Stock Prices and Incomplete Information: Implications for Monetary Policy

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April 2005
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LLEE Working Document No. 27
April 2005

Abstract

This paper derives optimal monetary policy when exogenous stochastic stock-price misalignments affect real activity, and analyzes what are the effects of incomplete information in pursuing such optimal policy.

Within a standard Dynamic New Keynesian (DNK) model with physical capital, two structural innovations are supposed to be driving stock prices dynamics: persistent productivity shocks affect the long-run component, while temporary stochastic misalignments are effective only in the short-term. The results suggest that, provided that it is able to identify in real time what is the source of the current dynamics of stock prices, an optimising Central Bank should not respond to productivity shocks while a stock-price misalignment calls for intervention.

In the case the Central Bankers have only access to limited information, they can't isolate fundamental movements in stock prices from exogenous misalignments. In this case, and provided that the information available is efficiently used, the optimal monetary policy bias is large when the underlying shock is on productivity, while turns out to be very small in the case of a stochastic misalignment.

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I wish to thank Giorgio Di Giorgio, Francesco Lippi, Fabrizio Mattesini, Gustavo Piga, Riccardo Tilli and seminar participants at the University of Urbino for very helpful comments and discussions. Any remaining errors are, of course, my sole responsibility.
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1 Introduction

The last decade of the twentieth century featured an extraordinary growth in stock prices and real activity, which at the time was imputed by many analysts to a productivity shock. From 1992 to the end of the century, the average annual growth rates of real GDP and real consumption were just a little higher than the post-WWII averages, but four times as stable; investment grew at an annual rate more than 4 percentage points above its post-war average; inflation was totally under control, and the target for the Federal Funds Rate displayed little volatility.

After a few years the boom reached its peak and both financial and real indexes experienced a U-turn: over the first three years of the new millennium, real GDP’s growth rate dropped by 45% of its value, its volatility doubled and investment fell at a rate of more than 1% per year. This made many observers believe that what had been pushing prices up was a speculative bubble, rather than simply a rise in productivity, and that the markets had already discounted any further future profitability linked to the technological push-up. The reaction of the Federal Reserve was a sudden intervention and a sequence of cuts in the Federal Funds Rate that within a little over two years took it down by 5 percentage points, for which “the collapse of the equity markets is undoubtably part of the motivation” (as Gilchrist and Leahy [20] maintain).

The question here is: should the Federal Reserve have moved earlier in preventing the bubble from getting overinflated? More precisely: was the conduct of the Federal Reserve during the late nineties optimal, at least in real time?

Whether or not a Central Bank involved in stabilizing inflation and, to some extent, output should also monitor—either targeting or reacting to—stock prices is a question that has recently gained much relevance in the economic literature, especially in the light of the mentioned recent events.

The seminal and most influential contribution to the debate is the one by Bernanke and Gertler [2], which addresses the question of how monetary policy ought to react to swings in stock prices. They implement such an analysis within a Dynamic New Keynesian (DNK) model augmented to allow for a financial accelerator channel to amplify the transmission of financial distress to the real activity, and to account for exogenous speculative bubbles. The main point they make is that a Central Bank which pursues a flexible inflation targeting as an overall strategy for monetary policy, should consider price and financial stability as
“highly complementary and mutually consistent objectives”, and simply grant an aggressive reaction to expected inflation. Such a rule would be sufficient to achieve both the goals of price and financial stability; the forward-looking behavior of private agents, moreover, should also provide the additional advantage of making future bubbles less likely.

On the other side, Cecchetti, Genberg, Lipsky and Wadhwani [8] maintain that reacting to stock-price misalignments would improve overall macroeconomic performance and reduce the likelihood of new bubbles forming. They draw these results from simulating the very same model as in Bernanke and Gertler [2], except that they consider a reaction function that embeds a response to a non-zero output gap, in addition to that to expected inflation and stock-price deviations from the steady state level.

Cogley [13], furthermore, stresses a point already mentioned by Bernanke and Gertler: one of the most important factors that would make undesirable a reaction to stock-price misalignments is the fact that it is almost impossible, in practice, to distinguish between swings in asset prices driven by fundamentals and those driven by speculative bubbles, triggered by short-termism and/or “irrational exuberance”.

This paper moves from this point and takes a stand on two issues. First, it derives Optimal Monetary Policy—which, therefore, is not restrained into Taylor’s form—when stock prices affect real activity and different shocks affect stock prices; second, it analyzes the effects of incomplete information on the actual outcomes of a Central Bank which pursues such an Optimal Monetary Policy.

Recent work by Svensson and Woodford ([36], [37]), Gerali and Lippi [19] and Cukierman and Lippi [14], develops a methodology to deal with incomplete information in designing optimal policy within rational expectations macro-models, and applies it to the analysis of the implications of unobservable potential output for optimal monetary policy. The main finding of this line of research, explicitly stated and proved in Cukierman and Lippi [14], is that retrospective mistakes in the conduct of monetary policy can occur, the more likely the more incomplete is the information available to policy makers, even if the latter is efficiently used and the policy is ex-ante an optimal one. This is due to the result that optimal monetary policy under partial information on the part of the Central Bank is generally and persistently biased, relative to the full information benchmark. A meaningful evaluation of

1Cecchetti, Genberg and Wadhwani [9] later carefully distinguish between reacting to asset prices and targeting asset prices, emphasizing that their recommendation is to react to them, within an overall strategy whose only direct targets are price stability and real output stability.
past ex-post “bad” policies, therefore, must take this into account, and distinguish between real time optimal policies that suffer from unavoidable mistakes due to the incomplete information, and policies which are not optimal, not even prospectively.

This paper applies such a methodology to a framework in which the informational problem concerns the stock-price dynamics, as well as the identification of demand and supply shocks.

Within a standard Dynamic New Keynesian model with variable physical capital, in which the classical Tobin’s Q channel is effective, stock prices are supposed to be driven by fundamental and non-fundamental factors. The focus is on technology as opposed to exogenous stochastic stock-price misalignments. While positive productivity shocks affect stock prices by raising their fundamental long-run component, non-fundamental shocks only imply temporary deviations from such a component.

It is highlighted how within a context with variable physical capital deviations of real marginal costs from their long-run level are not simply proportionate to output gap, but embed more complex a dynamics, which is affected also by the level of capital and by stock prices.

Within this framework, the Central Bank seeks to maximize social welfare, which is expressed in terms of price, output and interest rate stability. Optimal policy under full and incomplete information is derived. Under full information the Central Bank observes all the state variables of the system directly and with no measurement errors; under incomplete information, instead, only a subset of the variables are directly observable and the ones that are not are efficiently estimated through Kalman Filtering. One implication of this is that, whenever a random shock hits the economy, the perception that the Central Bank has of what is going on, is generally different from reality. As time goes by, the policy maker takes notice of the realizations of the variables it can measure and on their base it updates its estimates of the unobservable states.

To anticipate the main findings, optimal monetary policy under full information implies rising interest rates in the face of a stock-price misalignment, while no intervention is
prescribed following a productivity shock. A positive productivity shock, in fact, shifting both long-run and short-run components of all the variables in the same direction, does not require any intervention and the macroeconomic effects on output gap and inflation are nil. A non-fundamental shock to stock prices, on the other side, as well as demand and cost-push shocks, drives the economy temporarily farther away from the long-run path, calling for a possible intervention to reduce the macroeconomic effects on the goal variables.

When incomplete information is assumed, however, the Central Bank is affected by missperceptions of what shocks are actually driving the observed dynamics. As a consequence, a productivity shock—which yields higher output and stock prices—is only partially identified. For the remaining part, the signal is misinterpreted as resulting from a combination of fundamental and non-fundamental shocks. This missperception, therefore, although within an optimal policy strategy, yields a sub-optimal result, which is a contractionary shift in the monetary policy instrument. Such a sub-optimal result is the policy bias mentioned above.

Interestingly, however, in the case the boom in the stock prices is truly caused by an exogenous misalignment, the bias in the optimal policy is only quantitative and rather small. Efficient signal-extraction through Kalman Filtering, in fact, implies that only a small part of the observed rise in stock prices is attributed to a rise in productivity, which would call for no intervention. As to the rest, the Central Bank wrongly perceives also a combination of positive demand and supply shocks, in addition to a partially identified bubble. However, since the perceived combination of shocks and the underlying true shock hitting the system require the same reaction on the part of the monetary authorities, the resulting reaction is qualitatively similar, although quantitatively milder. Relative to the full information case, thus, an optimal behavior requires a slightly smaller and more temporary rise in the interest rate, which translates in a more persistent deviation of actual output gap from the zero-target, compared to the Central Bank’s estimation.

The remaining of the paper is structured as follows. Section 2 outlines the basic features of the theoretical model. Section 3 presents the methodology employed in Section 4, where a dynamic analysis of the optimal monetary policy in the framework proposed is provided. Section 5 finally summarizes and offers some concluding remarks.

\footnote{All the results are robust to different specifications of the loads in the Central Bank’s loss function. Specifically, to obtain an explicit concern about stock prices is not necessary to consider the variance of stock-price misalignments as a direct target in the objective function.}
2 The Theoretical Model

The theoretical model is a standard Dynamic New Keynesian model of the business cycle, with sticky prices and variable physical capital. Five agents act in the economy: a representative household consumes, saves and supplies labor and physical capital to the wholesale sector, which produces a continuum of intermediate goods whose prices are set facing quadratic costs of adjustment; a competitive retailer produces a final consumption good out of the intermediate goods; a third type of firm, also competitive, produces physical capital and faces convex costs of investing in new capital formation. Finally, a policy maker sets a short-term nominal interest rate to optimize social welfare, which is expressed in terms of output and prices variability.

A representative competitor capital producer makes new capital goods, choosing the optimal level of investment and exploiting a concave production function à la Uzawa [38]:

\[ \varphi \left[ \frac{I_t}{K_t} \right] K_t, \]  

where \( \varphi(\cdot) \) is increasing and concave in investments. Such a production function embeds convex costs of adjustment, which result in a variable real price for capital, \( Q_t \), at which the representative household purchases the capital goods.

Households have preferences over consumption and leisure. Each household chooses how much to consume, to work and how to allocate his savings between the two assets available: riskless bonds and physical capital. Households, thus, seek to maximize lifetime utility

\[ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ C_t^{1-\gamma} - \frac{1}{1-\gamma} - \eta N_t \right] \right\} \]

such that in each period real expenditures for consumption and the purchase of financial and real assets (such as bonds and capital) do not exceed total real revenues from labor, dividends and asset-holdings:

\[ C_t + \frac{B_t}{P_t} + Q_t K_t \leq W_t N_t + D_t + \frac{B_{t-1} R_{t-1}}{P_t} + \left[ R^k_t + Q_t (1 - \delta) \right] K_{t-1}, \]  

where \( \delta \) is the depreciation rate of physical capital \( K_t \), \( R_t \) is the nominal gross interest rate between period \( t - 1 \) and \( t \), \( W_t \) and \( R^k_t \) are real wage and real rental cost of capital, respectively. Finally, \( C_t, B_t \) and \( D_t \) are real consumption, nominal riskless bonds and real dividends from the producers of intermediate goods.
The linearity of the utility function with respect to labor follows Hansen [25], in what assumes an economy consisting of many identical consumers whose individual choice is either to work full time or not to work at all.

Labor and physical capital are rented to imperfectly competitive wholesalers. Each firm of this kind—indexed \( i \in [0, 1] \)—chooses the level of inputs to demand and the prices at which to sell its own good to maximize the expected discounted real stream of dividends

\[
E_0 \left\{ \sum_{t=1}^{\infty} \left[ \varrho_{t,0} D_t(i) P_t \right] \right\},
\]

where \( \varrho_{t,0} \equiv \beta_t \lambda_t / \lambda_0 \) is the stochastic discount rate to the household. In setting prices, the wholesalers face quadratic costs of adjustment, expressed in terms of the final good, \( Y_t \):

\[
\frac{\phi_P}{2} \left[ \frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right]^2 Y_t,
\]

where \( \Pi \) is the steady state gross inflation rate. The presence and magnitude of costs of adjusting prices, reflected by the parameter \( \phi_P \), makes the problem of the firm a dynamic one, and in equilibrium implies a short-term positively sloped Phillips Curve.

The continuum of differentiated perishable goods thus produced, are sold to the competitive retailer, whose optimality conditions yield the usual demand schedule for intermediate goods

\[
Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} Y_t
\]

and the aggregate price level

\[
P_t = \left[ \int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}},
\]

where \( \theta \) is the elasticity of substitution of the intermediate goods with one another in the production technology of the final good, and expresses the degree of imperfect competition of the market for wholesale goods.

The equilibrium conditions for the four agents above make up the macroeconomic dynamic system that the policy maker has to deal with. The steady state for such a system coincides with the flexible-price version of the model, which defines the potential level of output and implies a vertical Phillips Curve.
To see this more clearly, consider the optimality condition for setting the price of intermediate good $i$:

$$
(1 - \theta) \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} \frac{Y_t}{P_t} + \theta MC_t(i) \left[ \frac{P_t(i)}{P_t} \right]^{-\theta - 1} \frac{Y_t}{P_t} = \phi P \left[ \frac{P_t(i)}{P_t(i)} - 1 \right] \frac{Y_t}{P_t(i)} - \phi P E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \left[ \frac{P_{t+1}(i)}{P_t(i)} - 1 \right] \frac{P_{t+1}(i)}{P_t(i)} \right\} Y_{t+1},
$$

where $MC_t(i)$ is the lagrange multiplier on the technological constraint—a usual CRS, Cobb-Douglas production function—and represent the real marginal costs faced by wholesaler $i$.

The right-hand side (RHS) of equation (7) includes the only terms for which the proposed model differs from a frictionless environment, in which the equilibrium yields the potential level for all variables. This term is the parameter $\phi_P$. A frictionless environment in which prices are perfectly flexible is a special case of the model proposed, and corresponds to the case that $\phi_P = 0$. This case implies that the two terms on the RHS of the Euler equation above cancel out and immediately yields the standard result for a flexible-price economy that marginal costs under symmetric equilibrium are constant across firms and time and equal the inverse of the gross equilibrium mark-up rate\(^5\)

$$
MC_t(i) = MC_t = MC = \frac{\theta - 1}{\theta}.
$$

Within a sticky-price set up, nonetheless, the steady state similarly implies that the two terms on the RHS of equation (7) cancel out, since the equality

$$
\left[ \frac{P_{t+1}(i)}{P_t(i)} - 1 \right] = \left[ \frac{P_{t+1}}{P_t} - 1 \right] = \left[ \frac{\Pi}{\Pi - 1} \right] = 0
$$

holds in a symmetric equilibrium steady state. As a result, and notwithstanding a strictly positive parameter $\phi_P$, the long-run level of real marginal costs is the same as in the frictionless case.\(^6\)

\(^5\)The equilibrium mark-up by definition reflects the market power of each wholesaler, which is inversely proportionate to the elasticity of substitution of the intermediate goods with one another, $\theta$. One of the limiting cases is the perfect substitutability among such goods, with $\theta$ going to infinity and the equilibrium gross mark-up rate going to 1; this is the competitive case, in which there are no extra revenues beyond those determined by the marginal products of the inputs.

\(^6\)The only but notable difference is that under the flexible-price hypothesis the real marginal costs equal their potential level at every moment in time, while within a sticky-price framework the potential level is met only in the long-run.
In the long-run, therefore, the sticky-price model and its flexible-price counterpart yield the same levels of the relevant variables.

In absence of any kind of shocks, the system would rest in its theoretical steady-potential state. In the case of short-run temporary shocks the variables of the system would be shifted away from their long-run levels, to which however they would converge over time. The existence of such shocks injects some short-term dynamics and makes the short-run levels of all variables time-varying. Here I append to the structural model three short-run temporary disturbances, which affect current demand through consumption ($d_t$), current stock prices through an exogenous stochastic misalignment ($s_t$) and current inflation through cost-push expectations ($u_t$).

There are a number of reasons, though, to believe that also the long-run state of an economic system be non-constant and that the long-run levels of most variables be time-varying. The intuition is straightforward: changes in particularly important features of an economic system—the most intuitive to think of is a technological break-through—not only have immediate effects on the economy, affecting the current levels of the variables, but also shift the theoretical long-run state to which the economy would converge. It is common practice in theoretical and empirical economic models to assume a dynamic path for long-run potential output, which is affected by persistent structural shocks that affect the system’s overall capacity to grow.

In the present framework, as well as in any standard asset pricing model, the long-run level of real stock prices depends on the expected long-run rate of growth of dividends. Since the latter depend on the long-term growth rate of output, therefore, it is reasonable to assume that also the long-run level of real stock prices be time-variable and vulnerable to persistent structural shocks (see Goodhart and Hofmann [21]).

Here I assume that the long-run levels of output, $\bar{y}_t$, and stock prices, $\bar{q}_t$, can be affected by productivity shocks such as a technological break-through, and that the effects on the theoretical long-run state are persistent. This assumption translates into assuming the following dynamic behavior for $\bar{y}_t$ and $\bar{q}_t$:

$$\bar{y}_t = \rho_y \bar{y}_{t-1} + \varepsilon^p_t$$

$$\bar{q}_t = \rho_q \bar{q}_{t-1} + \varepsilon^p_t,$$

where $\varepsilon^p_t \sim WN(0, \sigma_p^2)$ is the shock on productivity, and $\rho_y$ and $\rho_q$ lie within the unit circle.
Thus, the complete linearized system about a zero-inflation steady state, ignoring uninteresting constants, consists of equations (9) and (10) plus the following ones:

\[
\hat{y}_t = \frac{C}{Y} \hat{c}_t + \frac{I}{Y} \hat{i}_t \tag{11}
\]

\[
\hat{c}_t = \hat{c}_{t+1|t} - \frac{1}{\gamma} (r_t - \pi_{t+1|t}) + d_t \tag{12}
\]

\[
\hat{i}_t = k_t + \psi \hat{q}_t \tag{13}
\]

\[
\hat{q}_t = \varpi \hat{q}_{t+1|t} + (1 - \varpi) (\hat{m}c_{t+1|t} + \hat{q}_{t+1|t} - k_{t+1}) - (r_t - \pi_{t+1|t}) + s_t \tag{14}
\]

\[
y_t = \bar{y}_t + (1 - \alpha) n_t + \alpha k_t \tag{15}
\]

\[
\hat{m}c_t = \gamma \hat{c}_t - (y_t - \bar{y}_t) + n_t \tag{16}
\]

\[
\pi_t = \beta \pi_{t+1|t} + \kappa \hat{m}c_t + u_t \tag{17}
\]

\[
k_{t+1} = \delta \hat{k}_t + (1 - \delta) k_t \tag{18}
\]

\[
d_{t+1} = \rho_d d_t + \varepsilon_{t+1}^d \tag{19}
\]

\[
s_{t+1} = \rho_s s_t + \varepsilon_{t+1}^s \tag{20}
\]

\[
u_{t+1} = \rho_u u_t + \varepsilon_{t+1}^u \tag{21}
\]

in which lower case variables are in logs; variables with the hat are log-deviations from their own long-run levels; \( \psi = -\frac{\varphi'(\frac{q}{p})}{\varphi'(\frac{q}{p})} \pi \) is the steady state elasticity of the investment-capital ratio with respect to stock prices, which reflects the adjustment costs borne by capital producers; \( \frac{C}{Y} \) and \( \frac{I}{Y} \) are steady state consumption and investment shares, respectively; \( \varpi = \beta (1 - \delta) \) and \( \kappa = \frac{\theta - 1}{\varphi p} \). Finally, \( x_{t+j|t} \) denotes the rational expectation—i.e. the best possible estimate—of the value that variable \( x \) will assume at period \( t+j \), conditional to the information available at time \( t \), \( E(x_{t+j|t}) \), for all \( j = 0, 1, 2, \ldots \).

By appropriate substitutions, the linearized system (9)–(21), can be reduced in terms of the three relevant forward-looking variables (output \( y_t \), stock prices \( q_t \) and inflation \( \pi_t \)) and the six state variables (the three temporary shocks \( d_t \), \( s_t \) and \( u_t \), the long-run levels of output and stock prices \( \bar{y}_t \) and \( \bar{q}_t \), and physical capital \( k_t \)). Such a reduced system consists
of equations (9) and (10), equations (19)–(21) and the following four:

\[ y_t = \bar{y}_t + \hat{y}_{t+1|t} - \sigma (r_t - \pi_{t+1|t}) + \frac{I}{Y} \left[ (1 - \delta)\hat{q}_t - \hat{q}_{t+1|t} \right] + \frac{C}{Y} d_t \]  \hspace{1cm} (22)

\[ q_t = \bar{q}_t + \nu_q \hat{q}_{t+1|t} + \nu_y \bar{y}_{t+1|t} - \frac{1}{\chi} (r_t - \pi_{t+1|t}) - \nu_k k_t + \frac{1}{\chi} s_t \]  \hspace{1cm} (23)

\[ \pi_t = \beta \pi_{t+1|t} + \kappa \mu_y (y_t - \bar{y}_t) - \kappa \mu_q (q_t - \bar{q}_t) - \kappa \mu_k k_t + u_t \]  \hspace{1cm} (24)

\[ k_{t+1} = k_t + \delta \psi \hat{q}_t, \]  \hspace{1cm} (25)

where \( \hat{y}_t = (y_t - \bar{y}_t) \) and \( \hat{q}_t = (q_t - \bar{q}_t) \) are the output and stock-price gaps, \( \sigma = \frac{C}{\gamma} \) is the interest rate sensitivity of demand and the coefficients \( \nu_y, \nu_q, \nu_k, \mu_y, \mu_q, \mu_k \) and \( \chi \)—all positive—are functions of the underlying deep parameters \( (\beta, \delta, \gamma, \alpha) \).

Equation (22) is a forward-looking IS-type schedule, which highlights that log-output is determined not only by potential output, demand shocks and real interest rates, but also by the current and expected level of the stock prices, through the channel of investments.

Equation (23) describes the dynamics of stock prices. Albeit coming from a General Equilibrium model, it is equivalent to Campbell and Shiller's [7] approximate present value model, in which stock prices are linked to expected variations in future dividends and discount factors. Here the relationship is explicitly stated in terms of underlying real variables like output and real interest rates. The last term of equation (23) captures the possible deviations of current stock prices from this fundamental-based valuation, to account for waves of optimism or short-termism.

Equation (24), finally uncovers the relevant macroeconomic variables that underlies the Phillips Curve in its general and familiar formulation of equation (17). This “explicit” Phillips Curve states that inflation dynamics is governed not only by expectations, cost-push shocks and the output gap, but also assigns an important role to deviations of stock prices from their potential value. Expressing real marginal costs of equation (16) in terms of the relevant variables, in fact, highlights that a positive stock-price gap, by stimulating over-investments in physical capital, reduces the marginal productivity of the latter and hence equilibrium real marginal costs, relative to their long-run level:

\[ mc_t - \log \left( \frac{\theta - 1}{\theta} \right) = \mu_y (y_t - \bar{y}_t) - \mu_q (q_t - \bar{q}_t) - \mu_k k_t. \]  \hspace{1cm} (26)

The dynamic relations illustrated above can be synthetically described by the following matrix system, whose notation recalls the one used in Svensson and Woodford [36] and
Gerali and Lippi [19], among the others:

\[
\begin{bmatrix} X_{t+1} \\ A^0x_{t+1|t} \end{bmatrix} = A^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + A^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + Br_t + \begin{bmatrix} C_e \\ \emptyset \end{bmatrix} \varepsilon_{t+1},
\]

and in which \(A^0, A^1, A^2, B \) and \(C_e \) are conformable and appropriate matrices, that reproduce the dynamic system, while \(X_t = [y_t \quad q_t \quad \pi_t] \) the \(n_X \) predetermined variables, \(x_t = [y_t \quad q_t \quad \pi_t] \) the \(n_x \) forward-looking non-policy variables, \(r_t \) the short term nominal interest rate—the policy variable—and \(\varepsilon_t = [\varepsilon_t^d \quad \varepsilon_t^a \quad \varepsilon_t^u \quad \varepsilon_t^p] \) the white noise innovations of the structural shocks to the economy, with covariance matrix \(\Sigma_e = \text{diag}(\sigma_d^2 \quad \sigma_s^2 \quad \sigma_u^2 \quad \sigma_p^2)\).

Given system (27) above, which describes the behavior of the private sector, the policy maker finally sets the net nominal interest rate \(r_t \) to minimize the welfare losses implied by price instability and persistent deviations of current from potential output, as reflected by the familiar period loss function

\[
L_t = \frac{1}{2} \left[ \lambda_r (r_t - r_{t-1})^2 + \pi_t^2 + \lambda_y (y_t - \bar{y}_t)^2 \right],
\]

in which \(\pi_t \) is the net inflation rate between period \(t-1 \) and \(t \), \(\lambda_r \) is the coefficient expressing the weight that the Central Bank assigns to interest rate smoothing and \(\lambda_y \) is the weight that the monetary authority assigns to real instability and reflects the degree of conservativeness of the Central Bank, in the sense of Rogoff [34].

3 Optimal Monetary Policy and Incomplete Information

In real-world policy making, what Alan Blinder calls “dark art” mostly refers to the fact that Central Banks inevitably have to deal with a substantial uncertainty about not only the timing and magnitude of the effects of its actions, but also about the state of the economy it is supposed to keep under control, and the real nature of the disturbances that might call for its intervention.

Svensson and Woodford ([36], [37]), moving from an earlier work by Pearlman [33], develop a methodology to deal with incomplete information on the part of a policy maker within linear-quadratic economies. By incomplete, or imperfect, information here it is meant

\footnote{According to Rogoff [34] a conservative Central Bank should be concerned with price stability only, with no regard on output dynamics. In this sense, the lower the parameter \(\lambda_y \) in equation (28) above, the more conservative the monetary authority.}
that the Central Bank does not know with certainty the true values of one or more of the states of the economy. Rather than just to optimize an objective function, then, the policy maker also needs to provide an efficient estimate of such states, through Kalman Filtering. The main results they illustrate and rationalize—for which an early discussion can be found in Chow [12]—consist of two principles: the certainty equivalence and the separation principles.

The first states that optimal policy, derived by minimizing a quadratic loss function subject to a system of linear constraints, is not dependent on the hypothesis about the information held by the policy maker. The optimal policy rule, therefore, which is a linear function of all the states of the economy, has the same response coefficients under both complete and incomplete information. Denoting $F$ the matrix of such coefficients, the optimal decision rule will, therefore, take the form

$$r_t = FX_t,$$

in which $F$ does not depend on the assumption about the information set available to the Central Bank.\(^8\)

This, of course, does not mean that incomplete information is not going to affect actual policy making and the dynamics of the system. Under imperfect information, in fact, the Central Bank is able to compute the optimal coefficients of the reaction function, collected by matrix $F$, but it applies them not to the real—and unknown—values of the states, but to the most efficient estimates of them that it is able to provide:

$$r_t = FX_{t|t}.$$  

Thus, the actual policy, although optimal conditionally to the knowledge available, may achieve sub-optimal and destabilizing outcomes, through no specific fault of the policy maker’s.

According to the second principle, moreover, the two tasks of the Central Bank—the determination of the optimal response coefficients (optimization) and the estimation of the states of the economy given the available observations (signal-extraction)—are separable and can be undertaken independently of each other.

The framework used in Svensson and Woodford [36] and Gerali and Lippi [19], among the others, to analyze optimal monetary policy with incomplete information consists of

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\(^8\)The present section adopts the same notation as in Svensson and Woodford [36] and Gerali and Lippi [19].
three building blocks and is expressed in terms of the relevant variables included in vectors $X_t$ and $x_t$: a model of the private sector, a model of the policy maker’s *modus operandi* and the structure of the Central Bank’s capability to observe the several variables at stake.

Within the current set-up, the first building block, i.e. the dynamics of the private sector, is represented by system (27).

As to the second, assuming that it does not make in advance any binding commitment over the future course of its monetary policy actions, the Central Bank sets a short-term interest rate, $r_t$, in each period $t$, by minimizing the expected discounted sum of period losses (28), conditionally to the information set currently available:

$$E \left\{ \sum_{\tau=0}^{\infty} \beta^\tau L_{t+\tau} \right\ | \mathcal{F}_t$$

and subject to the linear constraints of system (27).

The period loss function is a quadratic form of the goal variables $Y_t$, in which $W$ is a positive-semidefinite weight matrix:

$$L_t = Y_t'WY_t.$$  \hspace{1cm} (31)

The goal variables, on their part, are defined in terms of the relevant primitive policy and non-policy variables according to:

$$Y_t = C_1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + C_2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + C_r r_t.$$  \hspace{1cm} (32)

As regards the third block, the Central Bank is supposed to be able to directly observe and measure a number $n_Z \leq n_X + n_x$ of variables among all the ones involved:

$$Z_t = D_1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + D_2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + v_t,$$  \hspace{1cm} (33)

where $D_j$ with $j = 1, 2$ are matrices that select the elements of the vectors $X_t$ and $x_t$ that can actually be observed, and $v_t$ is a $(n_Z \times 1)$ iid stochastic vector of measurement errors, with mean zero and covariance matrix $\Sigma_v$.

Hence, the information set available at $t$ to the policy maker consists of the measurements of the observable variables $Z_t$ up to the current period, the structural parameters of the systems (27) and (31)–(33) and the covariance matrices of the disturbances:

$$\mathfrak{F}_t \equiv \{ Z_\tau, \tau \leq t; \ A^0, A^1, A^2, B, C_z, W, C^1, C^2, C_r, D_1, D_2, \Sigma_z, \Sigma_v \}.$$  \hspace{1cm} (34)
3.1 The Case of Complete Information.

Under complete, or full, information, the Central Bank is supposed to have a faultless perception of reality: it has direct access to precise measurement of all the variables involved in the dynamic system, both predetermined and forward-looking.

In this case, the power of the policy maker to achieve its tasks of stabilizing prices and output is maximum, and the policy actions it undertakes are most rightly effective in reality, in the sense that they take the economy right where they mean to. The reason for that is that at any given point in time the Central Bank knows exactly where the system stands; moreover, whenever any type of shock hits the economy, the policy maker is perfectly able to isolate what kind of shock it was, because it observes it directly and with precision. Thus, the signal-extraction problem is trivial, because the signal and the underlying true shock to be identified coincide. As a result, an optimizing Central Bank not only will undertake the best possible actions, but will also achieve in practice the best possible outcomes.

This is, therefore, the case chosen as a benchmark, and characterizes the optimal behavior that a Central Bank should pursue in the face of the shocks considered. Moreover, it allows to assess how incomplete information affects the actual results of an ex ante optimal policy, and to what extent the possible poor results of some policy-making might depend on imperfect information rather than on poor performance of the Central Banker.

Within the analytical framework presented above, complete information represents a special case, in which two requirements are met. First, the vector $Z_t$ is of dimensions $n_Z = n_X + n_x$ and collects all the forward-looking variables in $x_t$ and all the states in $X_t$: $Z_t^f = [X_t' \ x_t']'$, \[(35)\] where the superscript stands of course for full information.\footnote{This further implies that $D^1 = I_n$ and $D^2 = \emptyset_n$.} Second, not only the Central Bank can observe all the variables that play any role in the system, but it can also measure them with no errors whatsoever. This means that the vector of measurement errors, $v_t$, is an empty set, as well as its covariance matrix.

The information set available to the Central Bank under the most favorable case of complete information—denoted again by means of the superscripted $f$—is therefore:

$$\mathcal{I}_t^f \equiv \{ Z_t^f, \tau \leq t; \ A^0, A^1, A^2, B, C, W, C^1, C^2, C, D^1, D^2, \Sigma \}.$$ \[(36)\]
What denotes the full information case with respect to the general one of incomplete information is therefore basically the structure of system (33), which in the former case is restricted to have dimensions $n_Z = n_X + n_x$ and $v_t = \emptyset$. Ultimately, then, what makes the difference are matrices $D^1$, $D^2$ and $\Sigma_v$. The certainty equivalence principle, as will be shown below, implies that the solution of the optimization problem does not depend on such matrices.

3.2 The Optimization Problem.

Assuming that the information structure be the same across sectors of the economy, and that there are no binding commitments on monetary policy, the task of the Central Bank in each period is to choose the level of the short-term interest rate $r_t$, to solve the following optimal regulator problem, conditionally to the information currently available:\(^{10}\)

$$
\min_{r_t} \quad E\left\{ \sum_{\tau=0}^{\infty} \beta^\tau Y_{t+\tau} W Y_{t+\tau} \big| \mathcal{F}_t \right\}
$$

subject to the following constraints

$$
\begin{align*}
X_{t+1} &= A^1 X_t + A^2 X_{t|t} + B r_t + \left( C_x \emptyset \right) \varepsilon_{t+1} \\
Y_t &= C^1 X_t + C^2 X_{t|t} + C_r r_t \\
r_{t+1} &= F_{t+1} X_{t+1} \big| t+1 \\
x_{t+1|t+1} &= G_{t+1} X_{t+1|t+1}.
\end{align*}
$$

In the regulator problem above, two features seem worth noticing.

The first is the last set of constraints, equations (39) and (40). Their presence among the other constraints qualifies the desired solution as discretionary. Such a solution falls within the Nash Equilibria category, in what the policy maker re-optimizes every period, and takes as given both the private sector’s expectations and its own future actions. Since the model is linear-quadratic, equations (39) and (40) are linear guess solutions for the problem in period $t+1$, which, under discretion, are taken as given in period $t$.

\(^{10}\)For a more detailed derivation and discussion see Backus and Driffill [1], Currie and Levine [15], Söderlind [35] and Svensson and Woodford [36].
The second is the drawback with solving the problem above: the objective function involves endogenous forward-looking variables, \( x_t \), which depend on the expectations about the future values of other endogenous variables which, on their part, are going to be affected by the chosen value of the control variable, \( r_t \). Thus, the approach followed to find the solution matrices for the decision rule

\[
\begin{align*}
\bar{r}_t &= FX_{t|t} \\
x_{t|t} &= GX_{t|t}
\end{align*}
\]

and the linear relation

\[
\begin{align*}
x_{t|t} &= \tilde{A}_t X_{t|t} + \tilde{B}_t r_t
\end{align*}
\]

goes through re-writing the whole problem in terms of estimated state variables \( X_{t|t} \) only.

By means of equation (40) and of the lower block of equation (37), after some algebra we get:

\[
\begin{align*}
x_{t|t} &= \tilde{A}_t X_{t|t} + \tilde{B}_t r_t
\end{align*}
\]

where \( \tilde{A}_t \) and \( \tilde{B}_t \) depend only on matrices \( A_0, A, B \) and the guess solution matrix \( G_{t+1} \):

\[
\begin{align*}
\tilde{A}_t &= (A_{22} - A_0 G_{t+1} A_{12})^{-1}(A_0 G_{t+1} A_{11} - A_{21}) \quad (44) \\
\tilde{B}_t &= (A_{22} - A_0 G_{t+1} A_{12})^{-1}(A_0 G_{t+1} B_1 - B_2). \quad (45)
\end{align*}
\]

Similarly, taking expectations of the upper block of (37) and using (43) to eliminate the forward-looking variables, the perceived law of motion of the state variables is derived:

\[
\begin{align*}
X_{t+1|t} &= (A_{11} + A_{12} \tilde{A}_t) X_{t|t} + (B_1 + A_{12} \tilde{B}_t) r_t \\
&= A_1 X_{t|t} + B_1 r_t.
\end{align*}
\]

As to the objective function, letting \( Q \equiv C'W C, \ U \equiv C'W C_r \) and \( R \equiv C_r'W C_r \), the estimated period loss \( L_{t|t} \) can be written as:

\[
\begin{align*}
L_{t|t} &= \left[ X_{t|t} \right]' Q \left[ X_{t|t} \right] + 2 \left[ X_{t|t} \right]' U r_t + r_t' R r_t + 1_t,
\end{align*}
\]

in which

\[
\begin{align*}
1_t &= E \left\{ \left[ X_t - X_{t|t} \right]' C^1 W C^1 \left[ X_t - X_{t|t} \right] \right\} \quad (48)
\end{align*}
\]

\[11\] Henceforth let \( A \equiv A_1 + A_2, \ C \equiv C_1 + C_2 \) and \( D \equiv D_1 + D_2 \), and all due partitioning be conformable with \( X_t \) and \( x_t \) respectively.
does not depend on the control variable, \( r_t \), and accounts for the MSE in the estimation process. Applying equation (43) to (47) to substitute for the forward-looking variables, furthermore, allows to write the estimated period loss function in terms of the perceived state variables \( X_{t|t} \) only:

\[
L_{t|t} = X_{t|t} Q_t^* X_{t|t} + 2X_{t|t} U_t^* r_t + r_t R_t^* r_t + l_t,
\]

where

\[
Q_t^* = Q_{11} + Q_{12} \tilde{A}_t + \tilde{A}_t' Q_{21} + \tilde{A}_t' Q_{22} \tilde{A}_t
\]

\[
U_t^* = Q_{12} \tilde{B}_t + \tilde{A}_t' Q_{22} \tilde{B}_t + U_1 + \tilde{A}_t' U_2
\]

\[
R_t^* = R + \tilde{B}_t' Q_{22} \tilde{B}_t + \tilde{B}_t' U_2 + U_2' \tilde{B}_t.
\]

Given the transformations above, the policy maker’s problem now takes the form of a standard stochastic discounted linear optimal regulator problem with time-varying parameters but no forward-looking variables. From linear quadratic dynamic programming we know that such a problem fulfills the Bellman equation

\[
X_{t|t} V_t X_{t|t} + w_t = \min_{r_t} \left\{ L_{t|t} + \beta E \left[ X_{t+1|t+1} V_{t+1} X_{t+1|t+1} + w_{t+1} \right] \right\}
\]

subject to (46) and (49). In (53) \( V_t \) is the positive semidefinite matrix-value-function in \( t \), while \( w_t \) is a scalar reflecting the variability of the stochastic terms of the constraints.

From the first-order condition for a minimum of the right-hand side of equation (53) we can retrieve the solution of the problem for period \( t \): matrices \( F_t \) and \( G_t \), and the optimized value function, \( V_t \), which at this stage are all time-varying:

\[
F_t = -(R_t^* + \beta B_t''' V_{t+1} B_t^*)^{-1} (U_t^* + \beta B_t''' V_{t+1} A_t^*)
\]

\[
G_t = \tilde{A}_t + \tilde{B}_t F_t
\]

\[
V_t = Q_t^* + U_t^* F_t + F_t' U_t''' + F_t' R_t^* F_t + \beta (A_t^* + B_t^* F_t)' V_{t+1} (A_t^* + B_t^* F_t).
\]

The system of equations above defines a mapping from \( (F_{t+1}, G_{t+1}, V_{t+1}) \) to \( (F_t, G_t, V_t) \). This mapping calls for a numerical algorithm which starts off with some guesses of \( G_{t+1} \) and \( V_{t+1} \) and, through recursive computation of \( (F_t, G_t, V_t) \), eventually converges to the fixpoint \( (F, G, V) \). Such a fixpoint is the desired final solution of the original problem and fills in equations (41) and (42).

\[12\]Svensson and Woodford [36] prove this point.
Finally, it is worth noticing from equation (54) that the solution matrix $F_t$—and hence its time-invariant counterpart $F$—depends ultimately on matrices $A$, $B$, $C$ and $W$, but not on $D^1$, $D^2$, $\Sigma_e$ or $\Sigma_v$. This is sufficient to prove the Certainty Equivalence principle that the optimal decision rule is the same under both assumptions of complete and incomplete information.

The solution matrices thus found allows to derive the complete dynamics of the system in terms of predetermined ($X_t$) and observable ($Z_t$) variables only, as:

$$X_{t+1} = HX_t + JX_{t|t} + C\varepsilon_{t+1}$$

$$(57)$$

$$Z_t = LX_t + MX_{t|t} + v_t,$$

$$(58)$$

in which the coefficient matrices $H$, $J$, $L$ and $M$ are functions of structural matrices $A$, $B$ and $D$ and of solution matrices $F$ and $G$.

### 3.3 The Signal-extraction Problem.

If the assumption of complete information does not hold, the Central Bank has the additional need of estimating the variables that it does not directly observe, conditionally on the measurement of those whose observation is available. To these estimates it then applies the optimal response coefficient matrix computed in the first stage.

With respect to the methodology for providing such estimates there are of course—at least in principle—cases in which the policy maker might in fact be using a low-efficiency signal-extraction procedure; in this cases the, even ex-ante, systematic mistakes in interpreting the signals are primarily responsible for the poor performance of the policy actions.

In an optimal monetary policy context, however, which is going to serve as a benchmark for evaluating the importance of real-world imperfections—as incomplete information—in driving actual policy outcomes, we want this estimation procedure to be the most efficient, so that the benchmark results be truly optimal.

The Central Bank in $t$ observes a subset of variables $Z_t$ and on this base needs to estimate the vector of states $X_t$. To efficiently pursue this task, it needs to update the forecast of the previous period by applying an optimal filter to the new information borne by the signal just observed.

The most efficient estimate of vector $X_t$ will, then, take the form:

$$X_{t|t} = X_{t|t-1} + \tilde{K}(Z_t - Z_{t|t-1}),$$

$$(59)$$

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where $\tilde{K}$ is the steady state Kalman Gain Matrix relative to the dynamic system (57)–(58), in what it adjusts the last available prediction $X_{t|t-1}$ according to the latest observed one-period ahead forecast error $(Z_t - Z_{t|t-1})$. Since such a system is peculiar, because it involves the very same estimate that the optimal linear prediction procedure has to provide, the derivation of the Kalman Filter, which is presented and discussed by Svensson and Woodford [36] and Gerali and Lippi [19], in this case needs some more carefulness and departs from the standard version discussed in the appendix.

In particular, skipping all the formal derivation provided elsewhere, $\tilde{K}$ is defined according to:

$$\tilde{K} = PL' (LP L' + \Sigma_v)^{-1} \left[ I + MPL' (LPL' + \Sigma_v)^{-1} \right]^{-1},$$

(60)

where $P \equiv E[(X_t - X_{t|t-1})(X_t - X_{t|t-1})']$ is the steady state covariance matrix of the prediction errors with respect to the unobservable states, and fulfills the usual matrix Riccati equation

$$P = H'[P - PL' (LPL' + \Sigma_v)^{-1} LP] H' + C' \Sigma_{\epsilon} C_{\epsilon}.'$$

(61)

4 A Quantitative Evaluation.

This section provides a qualitative and quantitative evaluation of the model that delivers two main results. The first one indicates what policy is optimal in the face of different shocks affecting output and stock-price dynamics; the second result, instead, qualifies the impact of imperfect information in replicating in real world the outcomes implied by an Optimal Monetary Policy.

To this end, the theoretical model presented in Section 2 is first calibrated and then simulated under both assumptions of perfect and imperfect information. The analysis is further carried out through computing impulse response functions and assessing what is the dynamic response of the economy to different shocks and to what extent a Central Bank that ex ante follows Optimal Monetary Policy could ex post turn out to be achieving sub-optimal outcomes.\textsuperscript{13}

4.1 Calibration.

Table 3.1 displays the chosen parametrization of the theoretical model.

\textsuperscript{13} All the computations and the simulations in this Section are carried out by means of the MATLAB package developed and provided by Andrea Gerali and Francesco Lippi, and illustrated in Gerali and Lippi [19].
As to the structural preferences and technology parameters, the chosen values are taken from widespread convention. Thus, the quarterly discount rate is set to the value of 0.99; the elasticity of substitution of the intermediate goods with one another is fixed to 6, which implies a steady state mark-up rate of 20%; following Bernanke, Gertler and Gilchrist [4] and Gilchrist and Leahy [20], the adjustment costs for capital producers are taken to imply an elasticity $\psi$ equal to 4. The capital share in the production function $\alpha$ is set to one-third, while the coefficient of relative risk aversion $\gamma$ is taken to be 2. The quarterly depreciation rate of physical capital, again following convention, is assumed to be 2.5%. As to the parameter reflecting the costs of adjusting prices, $\phi_P$, the chosen calibrated value is 100, which yields a coefficient on marginal costs in the Phillips Curve, $\kappa = 0.05$, consistent with most related literature. Finally, to pin down the steady state shares of consumption and investments in the resources constraint, the average values across the past sixty years in the U.S. suggest 0.8 and 0.2, respectively.

When it comes to the structural shocks hitting the economy, the parameters that describe the persistence in the effects are chosen so that the temporary shocks fade out within around eight quarters, while shocks on productivity are supposed to last about twice as long. The parameters reflecting the variability of demand, cost-push and productivity shocks are taken from Ehrmann and Smets [16] to be 0.63, 0.42 and 0.13 respectively, while for the variance of the speculative bubble shock an intermediate value of 0.5 was chosen.
Finally, the period loss function was calibrated such that the Central Bank assigns equal weight to real output stabilization and interest rate smoothing.

4.2 Optimal Monetary Policy.

Within the complete information benchmark, the Central Bank observes all the state and the forward-looking variables with no measurement errors. The problem of the policy maker in this case is therefore to minimize the intertemporal loss

$$\Lambda_t = E\left\{ \sum_{\tau=0}^{\infty} \beta^\tau Y_{t+\tau}' W Y_{t+\tau} \left| \mathcal{F}_t \right. \right\},$$

conditional to the full information set

$$\mathcal{F}_t \equiv \{ Z^f_{\tau}, \tau \leq t; \ A^0, A^1, A^2, B, C_e, W, C^1, C^2, C_r, D^1, D^2, \Sigma_e \}.$$

In the quantitative model, the vector $Z^f_t$ of observables is defined as described by equation (35); the vector of forward looking variables $x_t$ collects output $y_t$, stock prices $q_t$ and inflation $\pi_t$, while the vector $X_t$ collects the lagged interest rate plus the six states commented on above (the three structural shocks, the long-run levels of output and stock prices, and physical capital):

$$X_t = [r_{t-1} \quad d_t \quad s_t \quad u_t \quad \bar{y}_t \quad \bar{q}_t \quad k_t]'$$

The presence of the lagged interest rate in vector $X_t$ is strictly technical and is required in order to define the change in interest rates as a goal variable in the loss function, according to:

$$Y_t = [\Delta r_t \quad (y_t - \bar{y}_t) \quad \pi_t]' = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{bmatrix} X_t \\ x_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r_t. \quad (62)$$

The application of the algorithm in the MATLAB Toolkit by Gerali and Lippi, yields the following optimal decision matrix:

$$F = [0.16828 \quad 0.42712 \quad 0.46639 \quad 0.48413 \quad 0 \quad 0 \quad -0.11175], \quad (63)$$

or, equivalently, the following optimal feedback rule under complete information:

$$r_t = FX_t = 0.17r_{t-1} + 0.43d_t + 0.47s_t + 0.48u_t - 0.11k_t, \quad (64)$$
which provides response coefficients whose signs and magnitude are consistent with theory and *a priori* belief, although the attitude towards the smoothing of interest rates is rather weakened relative to many other contributions.

Two considerations, though, seem especially worth underlining about equation (64) above. First, the optimal response coefficients on the long-run levels of both output and stock prices are nil; second, the coefficient on a stock-price misalignment is positive and of the same magnitude as those on the other short-term disturbances.

Let’s go over these in turn.

The second- and third-to-last terms in the vector $F$ of optimal response coefficients are zero and indicate that within an optimal monetary policy approach, the short-term interest rate should not be moved as a reaction to shocks in productivity that translate in a shift of the long-run levels of both output and stock prices. The reason for this lies in the fact that a shift in the long-run levels of output and equity prices have immediate effects in the short-run levels as well, as equations (22) and (23) clearly show, and hence no effects on the short-run deviations $(y_t - \bar{y}_t)$ and $(q_t - \bar{q}_t)$. Since, then, the task of the policy maker is to address the system towards the fastest convergence to such a long-run frictionless state, a shift in the latter does not call for any intervention whatsoever in itself, because it does not move the system any farther away from the long-run state than it was prior to the shock.

On the other hand, a speculative bubble shifts only and temporarily the short-run level of stock prices with no effects on the long-run value. The effect on stock prices then propagates to the other variables in the system through the channel of investments, pushing also real output and inflation away from their long-run targets. As a result, the effect is to temporarily shift the system out of the long-run state. Since this is exactly the target of the Central Banker, an optimizing one would move its instrument to minimize such short-run deviations.

A productivity shock and a speculative bubble, then, produce the same qualitative effect on observable current stock prices, but affect very differently the deviation of the system from the long-run target. The issue that arises, therefore, in evaluating real-time policy is what happens when the Central Bank is not able to distinguish between swings in stock prices driven by productivity shocks and those caused by stochastic misalignments, which call for different reactions; that is when incomplete information disables the policy maker’s perception of reality.
Incomplete information in the quantitative model above is characterized by the following two assumptions: first, the set of observables at \( t \) collects only the three forward-looking variables—i.e. \( Z_t = [y_t \quad q_t \quad \pi_t]' \); second, the inflation rate is observed with no error, while real output and real stock prices are subject to a measurement error with variance normalized to 1.\(^{14}\)

In such a context, the Central Bank observes noisy signals of what is happening, and is required to extract from those signals an efficient estimation of the value of the unobservable states, in order to apply the optimal decision rule that—due to the certainty equivalence—it was able to compute regardless of the informational deficiency. Although matrix \( F \) is the same as in the case of full information, therefore, the feedback rule that the nominal interest rate will actually follow is going to be \( r_t = FX_{\theta t} \).

As illustrated in the next sub-section, it might be the case that the actual achievements under incomplete and complete information be substantially different, although both resulting from an Optimal Monetary Policy strategy.

### 4.3 The Dynamic Response of the Economy.

In what follows I report and discuss the impulse-response analysis of the model, in which the emphasis is given to the role of the productivity shock as opposed to a stochastic stock-price misalignment in driving the system dynamics.\(^{15}\)

Figure 1 plots the dynamic response of several variables to a persistent productivity shock, under the assumption of full information.

Being the information available to the Central Bank complete, the productivity shock is perfectly identified and, as a result of optimal policy, the interest rate does not move at all (fifth box). The Central Bank, in fact, realizes that if the policy instrument were left untouched, the real effects of the shock would be to raise both current and long-run output, affecting neither the output gap nor the inflation rate. As a matter of fact, the actual result of such an optimal policy is that the dynamics of current real output mirrors the one of its long-term level—just as current stock prices follow the very same dynamic response of their long-run component—so that the output gap is not at all affected by the disturbance (last

---

\(^{14}\)The assumption that inflation is correctly measurable relative to the other variables is taken from Ehrmann and Smets [16].

\(^{15}\)The results of the simulations in the case of demand and cost-push shocks are of little, if any, innovative value, both for the scope of this paper and in comparison to existing literature, to which however they are totally consistent.
Figure 1: Macroeconomic dynamic response to a persistent productivity shock under Optimal Monetary Policy and Full Information

box). Moreover, zero-deviations of output and stock prices from their own long-run levels imply a flat dynamics for inflation, depicted in the fourth box; the target of zero-inflation is therefore perfectly hit, notwithstanding the only partial conservativeness of the Central Bank. Finally, with both inflation and nominal rates standing still, also the real interest rate remains unaffected, so that the dynamics of output and stock prices is just the one stimulated by the productivity shock alone; which in the end explains why the dynamic pattern of $\bar{y}_t$ is exactly followed by $y_t$.

Figure 2, on the other hand, displays what happens after a temporary stock-price mis-

\footnote{Typically, the less conservative the Central Bank, the more unlikely that the inflation target is hit, because of the short-run trade-off between price and output stability. The only case in which it can hit perfectly a target is when also the other ones can be met exactly, like in the case above, in which both the output gap and the interest rate change are nil throughout the transition.}
alignment, when the Central Bank pursues Optimal Monetary Policy and the information it has access to is full.

Again, the policy maker does not incur in any missperception of what is hitting the economy, so that the signal extraction not just is efficient but is in fact trivial, in what the true shock is fully and directly observable.

The observation of the shock allows the monetary authority to actually see the system moving out of the long-run state: it knows that higher stock prices relative to their potential level would push investments and output beyond their long-run dynamic path, which would cause a higher output gap, in addition to a higher stock-price-gap, and mild inflationary pressures; all of which results in the Central Bank moving the interest rate to correct the dynamics towards the fastest convergence to the steady-potential state.

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Given the chosen calibration, the optimal impact reaction would be to raise the short-term nominal interest rate by a factor of 0.47, as indicated by the optimal feedback rule. The dynamics thereafter involves an increase in stock prices, an almost equivalent increase in output, but also a mild depression of inflation.\textsuperscript{17} The effect on inflation is due to the degree of conservativeness assumed: since the Central Bank explicitly targets real stability and the inflationary pressures are relatively modest, the reaction of the interest rate goes beyond what would be sufficient to hit the target for the inflation rate. The policy maker is, in fact, prepared to pay a small price in terms of price stability to achieve a greater result in keeping the output gap close to target.

The outcome of such an optimal policy is a rather good performance of the Central Bank in keeping the output gap on target, for which a small price in terms of inflation stability is paid—the more so the less conservative is the Central Bank. As shown by the last box of Figure 2, in fact, just one period after the shock the output gap has fully absorbed the impact effect and is back on target, and also inflation, on which the impact effect was already four times smaller that the one on the output gap, is virtually back at its long-run level.

The first result, therefore, indicates that an exogenous stochastic stock-price misalignment, \textit{when perceived with certainty as such}, calls as an optimal response a monetary tightening, while a technology shock requires no intervention at all.

Turning to the effects of incomplete information, if the policy maker has only access to non-exhaustive and noisy signals, the overall \textit{ex post} achievements of an \textit{ex ante} optimal policy might be different from above.

Considering again a productivity shock as opposed to an exogenous misalignment in stock prices, the incompleteness of the information available to the policy maker does not allow to identify with precision what kind of shock actually hit the economy. The ultimate outcome is a 28\% welfare loss relative to the case of complete information (the value of the minimized intertemporal loss function is 106.7582 under incomplete information against 83.4432 if the information is full).

In the case of a positive productivity shock, the Central Bank measures a raise in both stock prices and output, and from those signals it has to extract some efficient information about the underlying true state of the economy. As Figure 3 shows, therefore, blindness

\textsuperscript{17}Since the speculative bubble does not affect any long-run level, the dynamics of real output is the same as that of the output gap, depicted in the last box of Figure 2.
about the states and measurement errors about the observables imply that the policy maker is, in fact, interpreting the signals only partially—and also marginally—as due to the true productivity shock (box four), assigning a substantial part of the responsibility for the available observations also to a positive demand shock and a stock-price misalignment (boxes one and two).

To these perceived states, the Central Banker applies the optimal policy he was able to derive, leading to the macroeconomic effects depicted in Figure 4.

The missperception of a combination of short-run shocks induces the policy maker to react through raising the interest rate, in the attempt to counteract the perceived deviation from the long-run state. As a consequence, the interest rate rises in a context in which it would be optimal for it to stay put, given the true state of the economy, and the difference

![Impulse Responses under Incomplete Information](image)

**Figure 3:** Central Bank’s perception following a productivity shock
between this reaction and the one shown under complete information is the “optimal policy bias”. As a result, the *erroneously tight money* holds back the level of current output relative to the raise in potential output generated by the productivity gain (current output grows by a factor just below 0.2, while the raise in potential output reflects the unitary productivity shock and is, therefore, 1). The level of output gap, hence, declines—see the second box from the bottom in Figure 4—as a direct consequence of the policy action undertaken, because it does not let current output grow as much as its long-run counterpart, and as much as it would in the case of no intervention; and the same happens to stock prices. The Central Bank, though, does not see this. What it does see—displayed in the last box—is a positive impact effect on the output gap, based on an underestimated raise in long-term output, and this further justifies the persistence in the restriction outlined by the dynamics of the interest rate.

Figure 4: The effects of a productivity shock under Incomplete Information
The comparison of the last two boxes in Figure 4 catches the main point made by the related literature: small imperfections in the structure of information may, by itself, lead to sub-optimal outcomes and a persistent bias even in the case that the signal-extraction is most efficient and monetary policy is optimally designed.

This is exactly the point stressed out by Cukierman and Lippi [14]: in the case innovations in potential output—as productivity shocks—are not directly observable, also an ex ante optimal monetary policy incurs in retrospective mistakes, which are not avoidable in real time, because they are entirely due to the informational deficiency and not to any inefficiency on the part of the policy maker.

Thus, in case of a positive productivity shock (which would not call for any intervention), since it is not distinguishable in real time from any positive short-run disturbance (which instead would), the Central Bank erroneously moves its policy instrument and achieves sub-optimal results through no fault of its own, because that policy is optimal in real time. Through the same argument, therefore, one may argue that in case of a bubble-driven boom in stock prices, since it cannot be distinguished from one driven by productivity gains, a real time optimal policy should be to react very slightly, or to not react at all.

As it turns out, this statement would be partially incorrect.

As Figure 5 shows, also a unitary innovation in the stochastic process that generates exogenous stock-price misalignments is perceived in real time as a combination of different short-run and long-run disturbances, which make up the estimate of the vector of states $X_{t|t}$. In particular, within the quantitative model proposed, the Central Bank extracts from the signals it observes only about 50% of the misalignment; as to the rest, it is taken as coming from a positive demand shock and a positive productivity shock. As a result, applying the matrix of optimal response coefficients $F$ to the estimated states yields the induced optimal dynamics for the interest rate, displayed in the third box of Figure 6.

As fairly clear from the plot, the impact effect on the policy instrument is qualitatively similar and quantitatively about 25% smaller and more temporary compared to the benchmark case depicted in Figure 2 (around 0.35 when the information is incomplete against about 0.47 in the case of full information). The optimal monetary policy bias then, in this case, is relatively smaller compared to the case of a productivity shock.

However, the argument made apropos of a shock hitting the long-run state, rather than

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18Almost no responsibility for the observed signals is assigned to cost-push shocks, primarily because of the assumption that prices are observable with no measurement errors.
just the current level of variables, still retains its validity, at least in principle. When the observed boom in stock prices is generated by an unobservable exogenous misalignment, in fact, the Central Bank suffers from the same deficiencies as in the case of a hidden technology shock: it is unable to properly and with certainty tell whether that was a bubble or a productivity push-up; it incurs in missperceptions that form the belief that that was a combination of different shocks; it applies the optimal response coefficients to an estimate of the states rather than to the actual vector $X_t$. In a word, the resulting optimal monetary policy is biased in this case as well. What makes the difference between the two cases, then, is that whatever a true productivity shock is confused with, it is going to be a kind of shock which prescribes the opposite optimal response, because any other assumed shock does, in fact, so; on the other hand, confusing a stock-price misalignment with a

\[\text{Impulse Responses under Incomplete Information}\]

\[\text{Periods}\]

\[0\quad 0.2\quad 0.4\quad 0.6\]

\[\text{From: bubble shock}\]

\[\text{To: } qbar|t\]

\[\text{To: outp}|t\]

\[\text{To: stock}|t\]

\[\text{Loss under discretion: 106.7582}\]

Figure 5: Central Bank’s perception following a stock-price misalignment
combination of three short-run shocks and one shock on the long-run state, can only yield a similar, though milder, optimal response, because any short-term shock requires a reaction that is qualitatively the same and quantitatively very close—see equation (63).

As to the *ex post* performance of the policy maker in terms of output gap and inflation, Figure 6 shows that under incomplete information optimal policy, although biased, performs actually better than under full information in keeping inflation close to the target, at the price of a more volatile output gap (this is due to the smaller and more temporary raise in the policy instrument).

The second finding, therefore, indicates that, given efficient signal-extraction, incomplete information implies a substantial bias in the optimal policy only when it comes to unobservable long-run shocks, while missperceptions about short-term disturbances do less harm. Within an optimal policy strategy, in fact, a stock-price misalignment, *even when*
only partially perceived as such, stimulates an active intervention on the level of the interest rate, which turns out to be only slightly biased with respect to the case of full information; a technology shock, on the other hand, results in a policy action which is in itself sub-optimal.

5 Concluding remarks.

The recent events that shook the stock markets around the world and started the first serious global economic slow-down since almost twenty years ago, animated further the debate about what are the links between equity prices and real activity and, more importantly for the present purposes, what should be the role of monetary policy in dealing with such interactions.

Most of the literature about monetary policy and stock prices is divided between two main positions: the first maintains that an explicit concern about financial stability would yield welfare gains and thus be desirable, while the second warns that swings in stock prices are either fundamental—and thus taken care of by a flexible inflation targeting strategy—or speculative—thus undetectable and however only temporary, either way leaving no room for any explicit concern on the part of the policy maker.

This paper analyzes the issue within an Optimal Monetary Policy framework, in which the Central Bank acts as an optimizing regulator, and qualifies the role of information in determining the actual outcomes of monetary policy. In particular, the analysis focuses on what happens when the Central Bank cannot detect with certainty whether the observed swings in stock prices are truly caused by fundamental factors like technology shocks or non-fundamental stochastic misalignments.

Under the benchmark assumption of full information an optimizing Central Bank is able to detect with no errors any source of disturbance to the economy, undertaking by definition the best possible actions and achieving the best possible outcomes. Under incomplete information, instead, the policy makers have only access to limited and noisy observation of a sub-set of variables, on the base of which they then have to form efficient estimates of what is not directly measurable.

Simulating a Dynamic New Keynesian model with variable physical capital and nominal rigidities, and comparing the Optimal Monetary Policy’s outcomes under complete and incomplete information, yield two interesting results.

When the Central Bank knows with certainty what shock hits the economy, Optimal
Monetary Policy prescribes and achieves rising interest rates in the face of a stock-price misalignment and no intervention at all in the case of a technology shock.

A disturbance on productivity, in fact, shifts both long-run and current levels of the relevant variables; when they know with certainty what shock they have to deal with, the policy makers realize that the effects on short-term deviations from the long-term state would be nil and as a consequence do not move their instrument. The outcome is the best possible: inflation does not move from the zero-target and neither does the output gap.

On the other hand, in the case of a stock-price misalignment, the fully informed Central Bankers are aware that the only effects would be on the current state of the economy and in no way on the long-run state; accordingly, then, they set the interest rate in order to counteract the pressures that higher stock prices would exert on inflation and output through the channel of investments. The outcome is a sudden return of the output gap back on target (it takes just one period given the assumed degree of conservativeness) and a mild and temporary reduction in inflation.

When the Central Bank only observes noisy signals and not the true shocks, Optimal Monetary Policy is generally and persistently biased, since the identification is only partial and the perception is typically a combination of different shocks, each possibly associated with a different optimal response.

The optimal policy bias, though, assumes different dimensions and relevance depending on the true underlying shock. In case of a positive productivity shift, the estimated vector of states assigns some role to three different kinds of disturbances which all require intervention, while only a minimal attribution of the observed rising output and stock prices is given to the true shock. Consequently, the policy maker perceives a higher output gap relative to the target, when the actual one is, in fact, declining. As a result, the optimal decision rule applied to the most efficient estimation of the state of the system prescribes a policy action that is optimal in real time but retrospectively erroneous: a raise in the nominal interest rate.

When the true shock is a stock-price misalignment, instead, the Optimal Monetary Policy bias is only quantitative and rather small. Imperfect information, in fact, does not allow a complete identification of the shock, but efficient signal-extraction yields an estimated vector of states which assigns little role to a productivity shock—the only one that could generate a substantial bias. As to the rest, the Central Bank perception involves
partially the true misalignment and partially a positive demand shock. Since both require as an optimal response a monetary tightening, then the misperception costs a smaller price than in the previous case. Relative to the full information benchmark, thus, the final outcome will be a smaller and more temporary raise in the policy instrument, which translates in a more persistent deviation of the output gap from the target in comparison with the policy maker’s perception.

The Central Bankers, therefore, in this case take the right decision (and get to achieve almost the best outcome) but for the wrong reason, or at least for a reason different than they think.

References


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