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SPATIO-TEMPORAL AUTOREGRESSIVE SEMIPARAMETRIC MODEL FOR THE ANALYSIS OF REGIONAL ECONOMIC DATA

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Spatio-Temporal Autoregressive Semiparametric Model for the analysis of regional economic data

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Abstract

In this paper we propose an extension of the semiparametric P-Spline model to spatio-temporal data including a non-parametric trend, as well as a spatial lag of the dependent variable. This model is able to simultaneously control for *functional form bias*, *spatial dependence bias*, *spatial heterogeneity bias*, and *omitted time-related factors bias*. Specifically, we consider a spatio-temporal ANOVA model disaggregating the trend in spatial and temporal main effects, and second and third order interactions between them. The model can include both linear and non-linear effects of the covariates, and other additional fixed or random effects. Recent algorithms based on spatial anisotropic penalties (SAP) are used to estimate all the parameters in a closed form without the need of multidimensional optimization. An empirical case compares the performance of this model against alternatives models like spatial panel data models.

JEL classification: C33, C14, C63.
Keywords: spatio-temporal trend, mixed models, P-splines, PS-ANOVA, SAR, spatial panel.

1 Introduction

A recent strand of the spatial econometric literature has proposed *Spatial Autoregressive Semiparametric Geoadditive Models* to simultaneously deal with different critical issues typically met when using spatial economic data, that is spatial dependence, spatial heterogeneity and unknown functional form (Montero et al., 2012; Basile et al., 2014). This approach combines penalized regression spline (PS) methods (Eilers et al., 2015) with standard cross-section spatial autoregressive models (such as SAR, SEM, SDM and SLX). An important feature of these models is the possibility to include within the same specification *i*) spatial autoregressive terms to capture spatial interaction or network effects, *ii*) parametric and nonparametric (smooth) terms to identify nonlinear relationships between the response variable and the covariates, and *iii*) a geoadditive term, that is a smooth function of the spatial coordinates, to capture a spatial trend effect, that is to capture spatially autocorrelated unobserved heterogeneity.

In this paper we propose an extension of the P-Spline spatial auto-regressive model (PS-SAR) to spatio-temporal data when both a large cross-section and a large time series dimensions are available. With this kind of data it is possible to estimate not only spatial trends, but also spatio-temporal trends in a nonparametric way (Lee and Durbán, 2011), so as to capture region-specific nonlinear time trends net of the effect of spatial autocorrelation. In other words, this approach allows to answer questions like: How do unobserved time-related factors (i.e. common factors), such as economic-wide technological or demand shocks, heterogeneously affect long term dynamics of all units in the sample? And how does their inclusion in the model affect the estimation of spatial interaction effects? In this sense, the PS-SAR model with spatio-temporal trend represents an alternative to parametric methods aimed at disentangling *common factors* effects (such as common business cycle effects) and *spatial dependence* effects (local interactions between regions generating spillover effects), where the former is sometimes regarded as 'strong' cross-sectional dependence and the latter as 'weak' cross-sectional dependence (Chudik et al., 2011).

Recently, Bai and Li (2015) and Shi and Lee (2016) have proposed the quasi maximum likelihood method (QML) to estimate dynamic spatial panel data models with common shocks, thus accommodating both strong and weak cross-sectional dependence. Spatial correlations and common shocks are also considered by Pesaran and Tosetti (2011), but they specify the spatial autocorrelation on the idiosyncratic errors, and not on the observable dependent variable. Bailey et al. (2016) and Vega and Elhorst (2016) have also proposed a two-step and one-step approach, respectively, to address both forms of cross-sectional dependence, but without including explanatory variables in the model. All these approaches are still parametric and do not properly allow for capturing nonlinearities. On the other hand, Su and Jin (2012) have considered the problem of estimating semiparametric panel data models with cross section dependence, where the individual-specific regressors enter the model nonparametrically, and the common factors enter the model linearly, thus extending Pesaran (2006)'s common correlated effects (CCE) estimator to a semiparametric framework. Nevertheless, they do not take spatial contagion effects (i.e. weak cross-section dependence) into account. By relying on the PS-SAR model with spatio-temporal trends, we handle simultaneously four main econometric issues which are relevant when modeling spatio-temporal data, namely *functional form bias*, *spatial* *dependence bias*, *spatial heterogeneity bias*, and *omitted time-related factors bias*.

The econometric model proposed here might seem complicated and computationally demanding. Nevertheless, we consider a decomposition of the spatio-temporal trend into several components (spatial and temporal main effects, and second and third order interactions between them) that gives further insights into the dynamics of the data. Furthermore, we use a mixed model representation that allows us to use the methods already developed in this area for estimation and inference, and to implementation of the necessary identifiability constraints in a straightforward manner. We also present an extension of the algorithm derived by Rodriguez-Alvarez et al. (2015) (for variance components estimation) to the case of PS-SAR model that dramatically reduces the computational time needed to estimate the parameters in the model. Also, the use of B-spline nested basis (Lee et al., 2013) for the interaction components contributes to the efficiency of the fitting procedure without compromising the goodness of fit of the model.

Using generated data, we illustrate the relative performance of the PS-SAR model with a spatio-temporal trend against other alternatives like spatial panel data models (in particular, the spatial CCEP model). Simulation results indicate that the spatio-temporal PS-SAR model outperformes the competing models in terms of information criteria, and it represents a robust alternative to estimate the slopes and spatial dependence parameter in the majority of scenarios analyzed.

We also apply the PS-SAR model with a spatio-temporal trend to regional unemployment data in Italy.[1] As well known, these data are characterized by spatial dependence, unobserved spatial heterogeneity and unobserved common effects. Substantive spatial dependence occurs due to interregional trade, labor migration and commuting, and knowledge spillovers; it can be captured by including spatial interaction effects in the model (Burridge and Gordon, 1981; Molho, 1995; Henry G. Overman, 2002; Patacchini and Zenou, 2007). Unobserved spatial heterogeneity is mainly due to a strong North-South spatial trend which can be hardly captured by the explanatory variables: regional unemployment rates largely increase moving from the North to the South, reflecting the well-known regional development divide within the Country. A time-invariant smooth spatial trend might be used to filtering out unobserved heterogeneity; thus, the spatial trend assumes the same role as the fixed regional effects. Several unobserved common factors (e.g. aggregate demand shocks, aggregate technological shocks, and global labor market policies) may also heterogeneously affect the level of regional unemployment. The econometric results suggest that the PS-SAR model with a spatio-temporal trend performs better than several parametric and nonparametric competing models both in terms of model fitting and diagnostics of the residuals. In particular, the spatio-temporal trend effectively captures the strong cross-sectional dependence (due to common factors), while the parameter associated to the spatial lag term reveals the existence of significant spatial spillovers net of the effect of the observed and unobserved common factors.

The plan of the paper is as follows. Section 2 sets out the PS-SAR model with a spatio-temporal trend and discusses various technical aspects related to its identification and estimation. Section 3 reports the results of Monte Carlo experiments, while Section 4 discusses the results of the application of the model to regional unemployment data. Section 5 concludes by identifying important areas for extensions and further develop-

[1] We implemented new functions in the R software to estimate PS-SAR models with spatio-temporal trends.

ments.

2 Spatio-Temporal Autoregressive Models

2.1 P-splines Mixed Models

Semiparametric models are a flexible tool to incorporate non-linear functional relationships into a regression model. The general form of a semiparametric model is given by:

$$\mathbf{y} = \mathbf{X}^*\boldsymbol{\beta}^* + f_1(\mathbf{x}_1) + f_2(\mathbf{x}_2) + f_{3,4}(\mathbf{x}_3, \mathbf{x}_4) + \ldots + \boldsymbol{\epsilon} \quad \boldsymbol{\epsilon} \sim N(0, \sigma^2) \tag{1}$$

where $\mathbf{y}$ is a continuous response variable, $\mathbf{X}^*\boldsymbol{\beta}^*$ is the linear predictor (containing the intercept and categorical and continuous covariates whose functional relationship with the response is linear), and $f_k()$ are unknown smooth functions of single covariates or interaction surfaces. Several approaches have appeared in the literature to fit such models (Green and Yandell, 1985; Hastie and Tibshirani, 1990). We use a penalized regression approach which combines a basis representation of the functions with a penalty to control the wiggliness of the curve/surface. In particular, we use the approach introduced by Eilers and Marx (1996) where each univariate smooth term is represented by:

$$f_k(\mathbf{x}_k) = \sum_{j=1}^{c_k} B_j(\mathbf{x}_k)\theta_j, \; j = 1, ..., c_k$$

with B_j a B-spline basis function and θ_j a vector of regression coefficients of length c_k. The smooth interaction terms are

$$f_{i,k}(\mathbf{x}_i, \mathbf{x}_k) = \sum_{j=1}^{c_i}\sum_{l=1}^{c_k} B_j(\mathbf{x}_i)B_l(\mathbf{x}_k)\theta_{jl}, \text{ with } j = 1, ..., c_i \text{ and } l = 1, ..., c_k,$$

where $B_j(\mathbf{x}_i)B_l(\mathbf{x}_k)$ is the tensor product of two marginal B-spline bases, and θ_{jl} is a vector of coefficients of length $c_ic_k \times 1$. In matrix notation, model (1) becomes:

$$\mathbf{E}[\mathbf{y}|\mathbf{X}^*, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4 \ldots] = \mathbf{B}\boldsymbol{\theta}, \tag{2}$$

where $\mathbf{B}$ is the full regression matrix, and $\boldsymbol{\theta} = (\boldsymbol{\beta}^*, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_{[3,4]}, \ldots)'$ is a vector of regression coefficients. The model matrix is defined by blocks:

$$\mathbf{B} = [\mathbf{X}^*|\mathbf{B}_1|\mathbf{B}_2|\mathbf{B}_{[3,4]}|\ldots], \tag{3}$$

with marginal bases of the covariates $\mathbf{B}_1 = \mathbf{B}_1(\mathbf{x}_1)$ and $\mathbf{B}_2 = \mathbf{B}_2(\mathbf{x}_2)$ and interaction basis $\mathbf{B}_{[3,4]}$ as the tensor product of the two marginals, i.e.

$$\mathbf{B}_{[3,4]} = \mathbf{B}_3 \Box \mathbf{B}_4 = (\mathbf{B}_3 \otimes \mathbf{1}'_n) * (\mathbf{1}'_n \otimes \mathbf{B}_4), \text{ of dimension } n \times c_3c_4. \tag{4}$$

where the symbols $\Box$, $\otimes$, and $*$ indicate the box product, the Kronecker product, and the element-by-element product, respectively. The size, c_k, of each individual basis should be large enough (in general, between 4 and 40) to identify nonlinearities, and the smoothness of each term is controlled by a quadratic penalty term, yielding the following penalized regression problem:

$$\|(\mathbf{y}-\mathbf{B}\boldsymbol{\theta})'(\mathbf{y}-\mathbf{B}\boldsymbol{\theta})\|^2+\boldsymbol{\theta}'\mathbf{P}\boldsymbol{\theta}.$$

Typically, the quadratic penalty term is equivalent to an integral of squared second derivatives of the function, but sometimes (especially in the case of interactions) its calculation is not straightforward. Thus, following Eilers and Marx (1996), we use second-order differences among adjacent coefficients. The penalty matrix (1) is therefore:

$$\mathbf{P}=\text{blockdiag}(\mathbf{P}_i) \tag{5}$$

with $\mathbf{P}_i=\lambda_i\mathbf{D}_i'\mathbf{D}_i$ or $\mathbf{P}_i=\lambda_{ij}\mathbf{D}_j'\mathbf{D}_j\otimes\mathbf{I}+\lambda_{ik}\mathbf{I}\otimes\mathbf{D}_k'\mathbf{D}_k$ in the case of interaction effects. This last penalty allows for a separate amount of smoothing per covariate (*anisotropy*).

Since the intercept of the model is contained in $\mathbf{X}^*\boldsymbol{\beta}^*$ and a column of 1's is also spanned by each basis $\mathbf{B}_i$, there is a problem of identifiability (common to any additive model). There are several ways to solve it, but we choose to re-parametrize model (1) as a mixed model:

$$\mathbf{y}=\mathbf{X}\boldsymbol{\beta}+\mathbf{Z}\boldsymbol{\alpha}+\boldsymbol{\epsilon}\quad \boldsymbol{\alpha}\sim N(\mathbf{0},\mathbf{G})\quad \boldsymbol{\epsilon}\sim N(\mathbf{0},\sigma^2\mathbf{I}) \tag{6}$$

by transforming the bases and the penalty. Several transformations are possible, but the most popular one is based on the singular value decomposition of the penalty matrix (see Lee, 2010b, for details). Matrix $\mathbf{X}$ will include parametric components such as the intercept, continuous covariates and categorical covariates, while $\mathbf{Z}$ includes all the nonlinear components of the smooth effects. The covariance matrix of random effects, $\mathbf{G}$, is a diagonal matrix which depends on the eigenvalues of the singular value decomposition and variance components τ_i^2, and the smoothing parameters become $\lambda_i=\sigma^2/\tau_i^2$.

The estimates of the coefficients $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ follow from the standard mixed model theory (see Searle et al., 1992):

$$\widehat{\boldsymbol{\beta}}=(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{y} \tag{7}$$

$$\widehat{\boldsymbol{\alpha}}=\mathbf{G}\mathbf{Z}'\mathbf{V}^{-1}(\mathbf{y}-\mathbf{X}\widehat{\boldsymbol{\beta}}), \tag{8}$$

where $\mathbf{V}=\sigma^2\mathbf{I}+\mathbf{Z}\mathbf{G}\mathbf{Z}'$.

Variance components (and, therefore, smoothing parameters) may be estimated by maximizing the residual log-likelihood (REML) of Patterson and Thompson (1971):

$$\ell(\tau_i^2,\sigma^2)=-\frac{1}{2}\log|\mathbf{V}|-\frac{1}{2}\log|\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}|-\frac{1}{2}\mathbf{y}'(\mathbf{V}^{-1}-\mathbf{V}^{-1}\mathbf{X}(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{-1})\mathbf{y}. \tag{9}$$

The estimated values of the observed variable are obtained as:

$$\hat{\mathbf{y}}=\mathbf{X}\hat{\boldsymbol{\beta}}+\mathbf{Z}\hat{\boldsymbol{\alpha}}=\mathbf{H}\mathbf{y}$$

where $\mathbf{H}$ is the hat matrix of the model given by:

$$\mathbf{H}=[\mathbf{X}:\mathbf{Z}]\begin{bmatrix}\mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{Z}\\ \mathbf{Z}'\mathbf{X} & \mathbf{Z}'\mathbf{Z}+\mathbf{G}^{-1}\end{bmatrix}^{-1}[\mathbf{X}:\mathbf{Z}]'. \tag{10}$$

The trace of this matrix is defined as the *effective dimension*, which is a measure of the complexity of the model. Also, confidence bands for the estimated values can be calculated using an approximation of the variance–covariance matrix of the estimation error given by $V(\mathbf{y}-\hat{\mathbf{y}})=\sigma_\varepsilon^2\mathbf{H}$.

2.2 Spatio-Temporal Smooth Models

When data are collected over space and time, models deal with these types of effects in different ways, but their formulation is constrained by the size of the data set, and the level of complexity used to estimate these models. Spatio-temporal smoothing is an approach computationally efficient, and, at the same time, it allows for the estimation of complex trends. The simplest example of this type of models is the additive model:

$$f(space) + f(time),$$

proposed by Kneib and Fahrmeir (2006) which ignores the space-time interaction, and cannot reflect important features in the data. This model also implies a spatio-temporal correlation structure given by separable covariance matrices for the spatial and temporal components. In most real applications, this approach is too simplistic, and its natural extension includes space-time interaction terms. Our proposal is based on the PS-ANOVA model introduced by Lee and Durbán (2011), including second- and third-order interactions between spatial and temporal components.

Let us assume that the data are collected at n spatial locations at t time points (the model could be easily generalized to the case in which the response variable is measured at different time points in each location), and no further covariates are available (in the next section we will combine this model with the semiparametric model introduced in the previous section). The smooth space-time model has the form:

$$\begin{aligned} \mathbf{y} \;=\;& \gamma + f_1(\mathbf{x}_{s_1}) + f_2(\mathbf{x}_{s_2}) + f_t(\mathbf{x}_t) + \\ & f_{1,2}(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}) + f_{1,t}(\mathbf{x}_{s_1}, \mathbf{x}_t) + f_{2,t}(\mathbf{x}_{s_2}, \mathbf{x}_t) + \\ & f_{1,2,t}(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}, \mathbf{x}_t) + \boldsymbol{\epsilon} \end{aligned} \tag{11}$$

where $\mathbf{x}_{s_1}$ and $\mathbf{x}_{s_2}$ are the geographical coordinates of the spatial location, and $\mathbf{x}_t$ the vector of time points. The B-spline basis for model (11) would be:

$$\mathbf{B} = [\mathbf{1}|\mathbf{B}_{s_1}|\mathbf{B}_{s_2}|\mathbf{B}_t|\mathbf{B}_{s_1}\square\mathbf{B}_{s_2}|\mathbf{B}_{s_1}\otimes\mathbf{B}_t|\mathbf{B}_{s_2}\otimes\mathbf{B}_t|(\mathbf{B}_{s_1}\square\mathbf{B}_{s_2})\otimes\mathbf{B}_t]$$

and the penalty matrix is block-diagonal with blocks corresponding to the different terms in the model. In this case, several constrained need to be imposed, since the space spanned by $\mathbf{B}_i \otimes \mathbf{B}_j$, already spans the space spanned by the marginal bases $\mathbf{B}_i$ and $\mathbf{B}_j$ (see Lee, 2010a, for more details). Again, the transformation of model (11) into a mixed model, using the singular value decomposition of marginal penalties, will solve the identifiability problem yielding matrices:

- $\mathbf{X} = [(\mathbf{X}_{s_1}\square\mathbf{X}_{s_2})\otimes\mathbf{X}_t]$
- $\mathbf{Z} = [(\mathbf{Z}_{s_1}\square\mathbf{X}_{s_2})\otimes\mathbf{X}_t|(\mathbf{X}_{s_1}\square\mathbf{Z}_{s_2})\otimes\mathbf{X}_t|(\mathbf{X}_{s_1}\square\mathbf{X}_{s_2})\otimes\mathbf{Z}_t|(\mathbf{Z}_{s_1}\square\mathbf{Z}_{s_2})\otimes\mathbf{X}_t$ $(\mathbf{Z}_{s_1}\square\mathbf{X}_{s_2})\otimes\mathbf{Z}_t|(\mathbf{X}_{s_1}\square\mathbf{Z}_{s_2})\otimes\mathbf{Z}_t|(\mathbf{Z}_{s_1}\square\mathbf{Z}_{s_2})\otimes\mathbf{Z}_t]$,

where $\mathbf{X}_{s_k}$, $\mathbf{Z}_{s_k}$ $(k = 1, 2)$, $\mathbf{X}_t$, and $\mathbf{Z}_t$ are the mixed model matrices obtained for the reparametrization of the marginal basis and penalties (Lee and Durbán, 2011). The covariance matrix of random effects,$\mathbf{G}$, is given by

$$\mathbf{G}^{-1} = \text{blockdiag}\left(\mathbf{0}, \frac{1}{\tau_1^2}\widetilde{\mathbf{\Lambda}}_1, \frac{1}{\tau_2^2}\widetilde{\mathbf{\Lambda}}_2, \frac{1}{\tau_3^2}\widetilde{\mathbf{\Lambda}}_3, \frac{1}{\tau_4^2}\widetilde{\mathbf{\Lambda}}_4 + \frac{1}{\tau_5^2}\widetilde{\mathbf{\Lambda}}_5, \frac{1}{\tau_6^2}\widetilde{\mathbf{\Lambda}}_6 + \frac{1}{\tau_7^2}\widetilde{\mathbf{\Lambda}}_7, \frac{1}{\tau_8^2}\widetilde{\mathbf{\Lambda}}_8 + \frac{1}{\tau_9^2}\widetilde{\mathbf{\Lambda}}_9, \frac{1}{\tau_{10}^2}\widetilde{\mathbf{\Lambda}}_{10} + \frac{1}{\tau_{11}^2}\widetilde{\mathbf{\Lambda}}_{11} + \frac{1}{\tau_{12}^2}\widetilde{\mathbf{\Lambda}}_{12}\right)$$

where

$$\widetilde{\boldsymbol{\Lambda}}_1 = \widetilde{\boldsymbol{\Sigma}}_{s_1}, \quad \widetilde{\boldsymbol{\Lambda}}_2 = \widetilde{\boldsymbol{\Sigma}}_{s_2}, \quad \widetilde{\boldsymbol{\Lambda}}_3 = \widetilde{\boldsymbol{\Sigma}}_t$$
$$\widetilde{\boldsymbol{\Lambda}}_4 = \widetilde{\boldsymbol{\Sigma}}_{s_1} \otimes \mathbf{I}_{c_{s_2}-q_{s_2}}, \quad \widetilde{\boldsymbol{\Lambda}}_5 = \mathbf{I}_{c_{s_1}-q_{s_1}} \otimes \widetilde{\boldsymbol{\Sigma}}_{s_2}, \quad \widetilde{\boldsymbol{\Lambda}}_6 = \widetilde{\boldsymbol{\Sigma}}_{s_1} \otimes \mathbf{I}_{q_{s_2}}$$
$$\widetilde{\boldsymbol{\Lambda}}_7 = \mathbf{I}_{c_{s_1}-q_{s_1}} \otimes \mathbf{I}_{q_{s_2}}, \quad \widetilde{\boldsymbol{\Lambda}}_8 = \widetilde{\boldsymbol{\Sigma}}_{s_2} \otimes \mathbf{I}_{c_t-q_t} \quad \widetilde{\boldsymbol{\Lambda}}_9 = \mathbf{I}_{c_{s_2}-q_{s_2}} \otimes \widetilde{\boldsymbol{\Sigma}}_t$$
$$\widetilde{\boldsymbol{\Lambda}}_{10} = \widetilde{\boldsymbol{\Sigma}}_{s_1} \otimes \mathbf{I}_{c_{s_2}-q_{s_2}} \otimes \mathbf{I}_{c_t-q_t}, \quad \widetilde{\boldsymbol{\Lambda}}_{11} = \mathbf{I}_{c_{s_1}-q_{s_1}} \otimes \widetilde{\boldsymbol{\Sigma}}_{s_2} \otimes \mathbf{I}_{c_t-q_t}, \quad \widetilde{\boldsymbol{\Lambda}}_{12} = \mathbf{I}_{c_{s_1}-q_{s_1}} \otimes \mathbf{I}_{c_{s_2}-q_{s_2}} \otimes \widetilde{\boldsymbol{\Sigma}}_t$$

and $\widetilde{\boldsymbol{\Sigma}}$ matrices correspond to the eigenvectors of the singular value decomposition of penalty matrices, and c_k and q_k are the dimensions of the bases and the order of the penalty used for the marginal smooth. It is worth noticing that the dimension of the matrices involved in interaction terms can increase very quickly if the size of the marginal bases is large, and so the estimation of the model can become very slow or intractable. We follow Lee et al. (2013) in using *nested* B-spline bases for the interactions terms, in order to reduce the computational burden without compromising the fit of the model. The idea is to use a matrix $\breve{\mathbf{B}}$ in the interaction, such that the space spanned by this matrix is a subset of the space spanned by $\mathbf{B}$. The use of this *simpler* matrix for the construction of the interaction terms will not be a problem since, in general, the information about the interaction usually is sparse. In the ANOVA context, the main effects are more important than the interactions, so in most situations this would be reasonable. The way to ensure that the new basis is nested relative to the original basis is to assume that the number of knots ($\texttt{ndx}^*$) in $\breve{\mathbf{B}}$ is a divisor of the number of knots used to construct the original basis ($\texttt{ndx}$):

$$\texttt{ndx}^* \text{ of } \breve{\mathbf{B}} = \frac{\texttt{ndx} \text{ of } \mathbf{B}}{\texttt{div}} \Rightarrow \mathsf{span}(\breve{\mathbf{B}}) \subset \mathsf{span}(\mathbf{B}).$$

Then, the number of parameters is dramatically reduced, but the model is still flexible enough to capture the complex space-time structure in the data. Matrices $\mathbf{Z}_{s_k}$ and $\mathbf{Z}_t$ would be modified by the corresponding $\breve{\mathbf{Z}}_{s_k}$ and $\breve{\mathbf{Z}}_t$ in $f_{(1,2)}$, $f_{(1,t)}$, $f_{(2,t)}$ and $f_{(1,2,t)}$. Estimation of fixed and random effects and variance components would proceed as in the case of a semiparametric model.

Before concluding this subsection, it is important to give some insights about the relevance and the meaning of the spatio-temporal components of the PS-ANOVA model 11 in applied econometric studies. First of all, as already pointed out in Basile et al. (2014), the geoadditive terms ($f_1(\mathbf{x}_{s_1})$, $f_2(\mathbf{x}_{s_2})$, and $f_{1,2}(\mathbf{x}_{s_1}, \mathbf{x}_{s_2})$) work as control functions to filter the spatial trend out of the residuals, and transfer it to the mean response in a model specification. Thus, they allow to capture the shape of the spatial distribution of $\mathbf{y}$, eventually conditional on the determinants included in the model. This control function also isolates stochastic spatial dependence in the residuals, that is spatially autocorrelated unobserved heterogeneity. Thus, it can be regarded as an alternative to individual regional dummies to capture unobserved spatial heterogeneity as long as the latter is smoothly distributed over space. Regional dummies peak significantly higher and lower levels of the mean response variable. If these peaks are smoothly distributed over a two-dimensional surface (i.e. if unobserved spatial heterogeneity is spatially auto-correlated), the smooth spatial trend is able to capture them.

The smooth time trend, $f_t(\mathbf{x}_t)$, and the smooth interactions between space and time - $f_{1,t}(\mathbf{x}_{s_1}, \mathbf{x}_t)$, $f_{2,t}(\mathbf{x}_{s_2}, \mathbf{x}_t)$, and $f_{1,2,t}(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}, \mathbf{x}_t)$ - work as control functions to capture the heterogeneous effect of common shocks, thus allowing for strong cross-sectional dependence in the data. In this sense, the PS-ANOVA model 11 works as an alternative to the Common Correlated Effects (CCE) method proposed by Pesaran (2006) based on the use of cross-sectional averages of the observations. Nevertheless, like the CCE model, also the spatio-temporal ANOVA model 11 (even if combined with the semiparametric model 1) neglects the presence of weak cross-dependence (i.e. spatial spillovers) in the data. While Bai and Li (2015), Shi and Lee (2016), Pesaran and Tosetti (2011), Bailey et al. (2016) and Vega and Elhorst (2016) have proposed parametric spatial panel extensions of common factors models to accommodate both strong and weak cross-sectional dependence, here we rely on a combination of the ANOVA spatio-temporal trend model described above and the PS-SAR model developed by Montero et al. (2012); Basile et al. (2014) to handle simultaneously spatial spillovers and strong cross-sectional dependence (see the next subsection).

2.3 Spatio-Temporal PS-SAR Models

As mentioned above, spatio-temporal models do not permit to distinguish between strong and weak cross-sectional dependence (Chudik et al., 2011), because all the correlation between spatial units is collected by the spatio-temporal trend and, perhaps, by the effect of covariates. In order to assess the presence of residual spatial spillovers net of the effect of common factors, we combine the PS-ANOVA model 11 described in the previous section with the spatial lag model. By including also linear ($\mathbf{X}^*$), and nonlinear ($\mathbf{z}_j$) additive covariates, the full model becomes:

$$\begin{aligned}
(\mathbf{A} \otimes \mathbf{I}_T)\mathbf{y} = {} & f_1(\mathbf{x}_{s_1}) + f_2(\mathbf{x}_{s_2}) + f_t(\mathbf{x}_t) + f_{1,2}(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}) + \\
& + f_{1,t}(\mathbf{x}_{s_1}, \mathbf{x}_t) + f_{2,t}(\mathbf{x}_{s_2}, \mathbf{x}_t) + f_{1,2,t}(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}, \mathbf{x}_t) + \\
& + \mathbf{X}^*\boldsymbol{\beta}^* + \sum_{j=1}^{l} f(\mathbf{z}_j) + \boldsymbol{\epsilon} \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_{NT}) \qquad (12) \\
\mathbf{A} = {} & \mathbf{I}_N - \rho \mathbf{W}_N
\end{aligned}$$

where $\mathbf{W}_N$ is a neighborhood spatial matrix, and ρ is the spatial parameter associated to $\mathbf{W}_N$ matrix. It measures the degree of the spatial weak dependence net of the strong dependence. This model is flexible enough to gather both types of cross-sectional dependence. Specifically, the inclusion of the ANOVA decomposition of the spatio-temporal trend helps interpret the evidence of significance spatial spillovers as weak cross-dependence net of the effect of common effects (strong dependence). In this sense, model 12 can be regarded as a valid alternative to Bai and Li (2015), Shi and Lee (2016), or Bailey et al. (2016) which consider a jointly modeling of both spatial interaction effects and common-shocks effects. Our framework is also flexible enough to control for residual spatial heterogeneity (due to a spatial trend) and for the linear and non-linear functional relationships between the dependent variable and the covariates.

To estimate all the parameters of the model it is possible to maximize REML function as in (9) slightly modified by the kronecker matrix product $(\mathbf{A} \otimes \mathbf{I}_T)$:

$$\begin{aligned} \ell(\tau_i^2, \sigma^2, \rho) = & -\frac{1}{2} \log |\mathbf{V}| - \frac{1}{2} \log |\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}| \\ & - \frac{1}{2} [(\mathbf{A} \otimes \mathbf{I}_T)\mathbf{y}]' (\mathbf{V}^{-1} - \mathbf{V}^{-1}\mathbf{X}(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{-1}) [(\mathbf{A} \otimes \mathbf{I}_T)\mathbf{y}] + \\ & + \log |\mathbf{A} \otimes \mathbf{I}_T| \end{aligned} \tag{13}$$

where the matrices $\mathbf{V}$ and $\mathbf{X}$ are obtained as described above (if linear and non-linear covariates have been added, $\mathbf{X}$ and $\mathbf{Z}$ matrices are augmented in an additive suitable way).

To get estimates for all the parameters, we need to maximize the REML function which is a very complex numerical problem. Recently, Rodriguez-Alvarez et al. (2015) have developed an algorithm named SAP (Separation of Anisotropic Penalties), which is based on the fact that the inverse variance-covariance matrix of the random effects, $\mathbf{G}^{-1}$, is a linear combination of precision matrices:

$$\mathbf{G}^{-1} = \sum_{i=1}^{12} \frac{1}{\tau_i^2} \boldsymbol{\Lambda}_i, \quad \boldsymbol{\Lambda}_i = \text{blockdiag}(\mathbf{0}, \ldots, \widetilde{\boldsymbol{\Lambda}}_i, \ldots, \mathbf{0}) \tag{14}$$

This expression allows to get closed estimates for all the variance component parameters τ_i^2 and σ^2 very efficiently. We have adapted this algorithm to include also the estimation of ρ parameter. The steps to apply SAP algorithm to optimize (13) can be summarized as follows:

1. Initialization. Set
   - Set $k = 0$
   - $\hat{\boldsymbol{\beta}}^{(k)} = \mathbf{0}; \quad \hat{\boldsymbol{\alpha}}^{(k)} = \mathbf{0}$
   - $\hat{\tau}_i^{2,(k)} = 1 \quad i = 1, 2, \cdots, 12, \cdots, z_j, \cdots$
   - $\hat{\sigma}^{2,(k)} = \text{var}(\mathbf{y})$
   - $\hat{\rho}^{(k)} = 0$
2. Compute $\hat{\mathbf{G}}^{(k)}, \hat{\mathbf{V}}^{(k)}, \hat{\mathbf{P}}^{(k)}, \hat{\mathbf{A}}^{(k)}$ matrices using next expressions:

$$\begin{aligned} \hat{\mathbf{G}}^{-1,(k)} &= \sum_{i=1}^{12} \frac{1}{\tau_i^2} \boldsymbol{\Lambda}_i^{(k)} \\ \hat{\mathbf{V}}^{(k)} &= \hat{\sigma}^{2,(k)} \mathbf{I}_{nT} + \mathbf{Z}\hat{\mathbf{G}}^{(k)}\mathbf{Z} \\ \hat{\mathbf{P}}^{(k)} &= \hat{\mathbf{V}}^{-1,(k)} - \hat{\mathbf{V}}^{-1,(k)}\mathbf{X}(\mathbf{X}'\hat{\mathbf{V}}^{-1,(k)}\mathbf{X})^{-1}\mathbf{X}'\hat{\mathbf{V}}^{-1,(k)} \\ \hat{\mathbf{A}}^{(k)} &= \mathbf{I}_N - \hat{\rho}^{(k)}\mathbf{W}_N \end{aligned}$$

3. Compute the estimates:

$$\begin{aligned} \hat{\boldsymbol{\beta}}^{(k)} &= (\mathbf{X}'\hat{\mathbf{V}}^{-1,(k)}\mathbf{X})^{-1}(\mathbf{X}'\hat{\mathbf{V}}^{-1,(k)}\hat{\mathbf{A}}^{(k)}\mathbf{y}) \\ \hat{\boldsymbol{\alpha}}^{(k)} &= \hat{\mathbf{G}}^{(k)}\mathbf{Z}'\hat{\mathbf{V}}^{-1,(k)}(\hat{\mathbf{A}}^{(k)}\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}^{(k)}) \\ ed_i^{(k)} &= \text{trace}(\mathbf{Z}'\hat{\mathbf{P}}^{(k)}\mathbf{Z}\hat{\mathbf{G}}^{(k)} \frac{1}{\hat{\tau}_i^{2,(k)}} \boldsymbol{\Lambda}_i \hat{\mathbf{G}}^{(k)}) \quad i = 1, 2, \cdots, 12, \cdots, z_j, \cdots \end{aligned}$$

where $\mathbf{\Lambda}_i \quad i = 1, \cdots, 12$ is defined in (14) and $\mathbf{\Lambda}_{z_j} = \text{blockdiag}\left(\mathbf{0}, \cdots, \widetilde{\Sigma}_{z_j}, \cdots, \mathbf{0}\right)$.

4. Estimate the variance components:

$$\hat{\tau}_i^{2,(k+1)} = \frac{\hat{\boldsymbol{\alpha}}^{(k)\prime} \mathbf{\Lambda}_i \hat{\boldsymbol{\alpha}}^{(k)}}{ed_i^{(k)}} \quad i = 1 \cdots, 12, \cdots, z_j \cdots$$

   Estimate the variance of the noise as:

$$\hat{\sigma}^{2,(k+1)} = \frac{(\hat{\mathbf{A}}^{(k)}\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}^{(k)} - \mathbf{Z}\hat{\boldsymbol{\alpha}}^{(k)})'(\hat{\mathbf{A}}^{(k)}\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}^{(k)} - \mathbf{Z}\hat{\boldsymbol{\alpha}}^{(k)})}{n - \sum_i ed_i^{(k)} - \text{rank}(\mathbf{X}) - 1}$$

5. Estimate the spatial parameter $\hat{\rho}^{(k+1)}$ solving numerically the non-linear univariate equation obtained by equating to zero the score of REML function with respect to ρ (this additional step is the only difference with respect to the usual SAP algorithm):

$$\frac{\partial \ell(.)}{\partial \rho} = -\frac{1}{2}\left[2\hat{\mathbf{P}}^{(k)}\left((\mathbf{A} \otimes \mathbf{I}_T)\mathbf{y}\right)\right]'\left(\frac{\partial(\mathbf{A} \otimes \mathbf{I}_T)}{\partial \rho}\mathbf{y}\right) + \text{trace}\left((\mathbf{A} \otimes \mathbf{I}_T)^{-1}\frac{\partial(\mathbf{A} \otimes \mathbf{I}_T)}{\partial \rho}\right) =$$
$$= \mathbf{y}'(\mathbf{A} \otimes \mathbf{I}_T')\hat{\mathbf{P}}^{(k)}(\mathbf{W}_N \otimes \mathbf{I}_T)\mathbf{y} - T\text{trace}(\mathbf{A}^{-1}\mathbf{W}_N) = 0$$

6. Set $k = k + 1$ and go to step (2) until convergence.

Once the convergence is obtained, the effective degrees of freedom of the model can be estimated as:

$$\text{edf} = \sum_i ed_i^{(k)} + \text{rank}(\mathbf{X}) + 1$$

This quantity is increased in one unit with respect to spatio-temporal smooth models because of the need to estimate the ρ parameter.

The previous algorithm allows to get estimates of all the parameters of model (12) without the necessity to use numerical optimization and, as a consequence, a huge reduction of computational burden. Moreover, the inference techniques usually applied in REML estimation of mixed models can also be used for this case.

3 Monte Carlo experiments

This section provides Monte Carlo evidence on the small sample performance of alternative parametric and semiparametric estimators. This experiment aims at investigating the extent to which alternative specifications of spatio-temporal models (the fixed spatial and time effects, CCEP, PS-ANOVA, SAR-fixed and time effects, SAR-CCEP and PS-ANOVA-SAR) capture the effects of weak as well strong cross-section dependence. The analysis corresponds to a simplified version of the Monte Carlo study in Pesaran (2006), where the dependent variable and the regressors are assumed to depend on a linear combination of unobserved common factors. Furthermore, the experiment extends the data generating process (DGP) by including a spatial autoregressive term $\sum_{j=1}^{N} w_{ij,N} y_{jt}$, thus combining the two sources of cross section dependence, i.e. common factor and spatial dependence. Following Pesaran (2006) and Pesaran and Tosetti (2011), we consider the following DGP:

$$\begin{aligned} y_{it} &= \alpha_i + \beta_1 x_{1it} + \beta_2 x_{2it} + \gamma_{i1} f_{1t} + \gamma_{i2} f_{2t} + \rho \sum_{j=1}^{N} w_{ij,N} y_{jt} + \varepsilon_{it} \\ x_{ikt} &= a_{ik} + \gamma_{ik1} f_{1t} + \gamma_{ik3} f_{3t} + v_{ikt} \quad k = 1, 2 \end{aligned}$$

for $i = 1, ..., N$ and $t = 1, ..., T$.

We always assume homogeneous slopes, $\beta = (1, 1)'$, for the regressors x_{1it} and x_{2it}, and errors independently distributed across i:

$$\varepsilon_{it} \sim \text{iid}N(0, \sigma_i^2) \quad \sigma_i^2 \sim \text{iid}U[0.5, 1.5]$$

In the above equation, f_{1t}, f_{2t} and f_{3t} are unobserved common effects generated as:

$$\begin{aligned} f_{lt} &= 0.5 f_{lt-1} + v_{f_{lt}} \quad l = 1, 2, 3 \\ v_{f_{lt}} &\sim \text{iid}N(0, 0.75) \quad f_{l0} = 0 \end{aligned}$$

and

$$v_{ikt} \sim \text{iid}N(0, \sigma^2_{v_{ikt}}) \quad \sigma^2_{v_{ikt}} \sim \text{iid}U[0.5, 1.5]$$

The parameters α_i and a_{ik} are generated as:

$$\begin{aligned} \alpha_i &\sim \text{iid}N(1, 1) \\ (a_{i1}, a_{i2}) &\sim \text{iid}N(0.5\boldsymbol{\tau}_2, 0.5\mathbf{I}_2) \end{aligned}$$

where $\boldsymbol{\tau}_2 = (1, 1)'$ and $\mathbf{I}_2$ is a 2×2 identity matrix.

The parameters of the unobserved common effects in the x_{it} equation are generated as

$$\begin{pmatrix} \gamma_{i11} & 0 & \gamma_{i13} \\ \gamma_{i21} & 0 & \gamma_{i23} \end{pmatrix} \sim \text{iid} \begin{pmatrix} N(0.5, 0.5) & 0 & N(0, 0.5) \\ N(0, 0.5) & 0 & N(0.5, 0.5) \end{pmatrix}$$

and the parameters of the unobserved common effects in the y_{it} equation are

$$\begin{aligned} \gamma_{i1} &\sim \text{iid}N(1, 0.2) \\ \gamma_{i2} &\sim \text{iid}N(1, 0.2) \\ \gamma_{i3} & \;\; 0 \end{aligned}$$

Finally, ρ is the spatial autoregressive coefficient associated to the spatial lag of the dependent variable and w_{ij}, for $i, j = 1, ..., N$, are elements of a spatial weight matrix $\mathbf{W}$, assumed to be time-invariant. The neighborhood criterion corresponds to five closest observations and matrix $\mathbf{W}$ has been row-normalized in the usual way. We have chosen three different values of $\rho = (0, 0.2, 0.7)$, which correspond to non-dependence, moderate spatial dependence and a sizable level of spatial dependence, respectively. We have also chosen $N = 200$ and $T = 30$ for the size of the spatio-temporal panels and 200 simulations have been generated for each combination of the parameters (except when $\rho = 0$ in which case 400 simulations were generated).

For each DGP we have estimated 8 competing spatio-temporal models ranging from classical parametric panel data models (with fixed spatial and temporal effects), spatial panel data models with a spatial lag of the dependent variable (Millo, 2014), pooled common correlated estimators (CCEP) described in Pesaran (2006), CCEP including a spatial lag of the dependent variable (Vega and Elhorst, 2016), and the spatio-temporal PS and PS-SAR nonparametric specifications proposed in this paper, with the ANOVA decomposition for the spatio-temporal trend (Table 1).

Table 1
Spatio-Temporal competing models

Model I	Linear model with spatial fixed effects (FE)
Model II	Linear model with spatial and time fixed effects (FE/TE)
Model III	Linear model with unobserved common effects (CCEP)
Model IV	Model with ANOVA spatio-temporal trend and linear terms (PS-ANOVA)
Model V	SAR model with spatial fixed effects (SAR-FE)
Model VI	SAR model with spatial and time fixed effects (SAR-FE/TE)
Model VII	SAR model with unobserved common effects (SAR-CCEP)
Model VIII	SAR model with ANOVA spatio-temporal trend (PS-ANOVA-SAR)

Table 2 and figures 1-3 show the results of the simulation. The main conclusions emerging can be summarized as follows:

1. With respect to the estimates of the slope parameter β_1, Figure 1 shows that all models not including a spatial lag of the dependent variable (Models I to IV) are upward biased. For models II to IV this bias is high only for $\rho = 0.7$ (Table 2), while for model I (FE) this bias is large even when $\rho = 0$. On the other hand, the bias is negligible for all spatial lag models (from V to VIII). Nevertheless, the lowest bias and RMSE is regularly reached by model VI (SAR-FE/TE). Furthermore, the performance of model V (SAR-FE) seems to be worse for small values of ρ, while model VII (SAR-CCEP) gets worse for $\rho = 0.7$. Finally, model VIII (PS-ANOVA-SAR) improves its results when ρ increases.

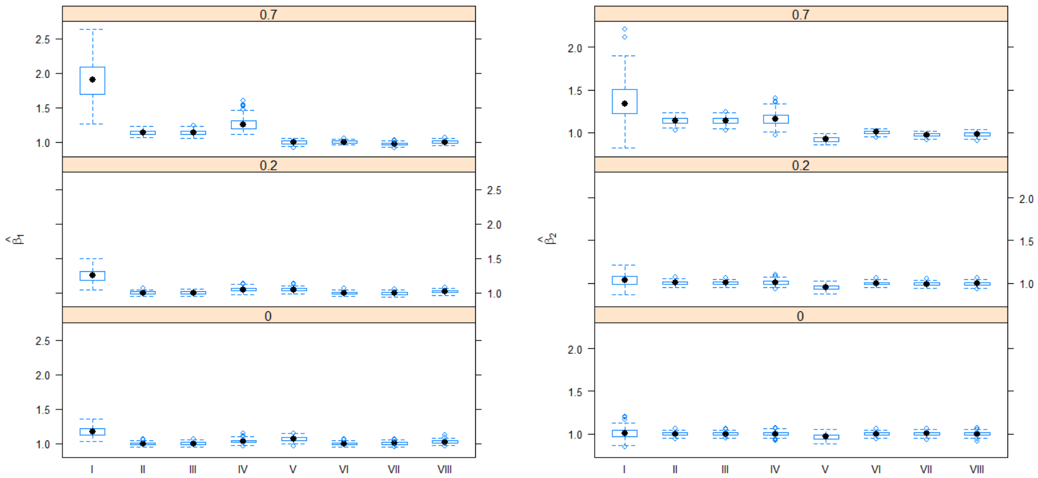
   Similar results emerge for the β_2 parameter, except for the better behaviour of model I which improves considerably. An improvement is also evident in the performance of models IV (PS-ANOVA) and, particularly, VIII (PS-ANOVA-SAR) which now behaves very similarly to model VI (SAR-FE/TE) for all the values of ρ. To sum up, the best results for the estimation of the two slope parameters are obtained with the SAR-FE/TE and the PS-ANOVA-SAR models. These models include the spatial lag of the dependent variable and a mechanism to control for spatial and temporal heterogeneity.

2. With respect to the estimates of parameter ρ, Figure 2 and Table 2 show that the only unbiased estimates correspond to model VI (SAR-FE/TE). The other

models are upward biased, but the lower bias is for model VIII (PS-ANOVA-SAR). Moreover, the distance between models VI and VIII decreases as the value of ρ increases. It should be noted the poor results of model VII (SAR-CCEP) which systematically overestimates the value of ρ, similarly to model V (SAR-FE).

3. To evaluate the general performance of the different models, we compare the values of the log(BIC) (Figure 3). It clearly emerges that model VIII (PS-ANOVA-SAR) outperforms all the other models except when $\rho = 0$, in which case the performances of PS-ANOVA-SAR and PS-ANOVA (model IV) are very close to each other, as expected. These two semiparametric models share a value of estimated degrees of freedom (EDF) much smaller than the others (particularly comparing with CCEP), and this parsimony explains the best scores in terms of information criteria. Furthermore, if the number of covariates were greater, the distance of these parsimonious specifications is likely to increase compared with CCEP models and, on the other hand, if N and/or T would increase the distance of PS-ANOVA-SAR model with respect to the panels with fixed effects (models I, II, V and VI) probably would also increase.

Summing up, the spatio-temporal PS-ANOVA-SAR model proposed in this paper achieves the best performance in terms of information criteria, and it is a valid alternative to the SAR-CCEP model for capturing the effects of weak as well strong cross-section dependence the in the majority of analyzed scenarios including common factors and some kind of spatial dependence.



(a) Estimates of β_1 (b) Estimates of β_2

FIGURE 1
Estimates of β parameters for Models I to VIII and $\rho = (0, 0.2, 0.7)$. True values of $(\beta_1, \beta_2) = (1, 1)$

TABLE 2
Table of bias and root of mean squared error (RMSE) of β_1, β_2 and ρ parameters for Models I to VIII in Monte Carlo simulation. True values of β_1 and β_2 are (1,1) and true values of ρ are (0,0.2,0.7), respectively

	Model	I	II	III	IV	V	VI	VII	VIII
					β_1				
$\rho = 0$	Bias	0.174	0.000	0.001	0.033	0.064	0.000	0.004	0.027
	RMSE	0.186	0.019	0.021	0.041	0.070	0.019	0.022	0.035
$\rho = 0.2$	Bias	0.253	0.004	0.004	0.049	0.045	-0.001	-0.007	0.020
	RMSE	0.267	0.020	0.021	0.057	0.052	0.020	0.023	0.030
$\rho = 0.7$	Bias	0.904	0.141	0.142	0.264	-0.001	0.004	-0.027	0.005
	RMSE	0.943	0.145	0.147	0.279	0.025	0.019	0.034	0.020
					β_2				
$\rho = 0$	Bias	0.003	-0.001	0.000	-0.000	-0.035	-0.001	0.003	-0.002
	RMSE	0.056	0.020	0.020	0.023	0.048	0.020	0.021	0.021
$\rho = 0.2$	Bias	0.035	0.003	0.004	0.008	-0.048	-0.001	-0.008	-0.007
	RMSE	0.078	0.021	0.021	0.028	0.057	0.019	0.021	0.023
$\rho = 0.7$	Bias	0.352	0.142	0.140	0.161	-0.080	0.003	-0.029	-0.020
	RMSE	0.418	0.147	0.145	0.176	0.085	0.020	0.035	0.029
					ρ				
$\rho = 0$	Bias					0.414	-0.016	0.322	0.148
	RMSE					0.418	0.027	0.329	0.158
$\rho = 0.2$	Bias					0.354	-0.015	0.281	0.141
	RMSE					0.358	0.026	0.286	0.148
$\rho = 0.7$	Bias					0.161	-0.013	0.149	0.075
	RMSE					0.161	0.016	0.151	0.079

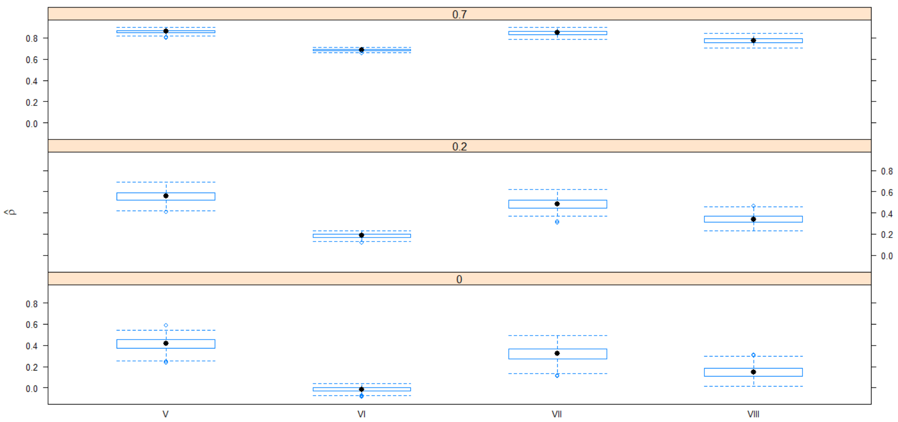


FIGURE 2
Estimates of ρ parameter for Models V, VI, VII and VIII. True values of $\rho = (0, 0.2, 0.7)$.

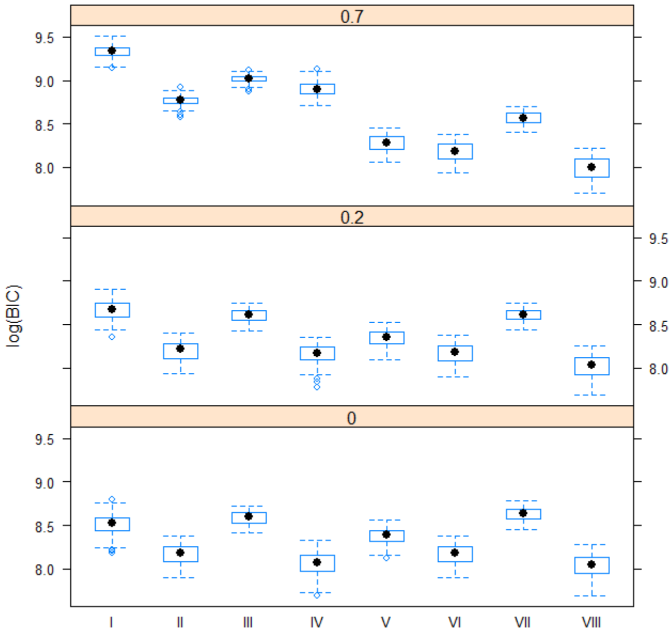


FIGURE 3
log(BIC) for Models I to VIII and $\rho = (0, 0.2, 0.7)$

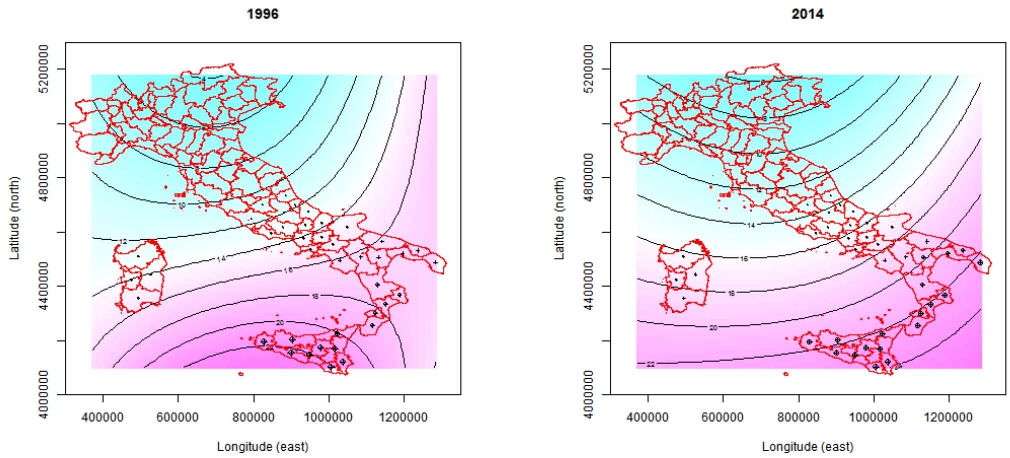
4 Empirical Case

Starting from Partridge and Rickman (1997) and Taylor and Bradley (1997), regional unemployment differentials have been subject of intensive research in the literature. Recent contributions apply spatial econometric models both in a cross-sectional setting (Molho, 1995; Aragon et al., 2003; Cracolici et al., 2007) and in a spatial panel framework (Lottmann, 2012; Basile et al., 2012; Rios, 2014). Here, we analyze the performance of the PS-SAR model with a spatio-temporal trend against different competing parametric and semiparametric models using panel data on regional unemployment in Italy. We first describe these data and their features in terms of spatial and temporal trend (section 4.1). Then, we briefly discuss the theoretical background and the set of variables used to explain regional unemployment differentials (section 4.2). Finally, we report the results of the econometric analysis (section 4.3).

4.1 Regional unemployment data

The data on regional unemployment rates ($unrate_{i,t}$) for each Italian province $i = 1, ..., N$ (N=103) which corresponds to an Italian NUTS-3 region, and for each time period $t = 1996, ..., 2014$ (T=19) used in this analysis are provided online by the Italian National Institute of Statistics (ISTAT). They are defined as $unrate_{i,t} = 100 \times \frac{U_{i,t}}{LF_{i,t}}$, where $U_{i,t}$ is the number of unemployed and $LF_{i,t}$ is the labor force.

Regional unemployment rates differ strongly in Italy, especially between northern and southern provinces. The North-South divide can be depicted by mapping the predicted



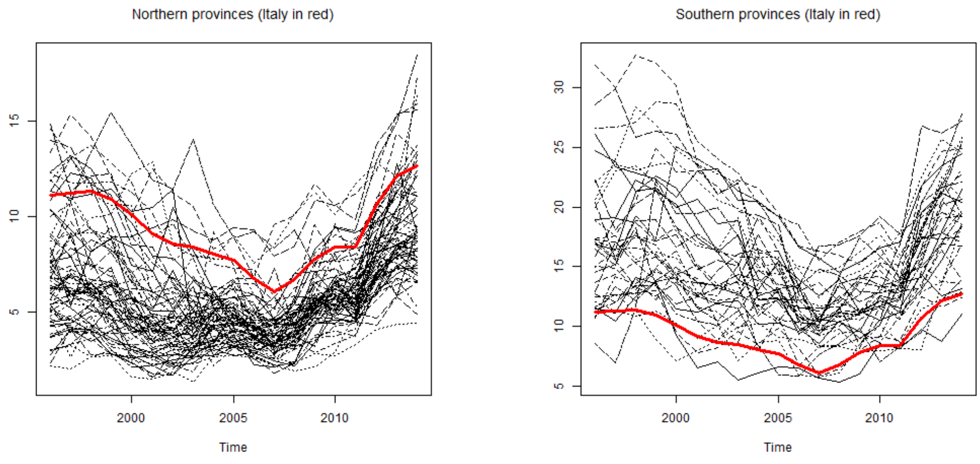
(a) Year: 1996

(b) Year: 2014

FIGURE 4
Spatial trend of provincial unemployment rates

values of a simple regression of provincial unemployment rates on the smooth interaction between longitude and latitude (figure 4). A clear spatial trend emerges and is persistent over time. These findings might suggest that the nature of regional unemployment disparities in Italy is the result of a long-run equilibrium rather than a short-term disequilibrium caused by temporary shocks and, as Marston (1985) points out, "*If unemployment is of equilibrium nature, any policy oriented to reduce regional disparities is useless since it cannot reduce unemployment anywhere for long*". Nevertheless, we cannot exclude that the strong persistence of regional unemployment differentials is caused by both structural problems in the economy and the inability of Italian regions to absorb specific shocks (on the demand or on the supply side).

A nonlinear time trend also characterizes unemployment data. The national unemployment rate (red line in Figure 5) shows a fall from 1996 (11.2%) to 2007 (6.1%); with the outbreak of the financial crisis and its extension to the productive economy in the subsequent years, it picked up reaching 12.7% in 2014. Both Northern and Southern provinces followed a similar time path, thus suggesting that common business cycles factors affect all the regions. However, there are relevant differences across provinces, thus indicating that regions may differ in their elasticity to common shocks. This feature is rather usual in regional unemployment studies. Thus, in order to obtain coefficients of the determinants that measure their impact on regional unemployment rates net of aggregate cyclical factors, these studies adopt one of two main approaches. The first one is to include time-period fixed effects in the model (Elhorst, 1995; Partridge and Rickman, 1997). However, this is a homogeneous approach since it assumes that the impact of common factors is the same across regions, while the usual finding in many applied settings is that some regions are more sensitive than others to aggregate fluctuations. The alternative approach is to take the difference between the regional and national unemployment rates as a way to appraise dispersion and factor out country-specific dynamics (Thirlwall, 1966; Blanchard et al., 1992; Decressin and Fatas, 1995). This 'factoring out' of aggregate cyclical factors has also a clear resemblance to the common factor approach proposed in



(a) North (b) South

FIGURE 5
Time trend of provincial unemployment rates: 1996-2014

Pesaran (2007), where common factors are modeled by cross-sectional averages of the variables at each point in time.

The presence of common cyclical factors is expected to generate significant cross-sectional correlation in the data ('strong' cross-section dependence). This hypothesis can be assessed using th CD test developed by Pesaran (2004, 2015). This test uses the pair-wise correlation coefficients between the time-series for each panel unit. The CD statistics computed on our sample of regional unemployment rates is highly significant, confirming the existence of cross-sectional dependence (Table 3). Applying the same test on the residuals of an AR(2) model (to accommodate for serial correlation), we obtain a CD value of 87.4 still highly significant. Moreover, the result of the estimation of the exponent of cross-section dependence (introduced by Bailey et al., 2016) provides a clear evidence in favor of strong cross-sectional dependence (the parameter is equal to 1). This result has relevant implications in terms of econometric modeling and suggests that a CCEP approach or a PS-ANOVA approach is preferable to a SAR model.

	Test statistic	p-value
Without control for serial correlation	180.7	0.00
With control for serial correlation	87.4	0.000

TABLE 3
Cross-sectional dependence test

Nevertheless, significant cross-sectional correlation in the data could also be generated by spatial autocorrelation ('weak' cross-section dependence). From a theoretical point of view, spatial autocorrelation in regional unemployment rates can be justified on the basis of a framework which builds on Blanchard et al. (1992) regional labor market model, including neighboring effects due to interregional trade, migration, and knowledge spillovers (Zeilstra and Elhorst, 2014). Starting from a steady state pattern of regional unemployment, a region-specific shock will not only affect the respective labor market, but spills over to neighboring regions. Given this interdependence, the induced changes of unemployment in neighboring areas may spill over again to adjacent labor markets, including the location where the shock originated. This implies that the unemployment rate of a particular region is affected not only by its own labor market characteristics, but also by the labor market performance of all other regions. Thus, in principle, we cannot exclude that potential sources of interaction between regions are both weak due to for example commuting flows, and strong due to common factors. A joint modeling of weak and strong cross-sectional dependence is therefore necessary.

Finally, an important issue is to assess the stationarity of regional unemployment data. To this end, we use panel unit root tests. The results of both the standard and the cross-sectionally demeaned Im et al. (2003) (IPS) tests do not allow us to reject the null hypothesis of a unit root in regional unemployment rates. However, the robust cross-sectional dependence test proposed by Pesaran (2007) clearly rejects the hypothesis of a unit root at all reasonable significance levels. Hence, these results give a strong indication regarding stationarity of the data once cross-sectional dependence is taken into account.

Deterministic component	Standard IPS	Cross-sectionally demeaned IPS	Robust against cross-sectional dependence IPS
None	0.625	-0.626	-1.462*
Drift	-0.930	-1.603	-2.387**
Drift and trend	-1.603	-1.603	-10.109**

TABLE 4
Panel unit root tests for regional unemployment rates

4.2 Explanatory variables

The unemployment rate can be considered as a reduced form function of a variety of factors affecting labor demand, supply and wages. According to the pioneering work of Partridge and Rickman (1997), these factors can be broadly categorized as disequilibrium factors (e.g., employment growth rates), and market equilibrium factors (e.g., industry and services shares, demographic variables and amenities). For the choice of the actual variables in these categories, we take into account the empirical regional unemployment literature. However, the set of our variables is limited by data availability. Table 5 reports simple descriptive statistics of these variables.

In order to account for regional disequilibrium labor market dynamics, the employment growth rate ($empgrowth_{i,t}$)) is included in the set of explanatory variables. Obviously, it is expected to have a negative effect on unemployment.

The other variables are proxies of equilibrium variables. First of all, differences in the industrial mix should impact the geographical distribution of unemployment. Provinces specializing in a declining economic sector, such as agriculture and industrial activity, might show higher structural unemployment rates than provinces specializing in services and construction. The share of employment in agriculture ($agri_{i,t}$), in industry activity ($ind_{i,t}$), in services ($serv_{i,t}$), and in construction ($cons_{i,t}$) over total provincial employment are proxies of the provincial economic structure.

Variable	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Std. dev.
unrate	1.33	4.78	7.71	9.23	12.50	32.72	5.65
empgrowth	-14.68	-1.54	0.40	0.37	2.27	13.91	3.21
agri	0.05	3.52	6.80	7.77	10.99	30.57	5.30
cons	3.59	6.71	7.75	7.83	8.77	14.68	1.64
serv	45.09	58.75	63.96	64.27	69.51	86.09	7.88
partrate	27.04	37.73	42.85	41.52	45.16	53.19	4.70
lpopdens	-3.49	-2.24	-1.75	-1.76	-1.32	0.98	0.77

Table 5
Summary statistics

The labor force participation rate ($partrate_{i,t}$), i.e. the ratio between total labor force and the working population (population aged between 16 and 65 years), is used as an indicator of labor supply. The expected sign of its coefficient is not unambiguous. On the one hand, factors determining low participation rates in a particular region also reflect relatively low investments in human capital and low commitment to working life, resulting in higher risks for people with these characteristics to become unemployed. On the other hand, a positive effect may occur if a faster growth of the labour force (i.e., young people) is not compensated by an as mush faster growth of new jobs (or vacancies).

As stated above, amenities are considered as a compensating differential for the higher probability of unemployment. Variables used to proxy for producer and consumer amenities are largely conditioned by the availability of data and we only included the log of population density, that is the ratio between total population and the area surface of the province ($lpopdens_{i,t}$), as a proxy for urbanization following López-Bazo et al. (2005) and Cracolici et al. (2007). Also the expected effect of this variable is not unambiguous. On the one hand, a high urban density may increase the efficiency of matching workers to jobs (unemployed persons have more employment opportunities and the matching process is expected to be more efficient in urban areas), but on the other hand, it may increase the time spent by workers to collect information about vacancies on the job market.

Obviously, population density cannot capture all kinds of regional amenities explaining regional differences in unemployment rates. In addition, there are many other equilibrium and disequilibrium variables affecting regional unemployment differentials. These include workers migration and commuting, which are relevant in this type of spatial context, the age structure of the population and human capital variables. This means that there is a huge amount of spatial *unobserved heterogeneity* in modeling regional unemployment rates. The inclusion of a *spatial trend* in the model is a way to clean up the residuals. In other words, the spatial trend captures all time-invariant region-specific unobservable factors, simultaneously allowing these factors to be freely correlated with observable determinants of regional unemployment rates. The alternative approach used in the literature consists of introducing spatial fixed effects in the model to measure time-invariant unobservable equilibrium effects.[2]

[2]It is worth noticing that the spatial distribution of the fixed effects parameters estimated with different fixed effects models is far from being random. Rather, the maps reported in the Appendix clearly depict a smooth spatial distribution. Therefore, there is no reason to use N degrees of freedom to control for spatial heterogeneity, and a smooth interaction between latitude and longitude (the smooth spatial trend) is valid option.

4.3 Econometric results

4.3.1 Model selection and diagnostrics

We use the data described above to compare the performance of the spatio-temporal PS-SAR model against different competing parametric and semiparametric models in terms of model fitting and residual diagnostics, focusing on the test for cross-sectional dependence in the residuals (see Table 6 for the list of models considered.

A distance based spatial weights matrix (W) has been used to estimate spatial lag models. A general element of this matrix, w_{ij}, represents a combination of a binary spatial weight based on the critical cut-off criterion and a decreasing function of pure geographical distance, namely the inverse distance function, d_{ij}^{-1}:

$$\nu_{ij} = \begin{cases} d_{ij}^{-1} / \sum_{j \neq i} d_{ij}^{-1} & if \quad 0 < d_{ij} < d^* \\ 0 & if \quad i = j \quad or \quad if \quad d_{ij} > d^* \end{cases}$$

where d_{ij} is the great-circle distance between the centroids of provinces i and j.[3] The selected cut-off distance (d^*) corresponds to the minimum distance that allows all provinces to have at least one neighbor.

The most restricted specifications are the *parametric* a-spatial linear models with spatial fixed effects (**Model 1**) and with spatial and time fixed effects (**Model 2**), estimated using the standard fixed effects estimator. Clearly, they cannot capture the presence of cross-sectionally correlated error terms, either strong or weak, as indicated by the results of the CD test. Including interactions between individual fixed effects and cross-sectional averages of the data (**Model 5**) (that is using the pooled common correlated effects - CCEP - estimator proposed by Pesaran, 2006), the evidence of cross-dependence disappears (Table 7). These results strongly confirm the existing literature. However, using the CCEP method, we cannot disentangle strong and weak cross-dependence, that is we cannot assess the presence of spatial interaction (network) effects net of the effect of strong cross-sectional dependence. Moreover, using the CCEP estimator, the unobserved common factors are treated as unknown parameters but the factor loadings are interactive individual effects that induce an incidental parameter problem since their number grows with N. In other words, the CCEP estimator requires a huge number of degrees of freedom (*edf*) which is reflected in a high BIC value.[4]

[3]Geographic distance has frictional effects on labor market activity. Workers prefer to find a job in their closer environment because commuting and moving entail monetary and psychological costs. Therefore, we use great circle distances between centroids of provinces to define the entries of the spatial weights matrix.

[4]The EDF's values include the parametric (fixed part in mixed model) and non-parametric (random part in mixed model) for each additive covariate. Therefore, they correspond to the total value of estimated degrees of freedom for each variable.

Linear parametric panel data models	
Model 1	Fixed spatial effects model (FE)
Model 2	Fixed spatial and time effects model (FE/TE)
Model 3	SAR model with fixed spatial effects (SAR-FE)
Model 4	SAR model with fixed spatial and time effects (SAR-FE/TE)
Model 5	Model with unobserved common effects (CCEP)
Model 6	SAR model with unobserved common effects (SAR-CCEP)
Spatio-temporal penalized spline (PS) ANOVA models	
Model 7	Spatio-Temporal model with linear terms (PS-ANOVA-Linear)
Model 8	Spatio-Temporal SAR model with linear terms (PS-SAR-ANOVA-Linear)
Model 9	Spatio-Temporal model with nonlinear terms (PS-ANOVA-Nonlinear)
Model 10	Spatio-Temporal SAR model with nonlinear terms (PS-SAR-ANOVA-Nonlinear)

TABLE 6
List of models

Model	CD test	p-val	rho	LR test	p-val	EDF	σ^2	BIC
Parametric models								
Model 1	141.80	(0.00)				109	5.99	4225.3
Model 2	2.39	(0.02)				127	3.13	3074.8
Model 3	-0.58	(0.56)	0.54	31.6	(0.00)	110	3.52	3193.3
Model 4	0.08	(0.94)	0.25	11.02	(0.00)	128	2.87	2905.4
Model 5	-0.72	(0.47)				830	1.01	5230.5
Model 6	-0.88	(0.38)	0.08	3.48	(0.00)	849	1.01	5355.7
Spatio-temporal models								
Model 7	0.20	(0.84)				94.7	3.09	2837.4
Model 8	0.25	(0.80)	0.05	45.7	(0.00)	94.6	3.09	2837.8
Model 9	0.15	(0.88)				126.6	2.65	2744.5
Model 10	0.17	(0.87)	0.07	45.32	(0.00)	125.7	2.65	2738.1

TABLE 7
Model selection and diagnostics

On the other hand, with the spatial lag fixed effects models (SAR-FE and SAR-FE/TE; **Models 3 and 4**) widely used in the recent applied spatial panel data literature (Elhorst, 2014), we are implicitly assuming that only weak cross-dependence (i.e. spatial dependence) exists. The CD test for the residuals of models 3 and 4 reveals that the null cannot be rejected, but the ρ parameter is quite high, suggesting that likely the spatial lag term has captured all cross-dependence (both strong and weak). Combining the SAR-FE specification and the CCEP model (**Model 6**), that is estimating a linear spatial lag model with fixed effects and cross-sectional averaged of dependent and independent variables, in line with recent contributions (Bai and Li, 2015; Shi and Lee, 2016; Bailey et al., 2016; Vega and Elhorst, 2016), we allow for both strong and weak cross-dependence. Indeed, the value of the ρ parameter (0.08) now appears much lower than before, while still remaining statistically significant. This value seems to be much more plausible than those estimated with SAR models, since we believe that most of the cross-sectional dependence in local labor market performances is due to unobserved time-related factors which influence all regions, rather than to unobserved idiosyncratic shocks which propagate to all regions with a distance decay mechanism driven by network relationships. Obviously, model 6 shares with model 5 the problem of the large number of *edf* (and, thus, high BIC).

The results of the CD test on the residuals confirm that the smooth spatio-temporal trend (**Models 7-10**) is able to capture the unobserved cross-sectional dependence and, thus, it represents a valid alternative to the inclusion of cross-sectional averages in the model.[5] With respect to fixed effects models and to CCEP models, the PS-ANOVA models are less affected by the incidental (nuisance) parameter problem as a result of the effective penalizing estimation procedure described in section 2. Indeed, the BIC values of PS-ANOVA models are lower than those computed for any other model. In absolute terms, Model 10 (i.e. the spatio-temporal ANOVA SAR model with nonlinear terms) shows the best performance with a BIC value of 2,738, lower than its linear counterpart (model 8), suggesting that the functional form of the relationship between the response variable (regional unemployment) and the covariates cannot be assumed to be linear. Moreover, the ρ parameter estimated with Model 10 (0.07) is statistically significant and very much close in magnitude to the one estimated with model 6, confirming the existence of some weak dependence net of the effect of common business cycle effects.

4.3.2 Estimation results

Table 8 reports the estimated β parameters of the linear terms included in models 1-8, along with the associated standard errors. Obviously, these parameters can be interpreted as marginal effects only in the case of non-spatial models (models 1, 2, 5 and 7), while the interpretation of the various SAR specifications (models 3, 4, 6 and 8) requires the computation of direct and indirect marginal effects, reported in Table 9. We already pointed out the evidence of a ρ parameter rather high from SAR-FE (0.54) and SAR-FE/TE (0.25) with respect to the values obtained with SAR-CCEP (0.08) and PS-SAR-ANOVA-Linear (0.05). Now, it is also important to discuss the consequences of these differences in terms of the magnitude of direct and indirect effects. In particular, in the SAR-FE model the indirect (spillover) effect of any variable is very close to the corresponding direct effect. This would imply that, if there is an idiosyncratic shock in a specific province (for example an increase in the employment growth rate, i.e. and increase in labor demand), this shock would have the same impact on this province (direct effect) and in the rest of the country (spillover effect). Of course, this is unreasonable, since we expect a spillover effect much lower than a direct effect. By including a time fixed effect in the model (SAR-FE/TE), the spillover effect turns out to be about one third of the direct effect, but still it is very high. Much more reasonable amounts of indirect effects emerge once we control for the common factor effects (strong cross-sectional correlation) through either the SAR-CCEP or the PS-SAR-ANOVA-Linear model.

[5]As described above, the ANOVA decomposition of the spatio-temporal trend includes three main effects (i.e. the smooth effect of latitude, longitude and time), as well as three second-order and one third-order interactions. In practice, however, a researcher may wish to select, for the sake of parsimony, some of these seven smooth functions. In our case, after some trials, we decided to exclude from models 7-10 the interaction between longitude and time and the third-order interaction because they turn to be redundant in the process of capturing spatial and temporal heterogeneity.

Model	*empgrowth*	ln *popdens*	*partrate*	*agri*	*cons*	*serv*
$\hat{\beta}$ and sd($\hat{\beta}$) (between parenthesis) for linear terms						
Model 1	-0.24	5.67	0.58	-0.13	-0.97	-0.10
	(0.02)	(2.08)	(0.04)	(0.04)	(0.07)	(0.03)
Model 2	-0.17	16.16	0.52	0.00	-0.17	0.09
	(0.01)	(1.70)	(0.03)	(0.04)	(0.05)	(0.03)
Model 3	-0.16	2.93	0.41	-0.08	-0.60	-0.07
	(0.01)	(1.55)	(0.03)	(0.03)	(0.05)	(0.03)
Model 4	-0.16	12.78	0.47	-0.03	-0.19	0.07
	(0.01)	(1.60)	(0.03)	(0.03)	(0.05)	(0.03)
Model 5	-0.19	35.37	0.62	0.07	0.07	-0.03
	(0.01)	(9.78)	(0.04)	(0.06)	(0.08)	(0.05)
Model 6	-0.19	33.87	0.62	0.06	0.06	-0.03
	(0.01)	(7.42)	(0.03)	(0.05)	(0.06)	(0.04)
Model 7	-0.12	0.26	0.22	0.04	-0.08	0.05
	(0.01)	(0.14)	(0.03)	(0.02)	(0.04)	(0.01)
Model 8	-0.12	0.25	0.22	0.04	-0.08	0.05
	(0.01)	(0.14)	(0.03)	(0.02)	(0.04)	(0.01)
EDF for non-linear terms						
Model 9	4.69	4.71	6.35	4.65	5.16	5.81
Model 10	4.67	4.65	6.35	4.66	5.19	5.80

TABLE 8
Estimation results

The results suggest that there is a clear explanation of unemployment differentials in terms of spatial equilibrium and disequilibrium factors. In all non-spatial models, higher employment growth rates lowers provincial unemployment rates, as suggested by the disequilibrium approach. The magnitude of the estimated β parameter associated to the variable *empgrowth* ranges between -0.16 and -0.24. Both average direct and indirect marginal effects of this variable computed for the fixed effects SAR models (3 and 4) have a negative sign and are strongly significant indicating that an increase in the employment growth rate in one region reduces not only the unemployment rate of that region, but also the unemployment rate of the other regions with a distance decay effect. However, as observed above, spatial spillovers (indirect effects) appears much lower when we control for time-related factors, especially with models 6 (SAR-CCEP) and 8 (PS-SAR ANOVA linear).

Regional unemployment rates are positively influenced by labor force participation rates. Both direct and indirect marginal effects of this variable computed for the four SAR models have a positive sign, but again the indirect effects are much lower in those models which control for common time effects. The positive impact of the participation rate along with the negative effect of the employment growth rate suggests, in particular, that labor market conditions in the South have worsened as a result of a faster growth of the labor force (i.e., young people) in contrast to a lower growth of new jobs (or vacancies). Increasing population density exerts detrimental effects on local labor market performances; the parameters associated to the variable ln *popdens*, as well as its direct and indirect marginal effects, vary greatly among the different model specifications. Finally, the coefficients of the regressors related to the structure of the economy are not sable across the various model specifications. Furthermore, most of them loose significance once we control for the effect of common factors.

Model		*empgr.*	ln *popd.*	*partr.*	*agri*	*cons*	*serv*
Model 3	Direct	-0.184**	3.277*	0.459**	-0.087*	-0.674**	-0.081**
	Indirect	-0.178**	3.164*	0.443**	-0.084*	-0.651**	-0.078**
	Total	-0.361**	6.440*	0.901**	-0.170*	-1.325**	-0.159**
Model 4	Direct	-0.159**	13.028**	0.479**	-0.031	-0.194**	0.071**
	Indirect	-0.049**	3.994**	0.147**	-0.009	-0.059**	0.022*
	Total	-0.208**	17.022**	0.626**	-0.040	-0.253**	0.093**
Model 6	Direct	-0.191**	33.94**	0.620**	0.063	0.061	-0.031
	Indirect	-0.017**	2.99*	0.055**	0.006	0.005	-0.003
	Total	-0.208**	36.93**	0.675**	0.068	0.066	-0.034
Model 8	Direct	-0.118**	0.247	0.219**	0.043*	-0.079	0.053**
	Indirect	-0.006**	0.013	0.011**	0.002*	-0.004	0.003**
	Total	-0.124**	0.260	0.230**	0.045*	-0.083	0.055**

Table 9
Direct, indirect and total marginal effects in SAR models. ** (*) indicates significance at 1% (5%)

Table 8 also reports the *edf* for the nonlinear terms included in models 9 and 10, a broad measure of nonlinearity (an *edf* equal to 1 indicates linearity, while a value higher than 1 indicates nonlinearity). Focusing on model 10, that is the one with the best performance, we report the plots of direct and indirect effects of the smooth terms in figure 6. Starting from direct effects, nonlinearities in the relationship between regional unemployment rates and the covariates are clearly detected, although most of the figures display monotonic relationships. Specifically, an increase in the employment growth rate within a province is negatively associated with a reduction in the unemployment rate in the same province, but the direct effect vanishes for employment growth rates higher than 5%. The positive direct effect of the participation rate is particularly strong and highly significant at low levels of the variable, as indicated by the narrow confidence band. This means that, starting from very low levels of the participation rate (a status which characterizes Southern provinces), a faster growth of the labor force (due to the enter of young or previously discouraged people) is not compensated by an as much faster growth of new jobs. After a certain threshold, the detrimental effect of the participation rate decreases, probably because after such a threshold people entering the labor force have a lower risk to become unemployed.

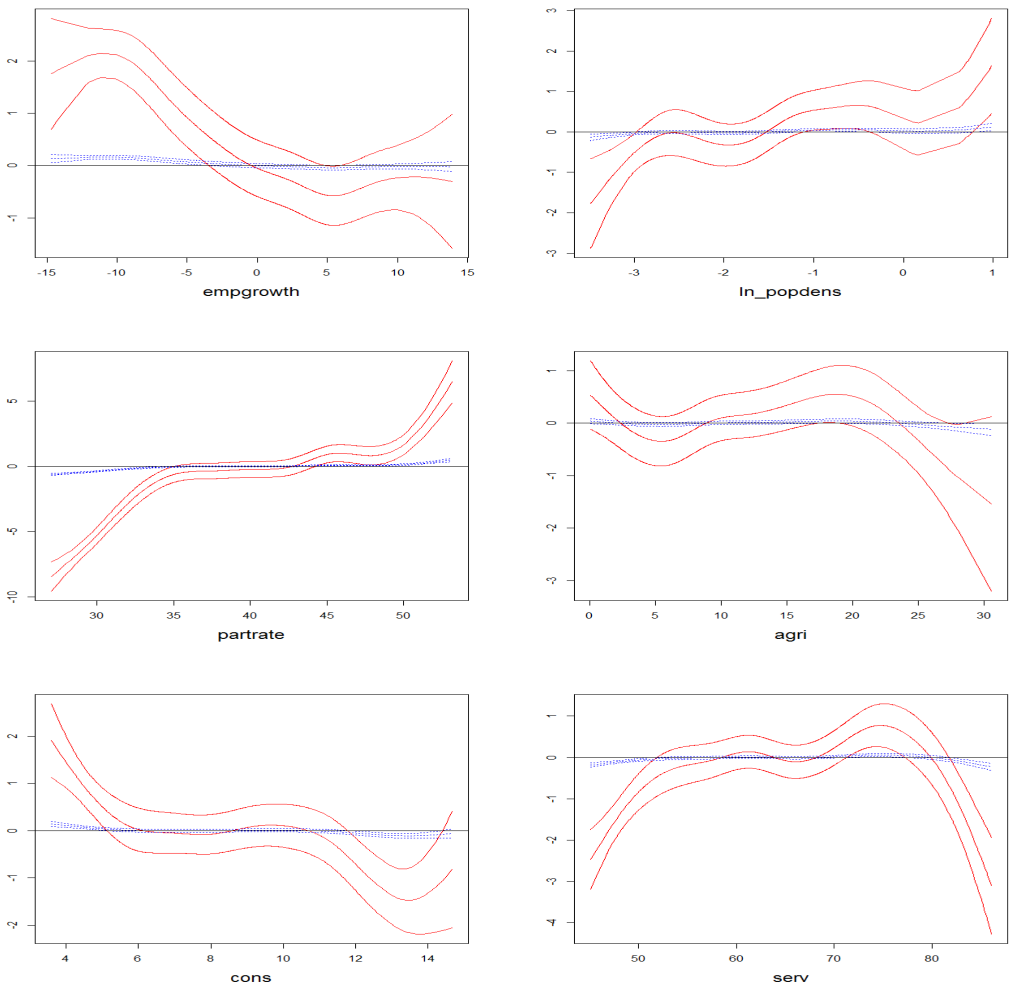


FIGURE 6
Direct (solid) and Indirect (dashed) functions for each covariate in Spatio-Temporal ANOVA SAR with nonlinear terms (Model 10). The intervals correspond to 95% of confidence

The level of uncertainty in the relationship between population density and unemployment rates is rather high (the confidence band contains the zero horizontal line for a large range of values of this explanatory variable) and does not allow us to make any ultimate statement. Similar considerations hold for the effect of *agri*. The direct effect of *serv* appears positive for low levels and negative for high levels of the variable, while the direct effect of *cons* turns out to be negative for low and high levels of the variable, confirming that provinces specialized in construction exhibit lower unemployment than provinces with a different sectoral structure. As expected, indirect effects are always much lower than direct effects; nevertheless, these effects remain statistically significant since the point-wise confidence interval crosses the zero horizontal line.

Finally, figures 7 and 8 report the yearly estimated spatial trend maps, and the regional specific time trends, respectively, from model 10. The map plots clearly show that, even after controlling for the role of equilibria and disequilibria factors, the spatial distribution of expected regional unemployment rates remains persistently characterized by a strong North-South spatial trend; the estimated regional specific temporal trends also confirm the presence of common business cycles factors heterogeneously affecting all the regions.

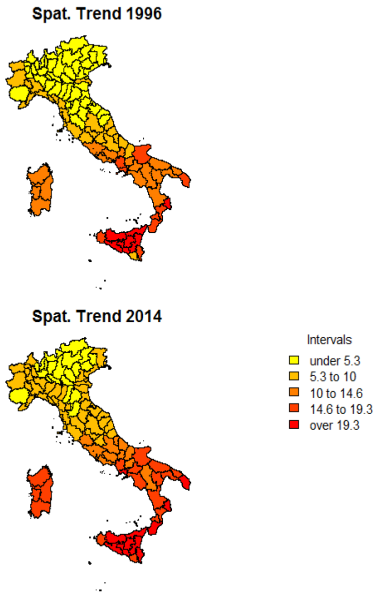


FIGURE 7
Spatial trends of unrate in 1996 and 2014 for Spatio-Temporal ANOVA SAR Model with nonlinear terms (Model 10)

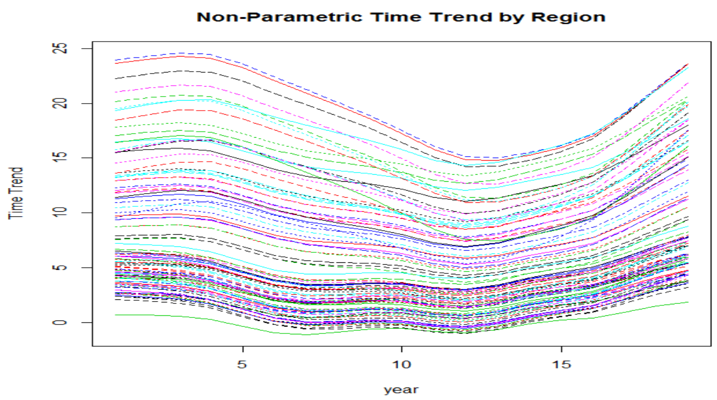


FIGURE 8
Regional time trends estimated by the Spatio-Temporal ANOVA SAR with nonlinear terms (Model 10)

5 Conclusions

Many large spatial panel data sets used in cross-regional and cross-country empirical analyses exhibit cross-sectional dependence which may arise from both spatial interactions (spatial spillovers) or common factors (aggregate shocks). Spatial spillovers are the results of local interactions and, thus, are classified as weak dependence effects; while common factors represent latent economic-wide technological and/or demand shocks, heterogeneously affecting all regions' dynamics and, thus, are classified as strong dependence effects.

Traditionally, each type of effect has been analyzed separately in the literature by the so-called 'factor' approach and 'spatial econometric' approach, respectively. Recently, however, some authors have proposed a joint modeling of both types to determine whether one or both of these effects are present (Bailey et al., 2016; Vega and Elhorst, 2016; Bai and Li, 2015; Shi and Lee, 2016). Specifically, Bailey et al. (2016) and Vega and Elhorst (2016) follow Pesaran (2006) in using cross-sectional averages of the observed variables as proxies for common factors.

In the present paper, we have shown that, the spatio-temporal trend can be interpreted as an alternative to cross-sectional averages of the observations to capture the heterogeneous effect of unobserved common factors. Specifically, the ANOVA decomposition of the spatio-temporal trend in a spatial trend, a time trend and second- and third-order interactions works effectively to control for both unobserved spatial heterogeneity and unobserved common factors. Thus, the inclusion of the ANOVA decomposition of the spatio-temporal trend helps interpret the evidence of significance spatial spillovers as weak cross-dependence net of the effect of common effects (strong dependence). In this sense, our proposed framework can be regarded as a valid alternative to parametric approaches which considers a jointly modeling of both spatial interaction effects and common-shocks effects. We have implemented this new framework using real data on unemployment rates in Italy. The results clearly suggest that the PS-SAR model with the ANOVA spatio-temporal trend outperforms parametric panel data SAR models with common effects.

As a concluding remark, it is worth noticing that regional unemployment rates, like many other regional and national economic variables, are typically characterized by strong persistence over time. Thus, a control for serial correlation is definitely needed. Our future research agenda will address this challenge by extending the PS-SAR model with the ANOVA spatio-temporal trend to a dynamic specification.

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