Monetary Policy and Stock-Price Dynamics in a DSGE Framework

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Abstract

This paper analyzes a small structural general equilibrium model in which stock prices have direct wealth effects on real activity, to assess what specific role (if any) they play in driving Optimal Monetary Policy.

It is shown that in the face of a given swing in stock prices, the optimal policy response of a Central Bank pursuing price stability by controlling a short-term interest rate, depends in principle on the shocks underlying the observed dynamics. If the driving forces are supply shocks like one to technology or the marginal disutility of labor then the real effects of stock prices do not require a dedicated response and the optimal policy-rate dynamics is the same as the one emerging from the standard Representative-Agent (RA) set up, in which stock prices are disregarded by definition, being ineffective. On the contrary, if the driving force is a demand shock like a shock to the marginal utility of consumption, government expenditure or a fad that affects the equity premium, then positive stock-wealth effects require a dedicated response on the part of the Central Bank, more and above the reaction which would result as optimal in the Representative-Agent set up.

Moreover, augmenting a standard Taylor Rule to include a reaction to stock-price growth reduces the monetary policy loss up to 65%, while reacting to deviations of real stock prices from a target level only yields minor gains, while making the economy more vulnerable to endogenous instability.

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1 Introduction

Over the past two decades, with inflation successfully kept under control after the tumultuous 1970’s, one of the major issues that Central Bankers had to learn to cope with was financial stability. The events of the past fifteen years in the United States, with the crash of the stock markets leading an economic and financial U-turn and the first serious recession that the U.S. experienced since 1982, gave new scope for a debate in the literature to determine what should be the best monetary policy response to the stock market dynamics.¹

The issue was analyzed in a variety of set-ups, both theoretical and empirical, and the consequent debate is still very controversial, under many respects. The main positions can be roughly represented by two main contributors, which engaged in a very lively debate over the past few years: Cecchetti et Al. (2000), (2002), (2003) though emphasizing the difference between targeting stock-price stability and reacting to stock-price misalignments, strongly recommend that a Central Bank that recognizes a bubble in the dynamics of the stock market react to it; Bernanke and Gertler (1999), (2001) on the other side, maintain that the only desirable reaction to stock prices is the one implicit in the response to expected inflation and output, and that a flexible inflation targeting is, in itself, enough to pursue also the aim of financial stability.²

The variety of models used and the differences in the conclusions drawn suggest the need of a simple structural framework that generalize the one usually employed for monetary policy analysis, with which to be compared.

This paper presents a cashless Dynamics Stochastic General Equilibrium (DSGE) framework for the analysis of Monetary Policy when the real economy is affected by the stock-price dynamics through wealth effects.

We move from the basic Dynamic New-Keynesian model widely used in the literature for monetary policy analysis, in which nominal rigidities assign a central role to monetary policy in driving output and inflation.³ The only but substantial extension to this baseline model proposed here is an explicit consideration and micro-foundation of stock-price dynamics and its effects on real activity, analyzing what are the implications for price stability and the role of monetary policy.

This extension is carried out by modifying the baseline model in two ways. The first one is by assuming that the firms in the monopolistic sector issue equity shares to the public, which then can choose to allocate their savings by either buying a riskless bond or a portfolio of private stocks. This assumption follows the structure of Lucas’ (1978) tree model, and allows for a micro-foundation of stock-price dynamics. Moreover, a stochastic, non-fundamental component of stock prices is also assumed, in order to capture the observed variations in the equity risk premium.

The second assumption modifying the baseline model concerns the demand-side of the model, which consists of an indefinite number of overlapping generations of finitely-lived consumers with no relevant bequest motives, along the lines traced by Yaari (1965) and Blanchard (1985). The demand-side hence takes the form of a stochastic “perpetual youth” model, for which a closed-form solution for aggregate consumption within a closed economy is derived by Chadha and Nolan (2003)

¹Mishkin and White (2002) highlight the difference between financial instability and stock market crashes, maintaining that the real concern of monetary policy makers should be the former, rather than the latter. The strong point they make is related to firms’ balance sheets conditions and seems weaker when it comes to the possible real effects through households’ wealth. Truth is, anyhow, that the stock market fragility is more often than not a highly sensitive indicator of financial instability, especially in periods of financial sophistication like the ones we live in.
³For models of this kind see, among the others, Rotemberg and Woodford (1997), (1999) and Yun (1996). For a discussion of the role of monetary aggregates for the analysis of monetary models see Woodford (1998). For a thorough analysis using this baseline model and a detailed discussion see Woodford (2003), Ch. 4.
This assumption assigns a crucial role to financial stability, since variations in the stock market wealth affect real activity through wealth-effects on private consumption.

Within this framework we analyze the role of stock prices in determining the optimal monetary policy to achieve price stability by comparing the implied Natural Rate of Interest with the one emerging from the benchmark case of infinitely-lived consumers, and discuss the implications for economic stability of augmenting a Taylor-type monetary policy rule to include an explicit reaction to stock prices.

The main results are twofold.

The first concerns the derivation of the optimal policy and its implementation. The real effects exerted by stock market wealth on consumption are shown to be not sufficient a condition for a Central Bank pursuing price stability to be required to take stock-price dynamics into account: the optimal response to a given swing in stock prices depends in fact on the structural shocks driving it. If the observed dynamics is driven by supply shocks like a shock to technology or the marginal disutility of labor, the optimal policy to achieve price stability is the same emerging from the standard Representative-Agent (henceforth RA) framework, in which financial wealth is ineffective, and does not require any dedicated consideration of the stock market performance. However, if the observed boom (or bust) in stock prices is driven by a demand shock like a shift in the marginal utility of private consumption, a shock to public expenditure or stochastic shock to the equity premium (for example driven by a fad), then the monetary policy makers pursuing price stability through the setting of a short-term interest rate are required to respond by moving their instrument in a more aggressive fashion, hence taking active account of the stock-price dynamics.

It is shown, moreover, that the implementation of this optimal policy requires a credible threat to vary the policy instrument in response to deviations of specific variables from their target. In this respect, and just as in the standard framework, the Taylor Principle ($\phi_{\pi}>1$) remains a sufficient condition for equilibrium determinacy. In contrast, threatening to respond actively to deviations of the stock-price level from a chosen target is shown to produce potential endogenous instability.

The second result concerns the macroeconomic implications of different simple rules within alternative monetary policy regimes. A standard Taylor-type rule is compared to a rule responding also to deviations in terms of the stock-price level (Gap-Rule), and the stock-price growth rate (Growth-Rule) from chosen equilibrium benchmarks. In particular, it is shown how moving from the standard rule to the Growth-Rule (and regardless of the regime adopted) implies gains in terms of overall stability as big as a 65% reduction in the monetary policy loss, which always entails a significant response to stock prices. In contrast, moving to a Gap-Rule only yields minor gains (if any), while making the system more vulnerable to sunspot fluctuations.

The remaining of the paper is structured as follows. Section 2 presents a DSGE model of the business cycle in which the micro-founded stock-price dynamics has real effects on consumption. Section 3 analyzes Optimal Monetary Policy, deriving the conditions for prices stability and discussing the issue of implementation. Section 4 analyzes the macroeconomic implications of adopting different instrument rules under alternative policy regimes. Section 5 finally summarizes and concludes.

2 A Structural Model with Wealth Effects.

2.1 The Demand-Side.

The demand-side of the economy is a discrete-time stochastic version of the perpetual youth model introduced by Blanchard (1985) and Yaari (1965).

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4Cardia (1991) derives an analogous solution within a small open economy framework.

5For other stochastic discrete-time versions of the perpetual youth model see Annicchiarico, Marini and Piergallini (2004), Cardia (1991), Chadha and Nolan (2001) and (2003), Piergallini (2004). For non-stochastic discrete-
Each period a constant-sized cohort of households is born, which faces a constant probability $\gamma$ of dying before the next period begins.\textsuperscript{6} To abstract from population growth the cohort size is set to $\gamma$.

Each household is assumed to have preferences over consumption and leisure, described by a log-utility function;\textsuperscript{7} moreover, we assume that such preferences are subject to aggregate, exogenous stochastic shocks shifting the marginal utility of consumption ($V_t \equiv \exp(\nu_t)$) and the marginal disutility of labor ($\mathcal{H}_t \equiv \exp(-\eta_t)$).

As suggested by Piergallini (2004), we use the suitable form of the equilibrium stochastic discount factor implied by the log-utility function to derive a closed-form solution for individual consumption.

Consumers demand consumption goods and two types of financial assets: state-contingent bonds and equity shares issued by the firms operating in the monopolistic sector, to which they also supply labor. Equilibrium of this side of the economy, along a state equation for consumption, also yields a pricing equation for the equity shares.

Consumers born in period $j$, therefore, seek to maximize their expected lifetime utility, discounted to account for impatience (as reflected by the intertemporal discount factor $\beta$) and uncertain lifetime (as reflected by the probability of survival across two subsequent periods, $(1-\gamma)$). To that aim, they choose a pattern for real consumption ($C_{j,t}$), hours worked ($N_{j,t}$) and financial-asset holdings. The financial assets holdings at the end of period $t$ consist of a set of contingent claims whose one-period ahead stochastic nominal payoff in period $t+1$ is $B_{j,t+1}$ and the relevant discount factor is $\mathcal{F}_{t,t+1}$, and a set of equity shares issued by each intermediate good-producing firm, $Z_{j,t+1}(i)$, whose real price at period $t$ is $Q_t(i)$.

At the beginning of each period, then, the sources of funds consist of the nominal disposable labor income ($W_tN_{j,t} - P_tT_{j,t}$) and the nominal financial wealth $\omega_{j,t}$, carried over from the previous period and defined as:

$$\omega_{j,t} \equiv \frac{1}{1-\gamma} \left[B_{j,t} + P_t \int_0^1 \left(Q_t(i) + D_t(i)\right) Z_{j,t}(i) di\right].$$

The financial wealth of an individual born at time $j$ includes therefore the nominal pay-off on the contingent claims and on the portfolio of equity shares, each of the latter paying a nominal dividend yield $P_tD_t(i)$ and being worth its own current nominal market value $P_tQ_t(i)$. Moreover, following Blanchard (1985), financial wealth carried over from the previous period also pays off the gross return on the insurance contract that redistributes among survived consumers (and in proportion to one’s current wealth) the financial wealth of the ones who died. Total personal financial wealth is therefore accrued by a factor of $\frac{1}{1-\gamma}$.

The optimization problem faced at time 0 by the $j$-periods-old representative consumer is therefore to maximize

$$E_0 \sum_{t=0}^\infty \beta^t (1-\gamma)^t \left[V_t \log C_{j,t} + \mathcal{H}_t \log(1-N_{j,t})\right]$$

subject to a sequence of budget constraints of the form:

$$P_tC_{j,t} + E_t\{\mathcal{F}_{t,t+1}B_{j,t+1}\} + P_t \int_0^1 Q_t(i) Z_{j,t+1}(i) di \leq W_tN_{j,t} - P_tT_{j,t} + \omega_{j,t},$$

\textsuperscript{6}We interpret the concepts of “living” and “dying” in the economic sense of being or not being operative in the markets, affecting economic activity through the decision-making process. In this perspective, the expected life-time $1/\gamma$ is interpreted as the effective decision horizon. See also Leith and Wren-Lewis (2000), Leith and von Thadden (2004) and Piergallini (2004).

\textsuperscript{7}The assumption that the utility function be logarithmic is necessary in order to retrieve time-invariant parameters characterizing the equilibrium conditions. See Smets and Wouters (2002) for a non-stochastic framework with CRRA utility.
where $\beta, \gamma \in [0, 1]$.

The first-order conditions for an optimum consist of the budget constraint (2) holding with equality, the intra-temporal optimality condition with respect to consumption and leisure

$$H_t C_{j,t} = \frac{W_t}{P_t} (1 - N_{j,t}) V_t,$$

(3)

and the inter-temporal conditions with respect to the two financial assets:

$$F_{t,t+1} = \beta \frac{U_c(C_{j,t+1}) V_{t+1}}{U_c(C_{j,t}) V_t} = \beta \frac{P_t C_{j,t}}{P_{t+1} C_{j,t+1}} \exp(\nu_{t+1} - \nu_t)$$

(4)

$$P_t Q_t(i) = E_t \left\{ F_{t,t+1} P_{t+1} \left[ Q_{t+1}(i) + D_{t+1}(i) \right] \right\}.$$  

(5)

From equation (4) we see that the equilibrium stochastic discount factor for one-period ahead nominal payoffs is the time-discounted stochastic growth in the marginal utility of consumption. For $k$-period ahead nominal payoffs will therefore be:

$$F_{t,t+k} = \beta^k \frac{U_c(C_{j,t+k}) V_{t+k}}{U_c(C_{j,t}) V_t} = \beta^k \frac{P_t C_{j,t}}{P_{t+k} C_{j,t+k}} \exp(\nu_{t+k} - \nu_t) = \prod_{i=0}^{k-1} F_{t+i,t+i+1}.$$  

(6)

The nominal gross return $(1 + r_t)$ on a safe one-period bond paying off one unit of currency in period $t + 1$ with probability 1 (whose price is therefore $E_t \{ F_{t,t+1} \}$) is defined by the following non-arbitrage condition:

$$(1 + r_t) E_t \{ F_{t,t+1} \} = 1.$$  

(7)

Taking conditional expectations on equation (4), using equation (7) and rearranging yields a stochastic Euler equation, defining the familiar forward-looking IS-type relation between individual consumption growth and the interest rate:

$$1 = (1 + r_t) \beta E_t \left\{ \frac{P_t C_{j,t} V_{t+1}}{P_{t+1} C_{j,t+1} V_t} \right\}.$$  

(8)

Equation (5), finally equates the nominal price of an equity share to its nominal expected payoff one period ahead, discounted by the stochastic factor $F_{t,t+1}$, and defines the stock-price dynamics.

One last requirement for an optimum of the problem above is that the possibility of Ponzi schemes be ruled out, that is the present value of last period’s financial wealth, discounted by the stochastic discount factor and conditional upon survival, shrink to zero as time diverges:

$$\lim_{k \to \infty} E_t \{ F_{t,t+k} (1 - \gamma)^k \omega_{j,t+k} \} = 0.$$  

(9)

Using equations (8) and (5), and recalling the definition of financial wealth (1), the equilibrium budget constraint (2) can be given the form of the following stochastic difference equation in the financial wealth $\omega_{j,t}$:

$$P_t C_{j,t} + E_t \left\{ F_{t,t+1} (1 - \gamma) \omega_{j,t+1} \right\} = W_t N_{j,t} - P_t T_{j,t} + \omega_{j,t}.$$  

(10)

Let’s now define the human wealth for cohort $j$ at time $t$ ($h_{j,t}$) as the expected stream of future disposable labor income, discounted by the stochastic discount factor and conditional upon survival:

$$h_{j,t} \equiv E_t \left\{ \sum_{k=0}^{\infty} F_{t,t+k} (1 - \gamma)^k (W_{t+k} N_{j,t+k} - P_{t+k} T_{j,t+k}) \right\}.$$  

(11)
As described in Piergallini (2004), using the equilibrium stochastic discount factor (6), the No-Ponzi-Game condition (9) and the definition of human wealth (11), allows a stochastic difference equation in the financial wealth like equation (10) to be solved forward, and to retrieve the equation that describes nominal individual consumption as a linear function of total nominal financial and human wealth:

\[ P_t C_{j,t} = \frac{1}{\Sigma_t} (\omega_{j,t} + h_{j,t}), \tag{12} \]

where \( \Sigma_t \equiv E_t \{ \sum_{k=0}^{\infty} \beta^k (1 - \gamma)^k \exp(\nu_{t+k} - \nu_t) \} \) is the reciprocal of the time-varying propensity to consume out of financial and human wealth, which is common across cohorts (being a function of the aggregate preference shocks).

A current positive innovation in the preference shock, therefore, by reducing the present value of future stochastic payoffs, has the effect of increasing the current propensity to consume out of wealth.

**2.1.1 Aggregation across Cohorts**

The aggregate value of the variables at stake is simply computed as a weighted average of the corresponding generation-specific counterpart, where the weights are given by the cohort sizes:

\[ X_t \equiv \sum_{j=-\infty}^{t} n_{j,t} X_{j,t} = \sum_{j=-\infty}^{t} \gamma (1 - \gamma)^{t-j} X_{j,t}, \tag{13} \]

for all \( X = C, N, B, T, h, Z(i) \).

The solution of the consumers’ problem provides three relevant equilibrium conditions specific to each generic cohort \( j \): the static labor supply of equation (3), the budget constraint holding with equality (equation (2)) and the relation linking personal consumption to total personal wealth, equation (12).

Note that these equilibrium conditions are linear in the cohort-specific variables. As a consequence, applying the aggregator (13) to such conditions yields a set of aggregate relations identical in the functional form to their generation-specific counterparts:

\[ \mathcal{H}_t C_t = \frac{W_t}{P_t} (1 - N_t) V_t, \tag{14} \]

\[ P_t C_t + B_t + P_t \int_0^1 Q_t(i) Z_t(i) di = W_t N_t - P_t T_t + \omega_t \tag{15} \]

\[ P_t C_t = \frac{1}{\Sigma_t} (\omega_t + h_t), \tag{16} \]

in which \( C_t, N_t, B_t, T_t, h_t \) and \( Z_t(i) \) fulfill equation (13).

Since the aggregate value of the gross return on the insurance contract must be 1 (it only has redistributive effects), aggregate financial wealth is defined as:

\[ \omega_t \equiv \left[ B_t + P_t \int_0^1 \left( Q_t(i) + D_t(i) \right) Z_t(i) di \right]. \tag{17} \]

Like its generation-specific counterpart, the aggregate budget constraint (15) can be given the form of a stochastic difference equation in aggregate wealth:

\[ P_t C_t + E_t \{ F_{t+1} + \omega_{t+1} \} = W_t N_t - P_t T_t + \omega_t. \tag{18} \]
Finally, the above equation (18) and equation (16) form a system whose solution is the equation describing the dynamic path of aggregate consumption, in which the first term represents the financial wealth effects, which fade out as the probability of exiting the market (γ) goes to zero:

\[(\Sigma_t - 1)P_tC_t = \gamma E_t\{F_{t,t+1}\omega_{t+1}\} + (1 - \gamma)E_t\{F_{t,t+1}\Sigma_{t+1}P_{t+1}C_{t+1}\}. \tag{19}\]

### 2.1.2 The Government and the Equilibrium

To derive the ultimate relations that describe the demand side of the economy, we finally need to impose that all markets clear.

Following Gali (2003), we assume a public sector which consumes a fraction \(\omega_t\) of total output, \((G_t = \omega_tY_t)\) financed entirely through lump-sum taxation to the households \((G_t = T_t)\). To show that our implications are not dependent on this choice and the implied intergenerational transfers, however, in the Appendix we analyze the distortionary case of proportional labor-income taxation \((T_t = \tau_t\frac{W_t}{Y_t}N_t)\).

In equilibrium, therefore, the net supply of state-contingent bonds is nil \((B_t = 0)\), while the equilibrium aggregate stock of outstanding equity for each intermediate good-producing firm must equal the corresponding total amount of issued shares, normalized to 1 \((Z_t(i) = 1\) for all \(i \in [0,1]\)).

Finally, let’s define total real dividend payments and the aggregate real stock-price index as the simple integration over the continuum of firms:

\[D_t \equiv \int_0^1 D_t(i) \, di \quad \text{and} \quad Q_t \equiv \int_0^1 Q_t(i) \, di. \tag{20}\]

Note that in equilibrium, given the pricing equation (5), the present discounted nominal value of future financial wealth, \(E_t\{F_{t,t+1}\omega_{t+1}\}\), is sufficiently represented by the current level of the nominal stock-price index:

\[E_t\{F_{t,t+1}\omega_{t+1}\} = E_t\left\{F_{t,t+1}\left[\frac{1}{1 - \gamma} + \int_0^1 (Q_{t+1}(i) + D_{t+1}(i)) \, di\right]\right\} = \int_0^1 E_t\{F_{t,t+1}P_{t+1}(Q_{t+1}(i) + D_{t+1}(i)) \, di\} = \int_0^1 P_tQ_t(i) \, di = P_tQ_t. \tag{21}\]

As a consequence of the above conditions, and given also condition (20), the demand-side of the economy is summarized by the following aggregate resource constraints

\[Y_t = C_t + G_t = C_t + \omega_tY_t \tag{22}\]

\[P_tY_t = N_tW_t + P_tD_t, \tag{23}\]

the labor supply

\[C_t = \frac{W_t}{P_t}(1 - N_t)\exp(\nu_t + \eta_t), \tag{24}\]

and the two aggregate Euler equations

\[(\Sigma_t - 1)C_t = \gamma Q_t + (1 - \gamma)E_t\{F_{t,t+1}\Pi_{t+1}\Sigma_{t+1}C_{t+1}\} \tag{25}\]

\[Q_t = E_t\{F_{t,t+1}\Pi_{t+1}\left(Q_{t+1} + D_{t+1}\right)\}. \tag{26}\]

\(^9\)For details refer to the Appendix, or to Piergallini (2004) for a framework with no preference shocks.

\(^{10}\)For the sake of precision, we should point out that equation (2?) actually defines the aggregate real market capitalization. However, given the assumptions regarding the mass of wholesalers and the total outstanding shares, both normalized to 1, the aggregate market capitalization is also equivalent to the aggregate stock-price index.
Equation (25) defines the dynamic path of aggregate consumption, in which an explicit and determinant role is played by the dynamics of stock prices. The latter is defined by equation (26), which is a standard pricing equation micro-founded on the consumers’ optimal behavior and derives from the aggregation across firms of equation (5).

Using $E[x y] = E[x]E[y] + \text{cov}[x, y]$, moreover, we can re-write equation (26) as

$$Q_t = E_t \{ F_{t+1} \} E_t \{ \Pi_{t+1} \left[ Q_{t+1} + D_{t+1} \right] \} - Q_t \mathcal{E}_t,$$

(27)

where $\mathcal{E}_t$ in the last term accounts for the risk implied by the covariance between the stochastic discount factor and the nominal gross rate of return on stocks,\(^{11}\) and generates a positive equity risk premium:

$$E_t \{ \Pi_{t+1} \left[ Q_{t+1} + D_{t+1} \right] \} - (1 + r_t) = (1 + r_t) \mathcal{E}_t,$$

(28)

Here we assume an exogenous stochastic component of the equity premium, to account for the observed average premium (in excess relative to the one implied by the log-utility) and fluctuations around it caused by non-fundamental factors (hence not directly linkable to the structure of the model, like fads or “irrational” exuberance):\(^{12}\)

$$\mathcal{E}_t = \mathcal{E} \exp(e_t).$$

(29)

Finally, note that the benchmark set-up of infinitely-lived consumers is a special case of the one discussed here, and corresponds to a zero-probability of death, $\gamma = 0$. In this case, in fact, equation (25) loses the term related to stock prices and collapses to the usual Euler equation for consumption, relating real aggregate consumption only to the long-run real interest rate:\(^{13}\)

$$(\Sigma_t - 1)C_t = E_t \{ F_{t+1} \Pi_{t+1} \Sigma_{t+1} C_{t+1} \}.$$

2.1.3 Steady State and Linearization.

In the long-run, the system converges to a non-stochastic zero-inflation steady state, in which the demand-side defines the following set of equilibrium relations:\(^{14}\)

$$\frac{(1 + r)^{-1}}{1 + \psi} = \beta,$$

(30)

$$\frac{D}{(1 + r)Q} = 1 + \mathcal{E} - \tilde{\beta},$$

(31)

$$\frac{Y}{D} = \frac{Y}{1 + \mathcal{E} - \tilde{\beta} Q},$$

(32)

where, as shown in the Appendix, we defined $\psi \equiv \gamma \frac{1 - \beta(1 - \gamma)}{\beta(1 - \gamma)} \frac{\omega}{PC}$.

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\(^{11}\)The term $\mathcal{E}_t$ is properly defined as the negative covariance between the discount factor and the rate of return on stocks, given the positive correlation between consumption and dividends.

\(^{12}\)See Smets and Wouters (2003) for an analogous ad hoc modelling choice.

\(^{13}\)In this case, moreover, equation (27) becomes completely redundant and can be disregarded, since any explicit dynamics of the nominal stock-price index does not affect in any way whatsoever not only the equilibrium level of real output but also the conditions for a determinate equilibrium of the macroeconomic system.

\(^{14}\)Note that in the long-run, the stochastic discount factor for one-period ahead stochastic payoffs converges to $(1 + r)^{-1}$, as implied by equation (7).
Log-linearization around such a steady state yields the following relations (refer to the Appendix for details):\(^{15}\)

\[ y_t = c_t + g_t \]  
\[ w_t - p_t = c_t + \varphi n_t - (\nu_t + \eta_t) \]  
\[ c_t = \frac{1}{1 + \psi} E_t c_{t+1} + \frac{\psi}{1 + \psi} q_t - \frac{1}{1 + \psi} (r_t - E_t \pi_{t+1} - \bar{\rho}) - (1 + \psi) E_t \Delta \nu_{t+1} \]  
\[ q_t = \frac{\bar{\beta}}{1 + \bar{\epsilon}} E_t q_{t+1} + \frac{1 + \bar{\epsilon} - \bar{\beta}}{1 + \bar{\epsilon}} E_t d_{t+1} - (r_t - E_t \pi_{t+1} - \bar{\rho}) - \frac{\bar{\epsilon}}{1 + \bar{\epsilon}} c_t \]  
\[ d_t = \frac{Y}{D} y_t - \frac{W N}{P D} (n_t + w_t - p_t), \]

in which \( \varphi \equiv \frac{N}{\bar{\beta}(1 - \gamma)^{\bar{\rho}_{\varphi}}} \) is the inverse of the steady-state elasticity of labor supply and let \( g_t \equiv -\log \left( \frac{1 - \xi_t}{1 - \bar{\xi}} \right) \) and \( \psi_{\nu} \equiv \frac{\psi(1 - \gamma)^{\rho_{\nu}}}{(1 + \nu)(1 - \gamma)^{\rho_{\nu}}}. \)

Equation (35) is the linear approximation of the Euler equation for consumption. Note that a positive probability of exiting the market in the next period (\( \gamma, \psi > 0 \)) affects the degree of smoothing in the inter-temporal path of aggregate consumption. The uncertainty regarding their existence in the next periods implies that the agents cannot fully spread over time the effects of current or expected future shocks, which then affect current consumption more than in the benchmark case. This makes the dynamics of aggregate financial wealth relevant for current aggregate consumption and the transmission of real and monetary shocks.

2.2 The Supply-Side and Inflation Dynamics.

The supply-side of the economy consists of two sectors of infinitely-lived agents: a retail sector operating in perfect competition to produce the final consumption good and a wholesale sector hiring labor from the households to produce a continuum of differentiated intermediate goods.

In the retail sector the final consumption good \( Y_t \) is produced out of the intermediate goods through a CRS technology,

\[ Y_t = \int_0^1 Y_t(i) \frac{1}{\epsilon} di, \]

where \( \epsilon > 1 \) reflects the degree of competition in the market for inputs \( Y_t(i) \). Equilibrium in this sector yields the input demand function:

\[ Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\epsilon} Y_t, \]

and the aggregate price index:

\[ P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \]

In this setting, any assumption about the property of this kind of firms is totally equivalent to one another, since the profits are driven to zero by perfect competition.

The firms in the wholesale sector produce a continuum of differentiated perishable goods out of hours worked, according to the following production function

\[ Y_t(i) = A_t N_t(i), \]

\(^{15}\)In what follows lower-case letters denote log-deviations from the steady state: \( x_t \equiv \log(X_t/X) \). Note that, \((1 + r_t)\) being the gross interest rate, \( r_t \) is (to first order) the actual net interest rate. The log-deviation of the gross interest rate from its steady state is therefore \( r_t - \bar{\rho} \), where we set \( \bar{\rho} \equiv \log(1 + r) = -\log \bar{\beta} \).
in which \( A_t \equiv \exp\{a_t\} \) reflects a labor-augmenting shock on productivity, which follow some stochastic process. Aggregating across firms and using equation (38) yields

\[
A_t N_t = Y_t \int_0^1 \left( \frac{P_t(i)}{F_t} \right)^{-\epsilon} di = Y_t \Xi_t,
\]

where \( N_t = \int_0^1 N_t(i) di \) is defined as the aggregate level of hours worked and \( \Xi_t = \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} di \) is an index of price dispersion over the continuum of intermediate goods-producing firms, whose log is of second-order, so that the linear aggregate production function is simply \( y_t = a_t + n_t \).

In choosing the optimal level of labor to demand, each firm enters a competitive labor market and seeks to minimize total real costs subject to the technological constraint (40). The equilibrium real marginal costs, therefore, are constant across firms and given by (in log-linear terms):

\[
mc_t \equiv \log(\mu MC_t) = w_t - p_t - a_t,
\]

where \( \mu = \frac{\epsilon}{1 - \epsilon} = (MC)^{-1} \) is the steady state gross markup and the reciprocal of steady state real marginal costs.

The price setting mechanism follows Calvo’s (1983) staggering assumption. When able to set its price optimally, each firm seeks to maximize the expected stream of future dividends (hence the real value of its outstanding shares, as can be seen iterating forward equation (5)), taking into consideration that the chosen price will have to be charged up until period \( t + k \) with probability \( \theta^k \).

The dynamic problem faced by an optimizing firm at time \( t \) can therefore be stated as:

\[
\max_{P_t(i)} E_t \left\{ \sum_{k=0}^{\infty} \theta^k \mathcal{F}_{t+k} \left[ P_t(i) \left( 1 - MC_{t+k} \right) Y_{t+k}(i) \right] \right\},
\]

subject to the constraint coming from the demand for intermediate goods of the retail sector (38).

The first-order condition for the solution of the above problem implies that all firms revising their price at time \( t \) will choose a common optimal price level, \( P^*_t \), set according to the following log-linear rule:

\[
p^*_t \equiv \log(P^*_t/P) = (1 - \theta)E_t \left\{ \sum_{k=0}^{\infty} \theta \tilde{\beta}^k (mc_{t+k} + p_{t+k}) \right\}.
\]

Finally, moving from the definition of the general price level (39) and considering that all firms revising their price at \( t \) (a fraction \( 1 - \theta \)) of all firms) choose the same price \( P^*_t \) and that all firms keeping the price constant (a fraction \( \theta \)) charge last period’s general price level, we can describe the inflation dynamics with a familiar New Keynesian Phillips curve:

\[
\pi_t = \tilde{\beta} E_t \pi_{t+1} + \frac{(1 - \theta)(1 - \theta \tilde{\beta})}{\theta} mc_t,
\]

where the households’ finite horizon (affecting the long-run interest rate) imply a lower weight on future inflation \( \tilde{\beta} \equiv \frac{\beta}{1 + \gamma} \) and a higher weight on the marginal costs compared with the standard case, to which anyhow we converge as \( \gamma \) goes to zero. To relate the inflation and real output dynamics, using the equilibrium in the labor market (equations (34) and (42)), the linear resource constraint (33) and the linear production function, we can write the equilibrium real marginal costs as:

\[
mc_t = (1 + \varphi)(y_t - a_t) - (g_t + \nu_t + \eta_t).
\]
2.2.1 The Flexible-Price Equilibrium.

The limiting case of full price flexibility arises as $\theta \rightarrow 0$. The price setting rule shows that in this case all firms set their price as a constant markup over nominal marginal costs: $P_t^n(i) = \mu P_t MC_t^n = P_t$.

As a consequence, real marginal costs match their long-run level at each point in time, implying $mc_t^n = \log MC_t^n - \log MC = 0$.

Imposing this condition on equation (45) allows to retrieve the equation for the natural level of output:

$$y_t^n = a_t + \frac{1}{1 + \varphi} (g_t + \nu_t + \eta_t),$$

and therefore to link short-run real marginal costs to the output gap $x_t \equiv y_t - y_t^n$:

$$mc_t = (1 + \varphi)x_t,$$

restating the usual result that in the absence of cost-push shocks real marginal costs are proportional to the output gap.

2.3 The Complete Linear Model.

Exploiting the results of the previous sections, the economic system described thus far reduces to the following three equations:

$$y_t = \frac{1}{1 + \psi} E_t y_{t+1} + \frac{\psi}{1 + \psi} q_t - \frac{1}{1 + \psi} (r_t - E_t \pi_{t+1} - \tilde{\rho}) - (1 + \psi) E_t \Delta \nu_{t+1} + \frac{1 + \psi - \rho q}{1 + \psi} g_t$$  

$$q_t = \tilde{\beta} E_t q_{t+1} - \frac{\lambda}{1 + \xi} E_t x_{t+1} - (r_t - E_t \pi_{t+1} - \tilde{\rho}) + \frac{1 + \xi - \tilde{\beta}}{1 + \xi} E_t y^n_{t+1} - \frac{\xi}{1 + \xi} e_t$$

$$\pi_t = \tilde{\beta} E_t \pi_{t+1} + \kappa x_t,$$

where we set $\lambda \equiv \left[ \tilde{\beta}\frac{1 + \varphi}{\rho} - (1 + \xi - \tilde{\beta}) \right]$ and $\kappa \equiv \left( \frac{1 - \theta}{\varphi} \frac{(1 - \theta \tilde{\beta})}{1 + \varphi} (1 + \varphi) \right)$.

Equation (48) defines a forward-looking IS-type relation that relates real output to its own expected future values, a short-term real interest rate and assigns an explicit and relevant role to real stock prices in driving its dynamics, through a wealth effect. The benchmark case of infinitely-lived consumers is a special case, featuring $\gamma(= \psi = \psi') = 0$, and reduces equation (48) to the usual forward-looking IS schedule:

$$y_t = E_t y_{t+1} - (r_t - E_t \pi_{t+1} - \rho) - E_t \Delta \nu_{t+1} - E_t \Delta y_{t+1}.$$  

Equation (49) describes the dynamics of real stock prices, which can therefore be driven by the supply shocks to productivity and labor supply and the demand shocks to public and private consumption and to the equity premium.

We will show below that in the face of a given observed swing in actual stock prices, the recognition of what shock was the actual cause of that swing plays a crucial role in defining the appropriate optimal response relative to the RA setup.

3 Optimal Monetary Policy and Stock-Price Dynamics.

In this Section we tackle the issue of what are the implications of the real effects exerted by stock prices when it comes to Optimal Monetary Policy on the part of a Central Banker seeking to

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**Footnote:** Henceforth we label the value that each variable takes in the flexible-price equilibrium as natural (or potential), and denote it by a superscript $n$. 

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minimize a period loss-function of the form \( L_t = E_t \{ \pi_t^2 + \alpha_y x_t^2 \} \), where \( 1/\alpha_y \) measures the degree of conservativeness of the Central Banker (in the sense of Rogoff (1985)).

Given the absence of cost-push shocks, there is no trade-off between price and output stabilization, so that full price stability \( (\pi_t = 0) \) drives the loss to zero regardless of the degree of conservativeness, and hence the flexible-price allocation is the proper target of monetary policy.

Assuming that the monetary policy instrument is the short-term interest rate \( r_t \), the value consistent with the flexible-price allocation is what Woodford (2003) calls the Wicksellian Natural Rate of Interest, henceforth \( r^*_{WR} \).

To assess the implications that the stock-wealth effects have in driving Optimal Monetary Policy, therefore, below we compare the Natural Interest Rate implied by the model at hand with the one arising in the benchmark RA setup.

In the RA setup (implying \( \gamma = \psi = \psi_q = 0 \)), from the IS schedule of equation (51) we obtain:

\[
rr^n_{RA,t} = \rho + E_t \Delta y^n_{t+1} - (E_t \Delta \nu_{t+1} + E_t \Delta g_{t+1})
= \rho + E_t \Delta a^n_{t+1} + \frac{1}{1 + \varphi} E_t \Delta \eta_{t+1} - \frac{\varphi}{1 + \varphi} (E_t \Delta \nu_{t+1} + E_t \Delta g_{t+1}),
\]

in which let \( \rho \equiv \log(1 + r) = -\log \beta_1^{19} \). In the benchmark case, then, the response of the natural interest rate is accommodative w.r.t. persistent supply shocks \( (a_t \text{ and } \eta_t) \) and restrictive w.r.t. persistent demand shocks (with response coefficients equal to \( \frac{\varphi}{1 + r} \)).

Within the framework with real effects of stock prices on real activity the Wicksellian natural rate of interest is retrieved as the solution of the two-equation system

\[
y^n_{t+1} = E_t y^n_{t+1} + \psi(g^n_{t+1} - y^n_{t+1}) - (rr^n_{t+1} - \bar{\rho}) - (1 + \psi)(1 + \psi_q) E_t \Delta \nu_{t+1} + (1 + \psi - \rho_y) g_t
\]

\[
q^n_{t+1} = \frac{\hat{\beta}}{1 + \hat{\epsilon}} E_t q^n_{t+1} - (rr^n_{t+1} - \bar{\rho}) + \frac{1 + \hat{\epsilon} - \hat{\beta}}{1 + \hat{\epsilon}} E_t y^n_{t+1} - \frac{\hat{\epsilon}}{1 + \hat{\epsilon}} e_t.
\]

Since the natural rate of output \( y^n_{t+1} \) is univocally determined by equation (46), the above system determines the potential level of stock prices \( q^n_t \), along with the natural rate of interest.

The potential level of stock market capitalization turns out to diverge from the natural rate of output in response to shocks to public consumption, the marginal utility of private consumption and the equity premium, all of which trigger a demand-driven dynamics in the stock prices:

\[
q^n_{t+1} - (1 + \hat{\epsilon})(1 + \psi - \rho_y) g_t
(1 + \psi)(1 + \hat{\epsilon}) - \hat{\beta} \rho_y
- (1 + \psi_q)(1 + \psi)(1 + \hat{\epsilon})(1 - \rho_y) y_t
(1 + \psi)(1 + \hat{\epsilon}) - \hat{\beta} \rho_{\nu}\]

\[
- \frac{\hat{\epsilon}}{(1 + \psi)(1 + \hat{\epsilon}) - \hat{\beta} \rho_{e_{t+1}}}. \quad (55)
\]

The potential level of stock prices, therefore, is decreasing in \( q \) and \( \nu \) (reflecting the reduction in private savings that both shocks imply) and increasing in non-fundamental shocks (like for example fads) that lower the equity premium.

Such a solution, together with equation (52), allows to retrieve the final reduced form for the Wicksellian natural rate of interest, which highlights the excess response implied by the stock-wealth

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\(^{17}\)Since in the flexible-price equilibrium the inflation rate is nil, \( rr^n_{RA} \) is both a nominal and a real rate.

\(^{18}\)See also Gali (2003).
effects:

\[ rr^n_t = \tilde{\rho} + E_t \Delta a^n_{t+1} + \frac{1}{1 + \psi} E_t \Delta \eta_{t+1} \]

\[ = \frac{\varphi}{1 + \varphi} (E_t \Delta \nu_{t+1} + E_t \Delta g_{t+1}) + \Psi_e e_t + \Psi_g g_t + \Psi_{\nu} \nu_t \]

\[ = \tilde{\rho} + rr^{RA,t}_t + \Psi_e e_t + \Psi_g g_t + \Psi_{\nu} \nu_t, \quad (56) \]

where \( \tilde{\rho} \equiv \rho - \rho = \log(1 + \psi) \) reflects the difference in the long-run interest rate relative to the RA setup, the latter being lower, as a consequence of the lower impatience due to the zero-probability of exiting the market.\(^1^9\)

Equation (56) shows that the optimal interest rate dynamics implied by a framework in which stock market performance has real effects on output and inflation, displays three terms which are direct implications of the stock-wealth effects, and require dedicated response to shocks to the equity premium \((e_t)\) and an over-restrictive response to shocks on public expenditures and the marginal utility of private consumption \((g_t \text{ and } \nu_t)\).

As derived in the Appendix, in fact, the coefficients are the following functions of underlying structural parameters:

\[ \Psi_g \equiv \frac{\psi \rho_g (1 + E - \tilde{\beta})}{(1 + \psi)(1 + E) - \tilde{\beta} \rho_g} \quad (57) \]

\[ \Psi_{\nu} \equiv (1 - \rho_{\nu}) \left[ \frac{\psi_{\nu}(1 + \psi)(1 + E) - \tilde{\beta} \rho_{\nu}[(1 + \psi)(1 + \psi_{\nu}) - 1]}{(1 + \psi)(1 + E) - \tilde{\beta} \rho_{\nu}} \right] \quad (58) \]

\[ \Psi_e \equiv -\psi E \frac{1}{(1 + \psi)(1 + E) - \tilde{\beta} \rho_e}. \quad (59) \]

It is straightforward to see that all three coefficients shrink to zero as the framework converges to the RA setup (in which \( \gamma = \psi = \psi_{\nu} = 0 \)); therefore the differences in the optimal interest rate’s dynamics that they imply with respect to the benchmark case can entirely be attributed to stock prices affecting the real economy.

Moreover, it is just as straightforward to see that, given the theoretical restrictions on the structural parameters, \( \Psi_g \) is always positive and \( \Psi_e \) always negative. This implies that the interest rate dynamics consistent with the flexible-price allocation entails an over-restrictive response both to shocks to government consumption \( g_t \) and to the equity premium \( e_t \), relative to the benchmark case (in which shocks to the equity premium are ineffective and remain unaccounted for in the optimal interest rate dynamics).

As to the reaction to \( \nu_t \), the sign and magnitude of the coefficient measuring the excess-response, \( \Psi_{\nu} \), is not univocal and depends on the specific structure of the economy. In case of a persistent stochastic shock shifting the marginal utility of consumption, in fact, the stock-wealth effect produces two additional and conflicting effects relative to the RA setup.

The first effect is positive and direct: a positive probability of exiting the market in the next period \((\gamma>0)\) reduces the degree of the desired smoothing in the inter-temporal path for consumption, and pushes current output above its natural level more than it would in the RA setup, thus asking for a higher raise in the rate of interest to offset the higher inflationary pressures.

The second effect is negative and indirect: the pressures towards an increase in current consumption tends to reduce the current demand for stocks, driving down their price and the value of aggregate wealth with it. But the latter effect, in a system where wealth matters for consumption

\(^{19}\)See the Appendix for details on the derivation.
decisions, has then the consequence of dragging output down, asking for an easier monetary policy compared to a system where wealth is ineffective.

The first effect dominates over the second (Ψν > 0) if the following condition holds:

\[ \psi_0 > \frac{\psi}{1 + \psi} + \beta \rho \frac{1}{1 + \psi} . \]  

(60)

The above condition, derived in the Appendix, imply therefore that if the direct excess-effect of the preference shock on real current consumption (ψν) is strong enough relative to the stock-wealth effect (ψ1 + ψ̂ βν), the optimal policy response should be over-restrictive compared to a framework in which the stock market performance has no real effects on consumption and output (ψν = ψ̂ βν = 0).

3.1 Implementation: the Equilibrium Determinacy.

Defining the stock-price gap, in analogy with the output gap, as the deviation of actual stock prices from their natural level (denoted by \( s_t ≡ q_t - q^n_t \)), and considering equation (56), we can write the linear model as:

\[ x_t = \frac{1}{1 + \psi} E_t x_{t+1} + \psi \frac{1}{1 + \psi} s_t - \frac{1}{1 + \psi} (r_t - E_t \pi_{t+1} - rr^n_t) \]  

(61)

\[ s_t = \beta E_t s_{t+1} - \frac{\lambda}{1 + \epsilon} E_t x_{t+1} - (r_t - E_t \pi_{t+1} - rr^n_t) \]  

(62)

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \]  

(63)

which highlights that a real interest rate matching its natural level (i.e. \( r_t - E_t \pi_{t+1} = rr^n_t \)) is required to achieve the Flexible-Price Equilibrium (i.e. \( x_t = s_t = \pi_t = 0 \)).

Albeit necessary, however, an interest rate matching its natural level in this class of models is usually not sufficient a condition for the flexible-price allocation to be the unique solution. Even if Central Bankers were able to commit themselves to the optimal rule (\( r_t = rr^n_t \)), then, they would still run the risk that self-fulfilling revisions in expectations might trigger potentially large and persistent (though stationary) endogenous fluctuations. The route to achieving a determinate equilibrium goes then through implementing a rule that entails a reaction to deviations of relevant variables from the desired target.

To analyze equilibrium determinacy, we assume here that the Central Bank can commit itself to a rule of the form:

\[ r_t = rr^n_t + \phi_\pi \pi_t + \phi_x x_t + \phi_s s_t. \]  

(64)

Using the assumed rule to substitute for the interest rate in system (61)–(63), the latter can be written in matrix form as

\[ E_t z_{t+1} = A z_t, \]  

(65)

where we set \( z_t ≡ [x_t \ s_t \ \pi_t]^\prime \). The conditions for the determinacy of the equilibrium implying \( z_t = 0 \) (and hence the flexible-price allocation) hinge then entirely on the magnitude of the three eigenvalues of matrix \( A \), which are required to be all outside the unit circle.21

For a given parameterization of the model, then, the conditions for determinacy translate into a set of conditions on the reaction coefficients \( \phi \)'s. If they meet those conditions all variables match their natural level and therefore the threatened variation of the interest rate in reaction to inflation and the gaps does not need to be implemented in practice.

20This is the case in most reasonable parameterizations.

21See Blanchard and Kahn (1980) for details.
Following the seminal work by Taylor (1993), Clarida, Gali and Gertler (2000) and Woodford (2003), among the others, summarize the results in the literature about determinacy in standard, Representative-Agent monetary models, where stock prices play no role. They move from the so-called *Taylor Principle*, according to which, in order for the system to show equilibrium determinacy, interest rates should respond more than proportionately to a sustained increase in the inflation rate ($\phi_\pi > 1$). The main results show that positive reactions to non-zero output gaps mildly relax the Principle, while the adoption of a forward-looking rule implies also an upper-bound on the parametric space for $\phi_\pi$ that ensures determinacy.

Within our framework, which implies a 3-by-3 coefficient matrix $A$ in system (65), retrieving analytical conditions for determinacy loses much of the usual appeal in terms of the power to draw clear conclusions. We found here much more powerful to plot the regions of determinacy by numerically simulating the model within a wide parameter sub-space for the policy rule’s coefficients.

Consequently, Figures 1 and 2, for different stances towards the output gap ($\phi_y$), show the regions in the parametric space ($\phi_q, \phi_\pi$) for which the conditions on the eigenvalues of matrix $A$ hold. The shaded areas indicate the regions in which at least one eigenvalue is within the unit circle, and therefore the equilibrium is indeterminate.

The exercise is conducted considering a quarterly calibration of the structural parameters, which is taken from widespread convention. Specifically, the steady-state net quarterly interest rate $r$ was calibrated at 0.01, implying a long-run real annualized interest rate of 4%, and the effective decision horizon was set at 15 years, implying a probability of exiting the markets $\gamma$ of 0.0167; accordingly, to meet the steady-state restrictions, the intertemporal discount factor $\beta$ was set at 0.994. $E$ was set at 0.015, implying an annualized steady-state equity risk premium of 6.24%; $\epsilon$, the elasticity of substitution among intermediate goods was set at 21, implying a steady-state net mark-up rate of 5%, and $\varphi$ was chosen to be 1/3, consistent with 6 hours worked per day; finally, the probability for firms of having to keep their price fixed for the current quarter was set at 0.75, implying that prices are revised on average once a year.

The analysis of determinacy confirms that the optimal policy ($\phi_\pi = \phi_y = \phi_q = 0$) implies an indeterminate equilibrium. It also confirms though that the optimal policy can be implemented through a credible threat to move the rate of interest if the actual allocation deviates from the potential one; i.e. if the appropriate conditions on the reaction coefficients are met. These conditions are consistent with both results mentioned above, concerning the Taylor Principle and the implications of forward-looking rules, and provide a further interesting insight about the links between monetary policy and stock prices.

As Fig. 1 shows, in fact, reacting to deviations of the stock-price level from the one prevailing in the Flexible-Price equilibrium reduces the determinacy space. In other words, for a given stance towards inflation, reacting too strongly to non-zero stock-price gaps might be destabilizing because it makes the system subject to potential endogenous fluctuations.

On the other hand, while no reaction at all yields endogenous instability, threatening a sufficiently aggressive reaction to inflation ($\phi_\pi > 1$) yields a determinate equilibrium even in the absence of any explicit concern about the output and stock-price gaps ($\phi_y = \phi_q = 0$).

As in the benchmark case, therefore, also within the proposed framework a commitment to the Taylor Principle alone qualifies as a sufficient condition for a determinate equilibrium, in which not only inflation but also output and stock prices end up matching their natural levels at all times.

This conclusion resembles the one drawn by Bernanke and Gertler (1999), insofar as it implies that an explicit reaction to stock-price deviations from a specific level might yield macroeconomic instability, the more so the less aggressive is the reaction to inflation. The policy ($\phi_\pi = 1.01, \phi_y = 0.1, \phi_q = 0$) that in Bernanke and Gertler (1999) yields a “perverse outcome”, in fact, here produces endogenous fluctuations triggered by innovations in the sunspot variable, while shifting to a more aggressive reaction to inflation ($\phi_\pi = 2.0, \phi_y = 0.1, \phi_q = 0$) ensures higher macroeconomic stability.
4 Simple Monetary Policy Rules and Stock-Price Dynamics.

While the results derived in the previous section may be taken as a warn against the perils of an excessive threat to control a specific stock-price level, they still don’t represent an exhaustive answer to the question of how an explicit concern about stock-price dynamics affects the macroeconomic implications of an operational monetary policy rule.

In this Section, therefore, we move to the analysis of two alternative simple rules, and assess their performance in terms of macroeconomic stability. The two rules considered here both modify the basic formulation proposed by Taylor (1993) to account for an explicit consideration of stock-price dynamics in monetary policy actions.

The first one augments a Taylor-type standard rule by having the interest rate respond also to deviations of a stock-price level from its “natural” counterpart (the “Gap-Rule”):

$$r_t = \tilde{\rho} + \phi_\pi \pi_{t} + \phi_y y_{t} + \phi_q q_{t} + u_{r,t}. \quad (66)$$

In the second rule considered, on the other hand, the concern about stock-price dynamics takes the form of changes in the policy rate in response to deviations of the stock-price growth rate from a given target, assumed here to be zero (the “Growth-Rule”):

$$r_t = \tilde{\rho} + \phi_\pi \pi_{t} + \phi_y y_{t} + \phi_q (q_{t} - q_{t-1}) + u_{r,t}. \quad (67)$$

The macroeconomic performance of the alternative instrument rules is measured in terms of their impact on a monetary policy loss function of the form

$$L_t \equiv \text{var}(\pi_t) + \alpha_y \text{var}(x_t) + \alpha_r \text{var}(r_t), \quad (68)$$

evaluated for different degrees of conservativeness ($1/\alpha_y$) and weight on interest rates’ variance ($\alpha_r$).

Before turning to their impact on overall stability, note that the two types of rules considered differ first of all with respect to their power to rule out endogenous instability.

As discussed in the previous section, in fact, a rule of the first type implies a reduction in the determinacy space, the more so the more aggressive the policy towards stock prices. Figure 2, in contrast, shows that a rule entailing a response to stock-price growth does not have any effect on the Taylor Principle, allowing for a stronger concern about stock-price dynamics without producing in itself endogenous instability.

As to the implications in terms of macroeconomic stability, Figures 3 and 4 plot the value of the loss function for different specifications of both the loss function and the policy rule. To evaluate the gains or losses of adopting an active concern about stock prices, we normalize to 1 the value of the loss function implied by the benchmark rule entailing a zero-reaction to stock prices ($\phi_q=0$).

We consider three alternative monetary policy regimes: a pure inflation targeting regime (PIT, $\alpha_y=\alpha_r=0$), a flexible inflation targeting regime (FIT, $\alpha_y=0.5$, $\alpha_r=0$) and a “smooth” flexible inflation targeting regime, featuring also an explicit concern about interest rates’ volatility (SFIT, $\alpha_y=\alpha_r=0.5$). Under each regime, we consider an aggressive response towards inflation ($\phi_q=2.0$) and four increasing values for the reaction coefficient on the output gap, ranging from 0 to 0.75. As to the stochastic shocks, since the aim is not to replicate the variables’ moments in the data, and in order to make the results not dependent on the relative weight of the single shocks, we calibrate the persistence of all shocks at .9 (except $u_{r,t}$ which is assumed white noise) and their variance at .01.

Since the Gap-Rule eventually yields equilibrium indeterminacy, in the bottom panels a flat line indicates that the model (hence the loss) displays endogenous instability.

The bottom-left panel of Figure 4 shows that under a PIT, responding to deviations of the stock-price index from its potential level produces a deadweight loss compared to the benchmark Taylor Rule, even in the subspace that ensures determinacy. In contrast, the top-left one shows that a
strong reaction to stock-price growth yields a stability gain as big as a 65% reduction in the loss function, relative to the benchmark rule.

A similar result arises also under the other two regimes considered. A small reaction to the stock-price gap yields in fact a very mild reduction in the monetary policy loss, not exceeding eight percentage points (two in case of an explicit target in terms of interest rates’ stability). On the other hand, adding the targets of output and interest rates’ stability to the objective function of a Central Bank adopting an instrument rule of the second type yields some interesting implications. Overall, it reduces the average gains implied by an explicit reaction to stock-price growth relative to the benchmark rule, the more so the more aggressive the stance towards the output gap. The qualitative result that loss minimization requires a significant response to stock prices, however, is not undermined and is robust to all specifications considered. Similarly robust to all specifications considered is the further result that a policy rule responding to variations in the growth rate of stock prices is superior in terms of implied overall stability compared to a rule responding to stock-price deviations from a benchmark level.

In particular, while a more aggressive stance towards the output gap increases absolute stability in all cases (see Figure 6) it also tends to reduce the relative gains of reacting to stock prices, since it grants a direct reaction to the targeted variables on which stock prices have real effects. Under a FIT and a Growth-Rule it also slightly reduces the value of $\phi_q$ that minimizes the loss function, while a Gap-Rule does not seem to be largely affected (although the latter still yields only small gains at the price of a higher vulnerability to endogenous fluctuations). Under a SFIT, on the other hand, higher reaction coefficients on the output gap drive to zero the (negligible) gains implied by responding to the stock-price gap, while those implied by the Growth-Rule are only slightly affected.

It is worth noticing that, as argued by Gali (2003), the concept of output gap in operational policy rules is generally different from the one arising in theoretical models like the one at hand, the latter being a function of unobservable structural shocks. As a consequence, a strictly operational policy rule in the analysis above would be obtained by omitting any unobservable variable (the output and stock-price gaps). This leaves two rules: one responding only to the inflation rate ($r_t = \tilde{\rho} + \phi_x \pi_{t+1} + \phi_y x_t + \phi_y s_t + u_{r,t}$) and another one responding also to stock-price growth ($r_t = \tilde{\rho} + \phi_x \pi_{t+1} + \phi_y (q_t - q_{t-1}) + u_{r,t}$). It seems interesting and meaningful that moving from the former to the latter yields the highest gains in terms of stability (see the dark blue lines in the charts).

In conclusion, common to all regimes and robust to different values of $\phi_y$ are three results. First, loss minimization requires a significant response to stock-price growth, while a very moderate (if any) reaction to non-zero stock-price gaps. Second, moving from a standard Taylor Rule to the Growth Rule yields considerable gains in terms of overall stability, especially considering strictly operational rules, while moving to the Gap-Rule only produces negligible gains (if any). Third, the Growth-Rule is superior to the Gap-Rule not only in terms of relative stability gains but also when it comes to absolute overall stability, as shown by Figure 6.

In order to assess whether the above results are in some way determined by the informational role that stock prices play with respect to future inflation, Figure 5 and the bottom panel of Figure 6 replicate the exercise for the forward-looking versions of the Gap-Rule:

$$ r_t = \tilde{\rho} + \phi_x E_t \pi_{t+1} + \phi_y x_t + \phi_y s_t + u_{r,t}, \quad (69) $$

and the Growth-Rule:

$$ r_t = \tilde{\rho} + \phi_x E_t \pi_{t+1} + \phi_y x_t + \phi_y (q_t - q_{t-1}) + u_{r,t}. \quad (70) $$

As fairly evident from the observation of Figures 5 and 6, the results discussed above are not noticeably affected by the difference in the policy rule’s specification.

22 The same exercise was conducted for other specifications of the loss function ($\alpha_y = 1, \alpha_r = 0$; $\alpha_y = 1, \alpha_r = 0.5$; $\alpha_y = 1, \alpha_r = 1$), without any effects on the discussed results.
Summary and Conclusions.

This paper enters the debate in the literature about the links between monetary policy and financial stability, and about the desirability that Central Banks be actively concerned about the stock market dynamics in the design of their monetary policy actions.

It presents the analysis of a small structural general equilibrium model in which stock prices have direct wealth effects on real activity, to assess what specific role (if any) they should play in driving Optimal Monetary Policy and what are the macroeconomic implications of adopting a policy rule that explicitly controls for stock-price dynamics.

It is shown that in the face of a given swing in stock prices, the optimal response of a Central Bank pursuing price stability depends on the shocks underlying the observed dynamics. If the driving forces are supply shocks (like to technology) then the real effects of stock prices do not require a dedicated response and the optimal policy-rate dynamics is the same as the one emerging from the standard Representative-Agent (RA) set up, in which stock prices are disregarded by definition, being ineffective. In contrast, if the driving force is a demand shock like a shock to the marginal utility of consumption, government expenditure or a fad affecting the equity premium, then positive stock-wealth effects require a dedicated response on the part of the Central Bank, more and above the reaction which would result as optimal in the RA setup.

Moreover, the macroeconomic implications of different policy rules within different policy regimes are derived. The main result in this respect is that a policy rule that has the interest rate vary in response to deviations of the stock-price growth rate from a target level proves to be superior to a “Gap-Rule”, which has the interest rate react to deviations of the stock-price level from its natural counterpart. This is shown along three different dimensions.

First, while the “Growth-Rule” does not affect the usual conditions for determinacy on the reaction coefficients on inflation and the output gap, the Gap-Rule narrows the determinacy space, implying that for a given stance towards inflation and the output gap, responding too aggressively to a stock-price gap yields potential endogenous macroeconomic instability triggered by self-fulfilling revisions in expectations. Second, under the three different monetary policy regimes considered the minimization of a monetary policy loss function requires a positive and significant reaction to stock-price growth under all specifications and parameterizations. In contrast, loss minimization adopting a Gap-Rule implies that a positive reaction to stock prices is optimal only within two regimes, conditional on a weak response to the output gap, and altogether quantitatively negligible. Third, if stock prices affect real activity, choosing the Growth-Rule over the benchmark Taylor Rule yields considerable stability gains under all regimes and parameterizations of the policy rule, in terms of reductions in the value of the loss function ranging from 15 to 65 percentage points (depending on the specific regime and the other reaction coefficients). On the other hand, moving from the benchmark rule to the Gap-Rule only yields minor gains (not higher than 8% and which become negative if the reaction to the output gap is sufficiently aggressive or the regime is the PIT) while making the system more vulnerable to endogenous fluctuations.

This paper limits its focus to the theoretical conditions for equilibrium determinacy and the macroeconomic implications of alternative policy regimes and rules. The presented framework, though, is suitable to address several other issues which are controversial in the literature, and which are tackled in two companion papers. In Nisticò (2004) an analogous framework is used to provide structural Maximum Likelihood estimates of the policy rule for the U.S. during the recent boom-bust cycle in the global financial markets, aiming at testing the hypothesis that the Federal Reserve did actively consider the financial crash in driving the Fed Funds Rate. Nisticò (2005), moreover, exploits the baseline model outlined here to derive optimal monetary policy under imperfect information about the true source of the observed dynamics in the stock market, and assess the role of such informational deficiencies in driving actual monetary policy outcomes.
References


A Appendix and Figures

Solving for Individual and Aggregate Consumption when allowing for Preference Shocks. Here we show how the methodology described in Piergallini (2004) can be applied to a framework with preference shocks.

The first-order conditions w.r.t. bonds and stocks reduce the budget constraint to equation (10):

$$P_t C_{j,t} + E_t \left\{ \mathcal{F}_{t,t+1}(1 - \gamma) \omega_{j,t+1} \right\} = W_t N_{j,t} - P_t T_{j,t} + \omega_{j,t}.$$  \hfill (A.1)

Solving forward and using the definition of human wealth (11) yields:

$$\omega_{j,t} = E_t \sum_{k=0}^{\infty} \mathcal{F}_{t,t+k}(1 - \gamma)^k P_{t+k} C_{j,t+k} - h_{j,t};$$  \hfill (A.2)

as in Piergallini (2004), substituting equation (6) for the stochastic discount factor allows to take current nominal consumption out of the sum, yielding the closed-form solution:

$$\omega_{j,t} = P_t C_{j,t} E_t \sum_{k=0}^{\infty} \bar{\beta}^k (1 - \gamma)^k \exp(\nu_{t+k} - \nu_t) - h_{j,t} = P_t C_{j,t} \Sigma_t - h_{j,t}$$  \hfill (A.3)

where in the second equality we $\Sigma_t \equiv E_t \sum_{k=0}^{\infty} \bar{\beta}^k (1 - \gamma)^k \exp(\nu_{t+k} - \nu_t)$ collects the effects of the growth in the preference shock $\nu$. Rearranging finally yields equation (12) in Section 2.1.

As to aggregate consumption, using equation (18) to substitute for $\omega_t$ in equation (16) yields:

$$P_t C_t = \frac{1}{\Sigma_t} P_t C_t + E_t \mathcal{F}_{t,t+1}\omega_{t+1} + h_t - (W_t N_t - P_t T_t).$$  \hfill (A.4)

Aggregating across cohorts the definition of human wealth (11) we get:

$$h_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \mathcal{F}_{t,t+k}(1 - \gamma)^k (W_{t+k} N_{t+k} - P_{t+k} T_{t+k}) \right\},$$  \hfill (A.5)

from which we can write:

$$h_t = (W_t N_t - P_t T_t) + E_t \left\{ \sum_{k=0}^{\infty} \mathcal{F}_{t,t+k+1}(1 - \gamma)^{k+1} (W_{t+k+1} N_{t+k+1} - P_{t+k+1} T_{t+k+1}) \right\} =
$$

$$= (W_t N_t - P_t T_t) + E_t \left\{ \mathcal{F}_{t,t+1}(1 - \gamma) \sum_{k=0}^{\infty} \mathcal{F}_{t+1,t+k+1}(1 - \gamma)^k (W_{t+k+1} N_{t+k+1} - P_{t+k+1} T_{t+k+1}) \right\} =
$$

$$= (W_t N_t - P_t T_t) + E_t \left\{ \mathcal{F}_{t,t+1}(1 - \gamma) h_{t+1} \right\}. \quad (A.6)
$$

Lead equation (16) forward one period, multiply by $\Sigma_{t+1} \mathcal{F}_{t+1,t+1}(1 - \gamma)$ and take conditional expectations:

$$E_t \Sigma_{t+1} \mathcal{F}_{t+1,t+1}(1 - \gamma) P_{t+1} C_{t+1} = E_t \mathcal{F}_{t,t+1}(1 - \gamma)(\omega_{t+1} + h_{t+1})$$  \hfill (A.7)

$$E_t \mathcal{F}_{t,t+1}(1 - \gamma) h_{t+1} = E_t \Sigma_{t+1} \mathcal{F}_{t+1,t+1}(1 - \gamma) P_{t+1} C_{t+1} - E_t \mathcal{F}_{t,t+1}(1 - \gamma) \omega_{t+1}. \quad (A.8)
$$

Using (A.6) to substitute the LHS yields:

$$h_t - (W_t N_t - P_t T_t) = E_t \Sigma_{t+1} \mathcal{F}_{t+1,t+1}(1 - \gamma) P_{t+1} C_{t+1} - E_t \mathcal{F}_{t,t+1}(1 - \gamma) \omega_{t+1}.$$  \hfill (A.9)

Plugging the above condition into equation (A.4) and rearranging finally yields equation (19) in Section 2.1.1:

$$\left( \Sigma_t \right)^{-1} P_t C_t = \gamma E_t \mathcal{F}_{t,t+1}\omega_{t+1} + E_t \Sigma_{t+1} \mathcal{F}_{t+1,t+1}(1 - \gamma) P_{t+1} C_{t+1}. \quad (A.10)
$$

Steady-State and Linear Equilibria. In the long-run the propensity to consume out of financial wealth converges to:

$$\left( \Sigma_t \right)^{-1} \equiv \sum_{k=0}^{\infty} \bar{\beta}^k (1 - \gamma)^k \exp(\nu - \nu) = \left\{ \sum_{k=0}^{\infty} \bar{\beta}^k (1 - \gamma)^k \right\}^{-1} = [1 - \beta(1 - \gamma)]. \quad (A.11)$$

21
As a consequence, its log-linear approximation $\sigma_t \equiv \log(\Sigma_t/\Sigma)$ is:

$$
\sigma_t = [1 - \beta(1 - \gamma)] \left[ \frac{1}{1 - \beta(1 - \gamma)\rho} - \frac{1}{1 - \beta(1 - \gamma)} \right] \nu_t = \frac{\beta(1 - \gamma)}{1 - \beta(1 - \gamma)\rho} E_t \Delta \nu_{t+1}. \quad (A.12)
$$

Given the assumption about the government consumption, the log-linear version of equation (22) is simply:

$$
y_t = c_t + g_t, \quad (A.13)
$$

where we set $g_t \equiv -\log \frac{1-w_t}{1-w_{t-1}}$, and we assume that $w_t$ follows a stochastic process such that $g_t = \rho g_{t-1} + a_{q,t}$, with $\rho_g \in [0, 1)$ and $a_{q,t} \sim N(0, \sigma_g^2)$.

Turning to the Euler equation for consumption, equation (19), a zero-inflation steady state defines the following condition:

$$
1 = \frac{\gamma \bar{p}_t \bar{D}}{(\Sigma - 1)(1 + r)} + \frac{(1 - \gamma)\Sigma}{(\Sigma - 1)(1 + r)} = \frac{\gamma \bar{p}_t \bar{D}}{(\Sigma - 1)(1 + r)} + \frac{1}{\beta(1 + r)}, \quad (A.14)
$$

where the second equality is implied by equation (A.11).

Rearranging the above equation, and using equation (A.11) to substitute for $\Sigma$, we obtain:

$$
\beta(1 + r) = \frac{\gamma(1 - \beta(1 - \gamma)) \omega}{(1 - \gamma) PC} + 1 = \psi + 1, \quad (A.15)
$$

where we set

$$
\psi \equiv \gamma \left[ \frac{1 - \beta(1 - \gamma)}{(1 - \gamma)} \right] \frac{\omega}{PC}. \quad (A.16)
$$

Overall, in the absence of stochastic shocks, equations (19), (17), (27), the production function, the equilibrium on the labor market and equation (23) define the following steady-state:

$$
\beta(1 + r) = \frac{\gamma(1 - \beta(1 - \gamma)) \omega}{(1 - \gamma) PC} + 1 = \psi + 1, \quad (A.17)
$$

$$
Q + D = (1 + r)Q(1 + \bar{E}) \quad (A.18)
$$

$$
Y = AN \quad (A.19)
$$

$$
MC = \frac{W}{\bar{P}A} = \frac{1}{\mu} \quad (A.20)
$$

$$
D = Y - \frac{W}{\bar{P}A} = Y(1 - 1/\mu) = Y - \frac{\mu - 1}{\mu}. \quad (A.21)
$$

Given values for the structural parameters $\beta, \gamma, \bar{E}, \mu$ and the steady-state share of consumption $C/Y$, the above system gives the steady-state level of the real wealth-to-consumption ratio ($\bar{p}_t \bar{D}$), the net interest rate $r$ and the implied parameter $\psi$.

As a consequence, the log-linear approximation of the Euler equation (25) for consumption reads:

$$
c_t = \frac{(1 - \gamma)\Sigma}{(\Sigma - 1)(1 + r)} \left[ E_t c_{t+1} - (r_t - \bar{\rho}) + E_t \pi_{t+1} \right] + \frac{\gamma \bar{p}_t \bar{D}}{(\Sigma - 1)(1 + r)} q_t - \frac{\Sigma}{(\Sigma - 1)} \left[ 1 - \frac{(1 - \gamma)}{(1 + r)} \rho_v \right] \sigma_t =
$$

$$
= \frac{1}{1 + \psi} E_t c_{t+1} + \frac{\psi}{1 + \psi} q_t - \frac{1}{1 + \psi} \left[ r_t - E_t \pi_{t+1} - \bar{\rho} \right] - (1 + \psi)E_t \Delta \nu_{t+1}, \quad (A.22)
$$

where let

$$
\psi_v \equiv \frac{\psi(1 - \bar{\gamma})}{(1 + \psi)(1 - \beta(1 - \gamma)\rho_v)}. \quad (A.23)
$$

Rearranging equation (A.18) gives equation (31) in Section 2.1.3:

$$
\frac{D}{(1 + r)Q} = 1 + \bar{E} - \frac{1}{1 + r}. \quad (A.24)
$$

As a consequence, the log-linear approximation of the pricing equation is

$$
(1 + \bar{E})q_t = \frac{1}{1 + r} E_t q_{t+1} + \frac{D}{(1 + r)Q} E_t c_{t+1} - (1 + \bar{E})(r_t - E_t \pi_{t+1} - \bar{\rho}) - \bar{E}e_t, \quad (A.25)
$$

which, rearranged and using definition (30), gives equation (36) in Section 2.1.3.

Consider now the dynamics of the real dividends, as described by equation (37). Using the linear production function and equation (42) to substitute for the second term yields:

$$
d_t = Y \frac{\bar{D}y_t - W N}{PD} (n_t + w_t - p_t) = Y \frac{\bar{D}y_t - W N}{PD} (y_t - c_t + w_t - p_t) = y_t - \frac{W N}{PD} mc_t. \quad (A.26)
$$
Moreover, using the steady state relations for the production function and the marginal costs and equation (32) we have

\[
\begin{align*}
\frac{WN}{PD} &= \frac{WY}{APD} = \frac{Y}{D_h} = \frac{\bar{\beta}}{1 + \xi - \bar{\beta}} \frac{Y}{\mu}, \\
normally, using the definition of the output gap and equation (47), we can finally write the equation for real dividends as a function of the output gap and potential output:
\end{align*}
\]

\[d_t = y^n_t - \frac{1}{\xi + \bar{\beta} \mu} \frac{y^n_t - \mu(1 + \xi - \bar{\beta})}{\xi + \bar{\beta} \mu}. \tag{A.28}
\]

Plugging the above equation into equation (36) yields equation (49) in Section 2.3.

**The Natural Rate of Interest.** From system (53)-(54) in Section 3, solving equation (53) for \((rr^n_t - \bar{\rho})\) yields

\[rr^n_t = E_t \Delta y^n_{t+1} - E_t \Delta q^n_{t+1} + \psi(q^n_t - y^n_t) - y_t^n (1 + \psi)(1 + \psi) E_t \Delta \nu_{t+1} + y_t^n \tag{A.29}
\]

and substituting into equation (54) gives

\[
\begin{align*}
q^n_t &= \frac{1}{1 + \xi} E_t q^n_{t+1} + \frac{1 + \xi - \bar{\beta}}{1 + \xi} E_t y^n_{t+1} - \frac{1}{1 + \xi} E_t \Delta y^n_{t+1} + E_t \Delta q^n_{t+1} - \psi(q^n_t - y^n_t) + (1 + \psi)(1 + \psi) E_t \Delta \nu_{t+1} - y_t^n = \\
&= \frac{1}{1 + \xi} (E_t q^n_{t+1} - E_t y^n_{t+1}) - (1 + \psi) y^n_t - \psi q^n_t + E_t \Delta q^n_{t+1} + (1 + \psi)(1 + \psi) E_t \Delta \nu_{t+1} - y_t^n - \frac{\xi}{1 + \xi} \xi. \tag{A.30}
\end{align*}
\]

The above equation takes the form of a stochastic difference equation in the capitalization ratio \((q^n_t - y^n_t)\):

\[
\begin{align*}
q^n_t - y^n_t &= \frac{1}{(1 + \psi)(1 + \xi)} E_t \{q^n_{t+1} - y^n_{t+1}\} + \frac{1 + \xi}{(1 + \psi)} E_t \Delta y^n_{t+1} \]
\[+ (1 + \psi) E_t \Delta \nu_{t+1} - \frac{\psi}{(1 + \psi)} y_t^n - \frac{\xi}{(1 + \psi)(1 + \xi)} \xi. \tag{A.31} \]
\]

Iterating forward, and using \(E_t \Delta \nu_{t+k+1} = \rho_v E_t \Delta \nu_{t+1}\), for \(v = g, \nu, \) yields

\[
\begin{align*}
q^n_t - y^n_t &= \frac{1 + \xi}{(1 + \psi)(1 + \xi)} \sum_{k=0}^{\infty} \frac{\xi}{(1 + \psi)(1 + \xi)} \frac{\xi}{(1 + \psi)(1 + \xi)} E_t \{q^n_{t+k} - y^n_{t+k}\} \]
\[+ (1 + \psi) \sum_{k=0}^{\infty} \frac{\xi}{(1 + \psi)(1 + \xi)} \frac{\xi}{(1 + \psi)(1 + \xi)} E_t \Delta \nu_{t+k} - \frac{\psi}{(1 + \psi)(1 + \xi)} \xi. \tag{A.32} \]
\]

Considering \(0 < \frac{1 + \xi}{1 + \psi(1 + \xi)} \xi < 1\), for \(v = g, c, \nu\) we get the final solution for the stock-market-to-GDP ratio under flexible prices:

\[
\begin{align*}
q^n_t - y^n_t &= \frac{1 + \xi}{(1 + \psi)(1 + \xi)} - \frac{\xi}{(1 + \psi)(1 + \xi)} E_t \Delta y^n_{t+1} - \frac{\psi}{(1 + \psi)(1 + \xi)} \xi. \tag{A.33} \]
\]

Plugging the above solution into equation (A.29) finally gives the reduced form for the Wicksellian Natural Rate of Interest:

\[
rr^n_t = \bar{\rho} + E_t \Delta y^n_{t+1} - E_t \Delta q^n_{t+1} - (1 + \psi)(1 + \psi) E_t \Delta \nu_{t+1} - y_t^n + \psi + \frac{1 + \xi}{(1 + \psi)(1 + \xi)} - \frac{\psi}{(1 + \psi)(1 + \xi)} E_t \Delta \nu_{t+1} - y_t^n \]
\[+ (1 + \psi)(1 + \psi) E_t \Delta q^n_{t+1} - y_t^n \frac{1 + \xi}{(1 + \psi)(1 + \xi)} - \frac{\psi}{(1 + \psi)(1 + \xi)} \xi. \tag{A.34} \]

Rearranging the terms conveniently and using equation (52) we can finally express the Natural Rate of Interest as:

\[
rr^n_t = \bar{\rho} + rr^n_{t+1} + \psi y_t^n - (1 + \psi - \rho_v) - \frac{1 + \xi}{(1 + \psi)(1 + \xi)} - \frac{\xi}{(1 + \psi)(1 + \xi)} \xi \]
\[+ (1 - \rho_v)(1 + \psi - 1) \psi - (1 - \rho_v)(1 + \psi)(1 + \xi) - \frac{\xi}{(1 + \psi)(1 + \xi)} \xi. \tag{A.35} \]

\]

23
Collecting the coefficients of the structural shocks we finally get

\[ rT^n_t = \bar{r} + rr^n_{R,t,t} + \Psi_e e_t + \Psi_g g_t + \Psi_r r_t, \]  
(A.36)

where the following holds:

\[ \Psi_e \equiv -\psi \frac{\mathcal{E}}{(1 + \psi)(1 + \mathcal{E}) - \beta \rho_e} < 0 \]  
(A.37)

\[ \Psi_g \equiv \psi \rho_g \frac{1 + \mathcal{E} - \beta}{(1 + \psi)(1 + \mathcal{E}) - \beta \rho_g} > 0. \]  
(A.38)

As to the coefficient of the preference shock \( \nu \), we get

\[ \Psi_\nu \equiv (1 - \rho_\nu)(1 + \psi)(1 + \psi_\nu) - (1 - \rho_\nu) - \psi(1 - \rho_\nu)(1 + \psi)(1 + \psi_\nu) \frac{1 + \mathcal{E}}{(1 + \psi)(1 + \mathcal{E}) - \beta \rho_\nu} \]

\[ = (1 - \rho_\nu)(1 + \psi)(1 + \psi_\nu) \frac{(1 + \psi)(1 + \mathcal{E}) - \beta \rho_\nu - \psi(1 + \mathcal{E})}{(1 + \psi)(1 + \mathcal{E}) - \beta \rho_\nu} - (1 - \rho_\nu) \]

\[ = (1 - \rho_\nu) \left( \frac{(1 + \psi)(1 + \psi_\nu)(1 + \mathcal{E}) - (1 + \psi)(1 + \psi_\nu)\beta \rho_\nu - (1 + \psi)(1 + \mathcal{E}) + \beta \rho_\nu}{(1 + \psi)(1 + \mathcal{E}) - \beta \rho_\nu} \right). \]  
(A.39)

Simplifying finally yields

\[ \Psi_\nu \equiv (1 - \rho_\nu) \left[ \psi \frac{(1 + \psi)(1 + \mathcal{E}) - \beta \rho_\nu}{(1 + \psi)(1 + \mathcal{E}) - \beta \rho_\nu} \right] \]

\[ \equiv (1 - \rho_\nu) \left[ \psi \frac{(1 + \psi)(1 + \mathcal{E}) - \beta \rho_\nu}{(1 + \psi)(1 + \mathcal{E}) - \beta \rho_\nu} \right]. \]  
(A.40)

To determine the sign of \( \Psi_\nu \), note that it is positive if and only if is the numerator. The latter condition translates into

\[ \psi > \frac{\beta \rho_\nu}{1 + \psi} \frac{1 + \mathcal{E} - \beta \rho_\nu}{1 + \psi(1 + \mathcal{E}) - \beta \rho_\nu}. \]  
(A.41)

Proof:

\[ (1 + \psi)(1 + \psi_\nu)(1 + \mathcal{E}) - (1 + \psi)(1 + \psi_\nu)\beta \rho_\nu - (1 + \psi)(1 + \mathcal{E}) + \beta \rho_\nu > 0 \]

\[ (1 + \psi_\nu) - (1 + \psi_\nu) \frac{\beta \rho_\nu}{(1 + \mathcal{E})} - 1 + \frac{\beta \rho_\nu}{(1 + \psi)(1 + \mathcal{E})} > 0 \]

\[ (1 + \psi_\nu) \frac{1 + \mathcal{E} - \beta \rho_\nu}{(1 + \psi)(1 + \mathcal{E})} + \frac{\beta \rho_\nu}{(1 + \psi)(1 + \mathcal{E})} > 1 \]

\[ (1 + \psi_\nu) > \frac{1}{(1 + \psi_\nu)} + \frac{\psi(1 + \mathcal{E})}{(1 + \mathcal{E}) - \beta \rho_\nu} \]

\[ \psi > \frac{\beta \rho_\nu}{1 + \psi} \frac{1 + \mathcal{E} - \beta \rho_\nu}{1 + \psi(1 + \mathcal{E}) - \beta \rho_\nu}. \]

The Case of Distortionary Taxation. Here we sketch the solution of the model in case the Government is assumed to run a balanced budget constraint levying a proportional tax on labor income.

In this case the individual tax burden in the budget constraint (2) takes the form

\[ P_t T_{t,t} = \tau_t W_t N_{t,t}, \]

which entails a number of modifications in the optimality conditions. The individual intra-temporal optimal condition w.r.t. consumption and leisure, equation (3) becomes now

\[ H_{t,C_{j,t}} = (1 - \tau_t) \frac{W_t}{P_t} (1 - N_{j,t}) \nu_t, \]  
(A.42)

where it can be seen the distortion that the tax rate imply on the static choice, reducing the effective real wage and thereby consumption. The same applies to the aggregate labor supply, equation (14), which takes the same form as equation (A.42), although relating aggregate hours worked to aggregate consumption.

Log-linearization of such an optimal condition modifies equation (34) as follows

\[ w_t - p_t = c_t + \varphi n_t + \frac{\tau}{1 - \tau} \tau_t - (\nu_t + \eta_t), \]  
(A.43)
where we denoted by $\hat{\tau}_t \equiv \log(\tau_t/\tau)$ the log-deviation of the tax rate from its steady state level.

Recall we assumed in Section 2.1.3 that public consumption is set as a share of total output, $G_t = \omega_t Y_t$, and thereby defined $g_t \equiv -\log\frac{1}{1-\omega_t}$. As a consequence, log-linearization of the income identity implies:

$$c_t = y_t - \frac{\omega}{1-\omega} \hat{\omega}_t,$$

where $\hat{\omega}_t \equiv \log(\omega_t/\omega)$, and from which we can conclude

$$g_t = \frac{\omega}{1-\omega} \hat{\omega}_t. \quad (A.44)$$

Moreover, the assumption that the Government runs a balanced budget constraint $(\omega_t Y_t = T_t = \tau_t \frac{W_t}{P_t} N_t)$ at each point in time implies that in the long-run the share of public consumption requires to be financed through a relatively higher tax burden, proportionally to the gross markup factor (since the latter measures the share of income coming from profits, which are tax-free):

$$\tau = \omega \frac{Y P}{NW} = \omega \frac{AP}{W} = \omega \mu. \quad (A.45)$$

Log-linearizing the balanced-budget constraint, and making use of equations (45), the linear production function and equation (A.44), yields

$$\hat{\omega}_t + y_t = \hat{\tau}_t + w_t - p_t + n_t = \hat{\tau}_t + w_t - p_t + y_t - a_t$$

$$\hat{\omega}_t = \hat{\tau}_t + w_t - a_t = \hat{\tau}_t + mc_t$$

and therefore

$$\hat{\tau}_t = \frac{1-\omega}{\omega} y_t - mc_t. \quad (A.46)$$

As a consequence, the equilibrium real marginal costs can be linearly approximated by

$$mc_t = (1 + \varphi)(y_t - a_t) - (g_t + \nu_t + \eta_t) + \frac{\tau}{1-\tau} \hat{\tau}_t$$

$$= (1 + \varphi)(y_t - a_t) - (g_t + \nu_t + \eta_t) + \mu \frac{1-\omega}{1-\tau} g_t - \frac{\tau}{1-\tau} mc_t$$

$$= (1-\tau)(1 + \varphi)(y_t - a_t) - (1-\tau)(g_t + \nu_t + \eta_t) + \mu(1-\omega) g_t, \quad (A.47)$$

and the natural rate of output fulfills:23

$$y^n_t = a_t + \frac{1}{1 + \varphi}(g_t + \nu_t + \eta_t) - \frac{1}{1 + \varphi} \frac{\tau}{1 - \tau} y^n_t = a_t + \frac{1}{1 + \varphi}(g_t + \nu_t + \eta_t) - \frac{\mu}{1 + \varphi} \frac{1 - \omega}{1 - \tau} \hat{\tau}_t. \quad (A.48)$$

The distortionary taxation, therefore, affects in the usual fashion both the dynamics of real marginal costs and the natural rate of output, but that is as far as it goes, when it comes to Optimal Monetary Policy.

To see this, note that equations (A.47) and (A.48) allows to write real marginal costs as proportional to the output gap, where the factor of proportionality now accounts for the steady-state tax rate as well:

$$mc_t = (1-\tau)(1 + \varphi)x_t. \quad (A.49)$$

The implication is that the complete linear model is the same as system (48)–(50) where, though, the composite parameters $\lambda$ and $\kappa$ are replaced by the following two, which reflect the distortions implied by the tax rate $\tau$:

$$\lambda^* \equiv \left[ \beta \frac{(1-\tau)(1 + \varphi)}{\mu} Y \right] \underbrace{Q}_{(1 + \varepsilon - \beta \varepsilon)}$$

$$\kappa^* \equiv \frac{(1-\theta)(1 - \theta \tilde{\beta})}{\theta} \underbrace{(1-\tau)(1 + \varphi)}.$$ 

When deriving the interest rate’s dynamics consistent with price stability, however, such distortions become effective only as far as the natural rate of output is concerned, with no effects whatsoever on the coefficients measuring the excess response to the several shocks directly implied by the stock-wealth effects. Imposing price stability to the system, in fact, reduces the linear model exactly to system (53)–(54), and therefore the Wicksellian Natural Rate of Interest fulfills equation (56)

$$rr_t^w = \tilde{\rho} + rr^w_{R,A,t} + \Psi_{S,\varepsilon t} + \Psi_{s,q t} + \Psi_{s,\nu t}, \quad (A.50)$$

in which the distortions due to the income tax are common to the RA set-up (affecting solely the potential output), therefore accounted for in the second term $rr^w_{R,A,t}$.

23Note that equation (A.46) implies that under flexible prices ($mc_t = 0$) the equilibrium tax rate is proportional to the fiscal shock: $\hat{\tau}_t^0 = \frac{1-\omega}{\omega} g_t$.  

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Figure 1: Analysis of Equilibrium Determinacy for contemporaneous rules. White (shaded) regions indicate determinacy (indeterminacy).

Figure 2: Analysis of Equilibrium Determinacy for forward-looking rules. White (shaded) regions indicate determinacy (indeterminacy).
Figure 3: Analysis of Equilibrium Determinacy for contemporaneous and forward-looking rules responding to stock-price growth. White (shaded) regions indicate determinacy (indeterminacy).
Taylor Rule: $r_t = \rho + \phi_\pi \pi_t + \phi_y x_t + \phi_q (q_t - q_{t-1}) + u_{r,t}$

Figure 4: Standardized loss implied by contemporaneous rules, for different parameterizations and specifications.
Figure 5: Standardized loss implied by forward-looking rules, for different parameterizations and specifications.
Figure 6: Absolute loss implied by contemporaneous and forward-looking rules, for different parameterizations and specifications.