-53.

[5] Ferguson, T. S. (1989). Who solved the secretary problem?. *Statistical science*, 282-289.

[6] Freeman, P. R. (1983). The secretary problem and its extensions: A review. *International Statistical Review/Revue Internationale de Statistique*, 189-206.

[7] Gth, W., & Weiland, T. (2011). Aspiration formation and satisficing in search with (out) competition. *New Zealand Economic Papers*, 45(1-2), 23-45.

[8] Kahneman, D., & Tversky, A. (1984). Choices, values, and frames. *American psychologist*, 39(4), 341.

[9] Kogut, C. A. (1990). Consumer search behavior and sunk costs. *Journal of Economic Behavior & Organization*, 14(3), 381-392.

[10] Mischel, W., Ayduk, O., Berman, M. G., Casey, B. J., Gotlib, I. H., Jonides, J., ... & Shoda, Y. (2010). Willpowerover the life span: decomposing self-regulation. *Social cognitive and affective neuroscience*, 6(2), 252-256.

[11] Phelps, E. S. (1968), Money-Wage Dynamics and Labor-Market Equilibrium, *Journal of Political Economy*, Vol. 76, 678-711.

[12] Presman, E. L. V., & Sonin, I. M. (1972). The best choice problem for a random number of objects. *Teoriya Veroyatnostei i ee Primeneniya*, 17(4), 695-706. Chicago

[13] Samuels, S. M. (1991). Secretary problems. *Handbook of sequential analysis*, 118, 381-405.

[14] Sauermann, H., & Selten, R. (1962). Anspruchsanpassungstheorie der unternehmung. *Zeitschrift fr die gesamte Staatswissenschaft/Journal of Institutional and Theoretical Economics*, (H. 4), 577-597.

[15] Seale, D. A. (1996). Sequential observation and selection with relative ranks: An empirical investigation of the Secretary Problem. *https://www.researchgate.net/publication*

[16] Seale, D. A., & Rapoport, A. (1997). Sequential Decision Making with Relative Ranks: An Experimental Investigation of the. *Organizational behavior and human decision processes*, 69(3), 221-236.

[17] Seale, D. A. & Rapoport, A. (2000). Optimal stopping behavior with relative ranks: The secretary problem with unknown population size. *Journal of Behavioral Decision Making*, 13, 391-411.

[18] Simon, H. A. (1995). Problem forming, problem finding and problem solving in design. *Design & systems*, 245-257.

[19] Stein, W. E., Seale, D. A., & Rapoport, A. (2003). Analysis of heuristic solutions to the best choice problem. *European Journal of Operational Research*, 151(1), 140-152.

[20] Szajowski, K. (2008). A rank-based selection with cardinal payoffs and a cost of choice. *arXiv preprint* arXiv:0812.3483.

[21] Todd, P.M., & Miller, G.F. (1999). From pride and prejudice to persuasion: Satisficing in mate search. In G. Gigerenzer, P.M. Todd, and the ABC Research Group, Simple heuristics that make us smart. *New York: Oxford University Press.*

[22] Wu, T., & He, Q. (2016). Secretary Problem with Cardinal Payoffs and Employment Costs. *Working Paper*

[23] Zwick, R., Rapoport, A., Lo, A. K. C., & Muthukrishnan, A. V. (2003). Consumer sequential search: Not enough or too much?. *Marketing Science*, 22(4), 503-519.

Appendix A - RN Optimal Strategy

The rational and risk neutral decision maker (DM) is sequentially presented with n "secretaries" with independent, non-negative qualities $X_1, X_2, \ldots, X_n$ with common continuous distribution F. At any trial i, $1 \leq i \leq n$, DM sees the quality realization $X_i = v_i$ and decides whether to recruit this "secretary" and stop searching or to reject the secretary and continue searching. Rejected "secretaries" cannot be recalled. DM maximizes the expected quality of the recruited "secretary". Thus, DM's objective when n secretaries are available is given by

$$\mathbb{E}_n = \sup_{1 \leq \tau \leq n} \mathbb{E}[X_\tau], \tag{1}$$

where $\mathbb{E}$ denotes the expectation operator, τ the stopping time with respect to the increasing sequence of σ-fields $\mathcal{F}_i = \sigma\{X_1, \ldots, X_i\}$ and the trivial σ-field $\mathcal{F}_0$.

The optimal stopping problem (1) can be solved by dynamic programming. To do so set $\underline{v}_0 = 0$, and let $\mathbb{E}_k$ denote the optimal expected quality of the recruited secretary when there are k secretaries still to be seen. The principle of optimal dynamic programming says that $\mathbb{E}_k$ obeys the following recursion

$$\mathbb{E}_k = \int_0^\infty \max\{v, \mathbb{E}_{k-1}\}\, dF(v) = \mathbb{E}[X_1] + \int_0^{\mathbb{E}_{k-1}} F(v)\, dv, \qquad \text{for } 1 \leq k \leq n. \tag{2}$$

The left term (2) $\mathbb{E}[X_1]$ is the pay-off that the decision maker obtains by recruiting the secretary currently under evaluation with quality $X_{n-k+1} = v_{n-k+1}$, while the right term is the pay-off for rejecting the current secretary and continuing to search optimally in the next trial $k-1$. Due to our special choice task the RN-optimal payoffs, when continuing search, coincide with the RN-optimal acceptance threshold, i.e. acceptance thresholds are also optimal aspirations. We also see from (2) that the "secretary" inspected, at time i with quality X_i, is an optimal recruit at time i if and only if

$$X_i > \underline{v}_{n-1} \qquad \text{for all } 1 \leq i \leq$$

and that the sequence of optimal thresholds is monotonically increasing in the number of (remaining) trails:

$$0 = \underline{v}_0 \leq \underline{v}_1 \leq \underline{v}_2 \leq \cdots \leq \underline{v}_n.$$

Moreover, if the random qualities $X_1, X_2, \ldots, X_n$ have the uniform distribution on $[0,1]$, then $F(v) = v$ for all $v \in [0,1]$ and

$$\underline{v}_0 = 0 \qquad \text{and} \underline{v}_k = \frac{1}{2}(1 + \underline{v}_{k-1}^2), \text{ for } 1 \leq k \leq n.$$

The experiment actually uses discrete integer qualities and one derives (2) when the cumulative distribution function F has discrete support. Specifically, we choose two integers $a \geq 0$ and $J \geq 0$ and suppose that the random qualities $X_1, X_2, \ldots, X_n$ have support on the integers $\{a+1, a+2, \ldots, a+J\}$. Setting $\underline{v}_0 = a$ and for any $\underline{v}_{k-1} \in [a, a+J]$ we have the recursion

$$\underline{v}_k = \mathbb{E}[\max\{X_{n-k+1}, \underline{v}_{k-1}\}] = \mathbb{E}[X_1] + (\underline{v}_{k-1} - \lfloor \underline{v}_{k-1} \rfloor) F(\lfloor v_{k-1} \rfloor) + \sum_{j=a+1}^{\lfloor v_{k-1} \rfloor - 1} F(j).$$

When F is the discrete uniform distribution on $\{a+1, a+2, \ldots, a+J\}$, then $F(j) = (\lfloor j \rfloor - a)/J$ for $a+1 \leq j \leq a+J$ and the right hand side becomes

$$\underline{v}_k = a + \frac{1}{2}(J+1) + \frac{1}{2J}(\lfloor \underline{v}_{k-1} \rfloor - a)(2\underline{v}_{k-1} - \lfloor \underline{v}_{k-1} \rfloor - a - 1), \qquad \text{for } 1 \leq k \leq \qquad (3)$$

Is optimal to accept the secretary, inspected at time i, if and only if

$$X_i > \underline{v}_{n-i}$$

The optimal thresholds (3) when $n = 15$, $a = 24$ and $J = 99$ are graphically shown in Figure 1 in the main text.

Appendix B - Instructions: Cold-Increasing

Welcome! This is an experiment on how individuals make decisions. We are only interested in your choices. Be careful how you take your decisions as your behavior will determine the amount of money that you will receive and which will be paid out at the end of the experiment. In addition, you will gain an amount of 5 as a show up fee.

The following instructions will explain what choices you make and the gains associated with each choice.

Expected gains from this experiment are defined in ECUs (Experimental Currency Unit), converted at the following rate:

1 ECU = 2cents ()

This experiment is computerized and is based on individual decisions. All decisions will be taken anonymously through the computer in front of you. It is forbidden to communicate in any way with other participants for the duration of the experiment. At the end of the experiment there will be a questionnaire. At the end of the questionnaire you will be called individually to receive the final payment. Please wait in silence until the experimenters call your number.

After reading the instructions by the experimenter you'll have some minutes to read:
If something is not clear please raise your hand and be silent, one of the experimenters will come to help you ind
Please do not disturb other participants during the experiment.

Design

In this game you will play 6 rounds. In each round there will be made a number of draws, namely:

- In round 1 and round 2 there will be up to 5 draws

- In round 3 and round 4 there will be up to 10 draws

- In round 5 and round 6 there will be up to 15 draws

Well call the drawn values (v): each value v is randomly drawn by the computer from a range of integers between 24 to 123, where all possible 100 integers are equally likely, i.e. every value v is chosen with probability 1In round 1 and round 2 a maximum of 5 values (v) will be drawn, values v_1, v_2, v_3, v_4, v_5, in round 3 and 4 a maximum of 10 values (v) will be drawn, that is, v_1, v_2, ..., v_9, v_{10} etc..

The round ends when a draw is accepted (so if you accept the first draw, remaining draws for that round will not be carried out); in case no extraction is accepted then the round will end at last draw for that round. The procedure for acceptance will be explained in detail below.

In each round your gain is defined by the accepted draw, or the last draw if no earlier draw has been accepted. In particular the ECU's (Experimental Currency Unit) is equal to the accepted value v in that particular round (so, in each round you can earn minimum 24ECU, and maximum 123ECU).

Acceptance and choosing acceptance thresholds

At the beginning of each round, before the draws begin, you will be asked to define your acceptance thresholds (t) for all potential draws that round. The acceptance threshold t is the value you choose, from 24 to 124, in order to define what value you are willing to accept for each draw. For example in rounds 1 and 2:

Draw	Acceptance Threshold
1	t_1
2	t_2
3	t_3
4	t_4

In particular: in round 1 and round 2 you define four acceptance thresholds t_1, t_2, t_3, t_4 for the first 4 draws (the last value, value v_5 is automatically accepted if no other draw has been accepted). In round 3 and round 4 you will have to define nine acceptance thresholds t_1, t_t, ..., t_8, t_9 for the first 9 draws, and in rounds 5 and 6 you will have to define fourteen acceptance thresholds t_1, t_2, ..., t_{13}, t_{14} for the first 14 draws.

After you define all the acceptance thresholds $t_1 - t_{n-1}$, the draw of the first value v1 is made and:

a) If the extracted value v1 is greater or equal than the threshold of acceptance ($v_1 \geq 5_1$), the draw is accepted and the round ends;

b) If the extracted value is below the threshold of acceptance ($v_1 < 5_1$), the draw is not accepted and you move on to the second draw for which you have already defined an acceptance threshold t_2. Since you may want to reject any value of a certain draw, we included the acceptance threshold $t = 124$ which automatically rejects any possible value drawn. $t = 24$, on the other hand, accepts every possible drawn value.

Please take your time in making your decisions, there is no point in rushing as the next round starts only when all participants concluded their decision and extractions.

The final gain from this experiment

Your final gain of the experiment will be determined by the sum of earnings for each round and the amount that you are paid for your participation, specifically:

- 5 as a fixed participation fee;

- the gain equal to the value v of the accepted draw (value v of the last draw if no earlier is accepted) in round 1;

- the gain equal to the value v of the accepted draw (value v of the last draw if no earlier is accepted) in round 2;

- the gain equal to the value v of the accepted draw (value v of the last draw if no earlier is accepted) in round 3;

- the gain equal to the value v of the accepted draw (value v of the last draw if no earlier is accepted) in round 4;

- the gain equal to the value v of the accepted draw (value v of the last draw if no earlier is accepted) in round 5;

- the gain equal to the value v of the accepted draw (value v of the last draw if no earlier is accepted) in round 6;

Appendix C - Figures

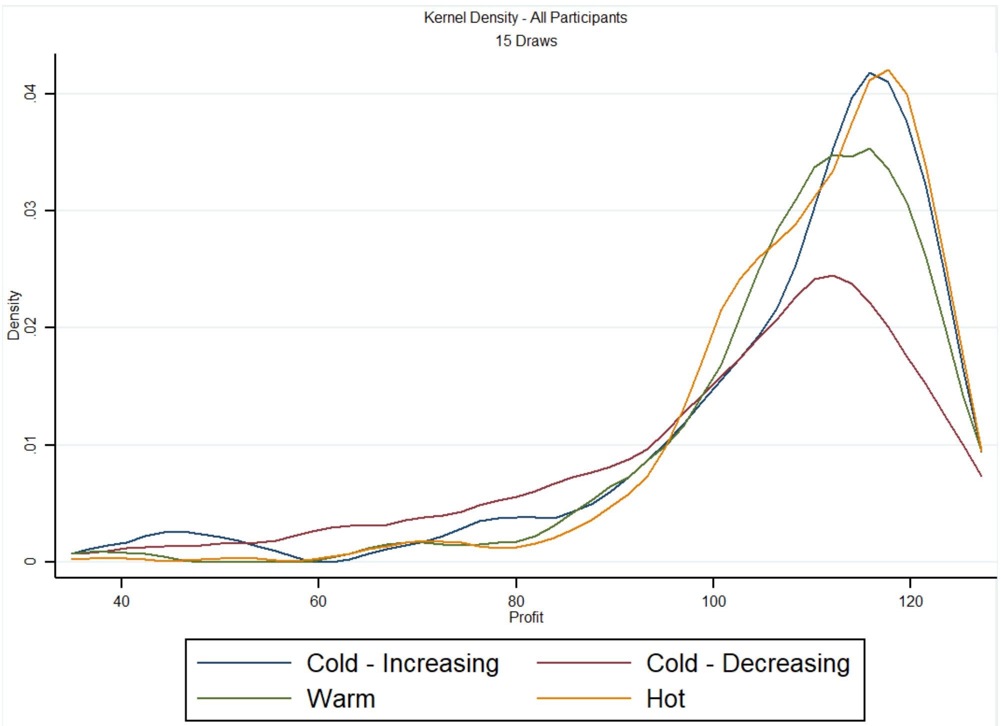

Figure 9 Kernel density of profits ($n = 15$) - all participants