Financial Liberalization: Efficiency Gains and Black-Holes*

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Abstract

We analyze the gains and costs of financial liberalization in the presence of systemic bailout guarantees. Empirically, financial liberalization spurs mean long-run growth, but it does so via lending booms that are punctuated by costly crises. These growth benefits have derived mostly from aggregate productivity gains associated with higher allocative efficiency. In some extreme cases, however, unfettered liberalization has lead to a break-down of financial discipline and large-scale funding of unproductive activities. We propose a framework that replicates these facts and helps understand how, in the presence of bailout guarantees, regulation shapes the outcome of financial liberalization. We consider a two-sector model under alternative regulatory regimes. Under financial repression, borrowing constraints in the input sector lead to underinvestment, which results in resource misallocation and low growth. Liberalization allows for new financing instruments, which relax the constraints and reduce misallocation. However, the use of new instruments generates new states of the world in which insolvencies occur, and so a riskless economy is endogenously transformed into an economy prone to rare crises. If only standard debt is allowed, liberalization preserves financial discipline and it brings the allocation closer to the Pareto efficient level, increasing average growth and consumption possibilities. By contrast, under an anything-goes regime where agents can issue option-like liabilities without having to post collateral, financial discipline collapses as unproductive projects are funded and production efficiency falls: a financial black-hole.

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1 Introduction

Financial liberalization tends to enhance growth, but it also generates greater crisis-volatility, induced by risk-taking and lending booms. Here, we analyze the gains and costs of financial liberalization in a setup that incorporates this growth-crisis trade-off.

The paper makes three contributions. The first contribution is positive. Given the availability of new micro-level data sets, we now know much more about the key empirical regularities associated with financial liberalization, crises and growth. This paper provides a theoretical framework to integrate them. The second contribution is normative. Our framework allows us to decompose the effects of financial liberalization into gains from higher production efficiency and losses associated with a higher incidence of crises. Third, the paper contributes to the debate on financial regulatory design. It helps understand how, in a world with systemic bailout guarantees, the regulatory environment shapes the outcome of financial liberalization.

We show that, even when taking into account the costs of crises and the existence of bailout guarantees, there are net gains from liberalization provided that regulatory limits on the types of issuable liabilities ensure borrowers risk their own capital. The micro-level risk-taking mechanism by which liberalization spurs growth—and which is motivated by firm-level evidence—generates aggregate booms that tend to be punctuated by rare crises. With regulatory limits, these booms tend to be associated with the funding of productive investment. Without regulatory limits, the proliferation of liabilities that concentrate most repayments in crisis states, can undermine and even overturn the gains from liberalization because there are bailout guarantees. In this case, liberalization turns into an anything-goes regime, in which the breakdown of financial discipline leads to large scale funding of unprofitable projects and to a reduction in production efficiency: a financial black-hole.

In this paper financial liberalization enhances growth and consumption possibilities because it improves allocative efficiency. This channel is important in economies where financial frictions hinder the growth of sectors that are more dependent on external finance. By allowing for new financing instruments and the undertaking of risk, liberalization relaxes the financing constraints. As a consequence, sectors more dependent on external finance invest more and grow faster. The rest of the economy benefits from this relaxation of the bottleneck via input-output linkages, and so there is an increase in aggregate growth, production efficiency and consumption possibilities. However, because financial liberalization induces credit risk-taking, it generates financial fragility and might lead to crises, which although rare, are severe.

Here, we analyze this trade-off between risk-taking, growth and production efficiency in a two-sector model with financial frictions. Our model is designed to capture three prominent empirical
regularities associated with financial liberalization. First, although crises have been costly, countries that have liberalized financially, and have experienced booms and busts have been, on average, growing faster than non-liberalized countries.\(^1\) Second, financial liberalization spurs aggregate growth mainly through TFP gains rather than aggregate capital accumulation.\(^2\) Such aggregate TFP gains are associated with a sectoral reallocation of resources. Typically, following liberalization sectors more dependent on external finance grow more, but crash more severely during a crisis and subsequently suffer a greater decline during the credit crunch.\(^3\) Third—implicit and explicit—guarantees to bailout lenders during systemic crises have been widespread the world over.\(^4\)

The argument relies on how, in the presence systemic bailout guarantees, the financial regulatory regime influences financing decisions, and on how financing constraints in one sector affect the performance of the whole economy via input-output linkages. In a financially repressed economy, there is misallocation because the input producing sector depends on external finance to fund its investment and faces borrowing constraints due to contract enforceability problems. These constraints generate a bottleneck that limits the supply of intermediate inputs for the final-goods sector, and thus impacts negatively the growth performance and the production efficiency of the economy as a whole.

Because both sectors compete every period for the available supply of inputs, when contract enforceability problems are severe, the input producing sector attains low leverage and commands only a small share of inputs for investment: there is a misallocation of inputs which results in a socially inefficient low aggregate growth path. A central planner would increase the input sector investment share to attain the Pareto optimal allocation. In a decentralized economy, the first best can be attained by reducing the enforceability problems that generate the financing constraints. However, if such a reform is not feasible, financial liberalization, may be seen as an alternative way to improve the allocation despite the financial fragility it entails.

Financial liberalization allows for new financing instruments, which leads to a relaxation of the constraints and the bottleneck. However, and this is key, the use of the new instruments generates new states of the world in which insolvencies occur, and so a riskless economy is endogenously transformed into a risky one. Our framework provides an internally consistent mechanism under which such transformation emerges, and helps understand how it can enhance long-run growth and consumption possibilities, even though occasional crises occur during which the input sector suffers

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\(^1\)Bekaert, et.al. (2005), Bekaert, et.al. (2011), Ranciere et.al. (2008), and Henry (2007).


\(^3\)Dell’Arricia, et.al. (2008), Klingebiel, et.al. (2007), Levchenko, et.al. (2009), Gupta et.al. (2009), and Galindo et al. (2002).

\(^4\)Jeanne and Zettlemeyer (2001), Ranciere, et. al.(2008), and Kelly et.al. (2011)
the costs associated with widespread bankruptcies.

In order to analyze the link between financial regulation and production efficiency, we consider two classes of one-period securities—standard bonds and catastrophe bonds—and three financial regulatory regimes: repression, liberalization and an anything-goes regime. With standard bonds a debtor must promise to repay the same nominal amount in all states, and if it fails to repay it must default. In contrast, with catastrophe bonds a debtor can promise to repay an arbitrarily large amount in bad states and nothing in good states.\(^5\)

Under financial repression firms can only issue standard bonds and must denominate repayments in the good which it produces—i.e., cannot take on insolvency risk. In a liberalized regime firms can only issue standard bonds, but can take on risk through a mismatch between the unit of the good they produce and the unit of the good on which they denominate their liabilities.\(^6\) Finally, under the anything-goes regime firms can issue both standard and catastrophe bonds, and can take on insolvency risk.

Under financial repression there exists only one equilibrium where insolvencies and crises never occur. If there are significant contract enforceability problems, along this safe equilibrium the input sector exhibits low growth because its investment is constrained by its cash flow. Financial liberalization allows the input sector to denominate debt in units of final goods.\(^7\) Such currency mismatch is individually profitable if there is a possibility of a sharp decline in the input price that would bankrupt a critical mass of borrowers and trigger a bailout (i.e., a systemic crisis). This micro-level risk taking generates systemic risk when a critical mass of agents undertakes it. Under such set of expectations, currency mismatch reduces interest costs and relaxes borrowing constraints. However, it also generates financial fragility, as a shift in expectations can cause a sharp fall in the relative price of inputs, bankrupt input producing firms and land the economy in a crisis.

In order to address the growth-stability trade-off the model captures two costs typically associated with crises: bankruptcy and financial distress costs. Bankruptcy costs are static and derive from the severe input price decline that leads to firesales and bankrupts input sector firms with mismatch on their balance sheets. Financial distress costs are dynamic and derive from the resultant collapse in internal funds and the reduction in risk taking in the aftermath of crisis, which

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\(^5\)The issuance of catastrophe bonds corresponds, for instance, to the sale of options and credit default swaps without collateral.

\(^6\)In the context of emerging markets, this mechanism corresponds to the famous currency mismatch by which firms in nontradables sectors, issue liabilities denominated in foreign currency.

depresses new credit and investment, hampering growth.

Our first result is that a liberalized–financially fragile–economy will on average grow faster than a repressed–safe–economy even if bankruptcy costs are arbitrarily large, provided that the dynamic crisis costs are not too severe. This result follows from the fact that crises must be rare in order for them to occur in equilibrium, and from the fact that not all the bankruptcy losses experienced by the input producing sector during crises are aggregate deadweight losses. The financial distress costs of crises can be far more significant than bankruptcy costs because they spread dynamically: the decline in internal funds and the reduction in risk taking translate into depressed leverage and investment in the input sector that reduces aggregate growth.

Our second result is that when contract enforceability problems are severe enough credit risk improves allocative efficiency and brings the allocation nearer to the Pareto optimal level. Furthermore, it increases the present value of consumption that the economy can attain, even when we net out the fiscal costs of bailout guarantees, as long as crisis are rare and the associated financial distress costs are not too large.

The efficiency benefits described above rely on the fact that the increase in leverage occurs without losing financial discipline. In the model this discipline comes about by limiting external finance to standard debt contracts under which agents must repay in all states to avoid bankruptcy. Because of contract enforceability problems, lenders require that borrowers risk their own equity by imposing borrowing constraints. In this way the incentives of borrowers and creditors are aligned in selecting only projects with a high enough expected return. Importantly, systemic bailout guarantees do not undermine this discipline because they are granted only in case of a systemic crisis, not if an idiosyncratic default occurs.

This discipline breaks down in an anything-goes regime if bailout guarantees are present. Our third result is that allowing for the issuance, without collateral, of catastrophe bonds that pay zero in good states and promise a huge amount if a–rare–crisis occurs, reduces production efficiency. This is because such bonds allow for the funding of unproductive projects with a negative contribution to national income. These inferior projects are privately profitable because they are a means to exploit the subsidy implicit in the guarantee. A firm with a non-profitable project could issue bonds that promise to repay only in a crisis state. Lenders would be willing to buy them without requiring collateral because they expect the promised repayment to be covered by the bailout. As a result the firm can fund inferior projects without risking its own equity, and bet that the project turns out a large profit in good states.

These theoretical results allow us to contrast the experience of emerging markets following financial liberalization, with the recent U.S. experience. Emerging markets’ booms have featured
mainly standard debt, and while they have experienced crises (the so called ‘3rd generation’ or balance-sheet crises) systemic risk-taking has been, on average, associated with higher long-run growth. In contrast, the recent U.S. boom featured a proliferation of uncollateralized derivatives that supported large-scale funding of negative NPV projects in the housing sector.\textsuperscript{8} According to our model, these contrasting experiences highlight the key role of regulatory limits in a world with systemic bailout guarantees. In the absence of limits on the type of liabilities that can be issued without collateral, financial discipline vanishes and the efficiency gains of financial liberalization are overturned.

The rest of the paper is structured as follows. Section 2 relates our paper to the literature. Section 3 presents the model. Sections 4 and 5 analyze long-run growth and production efficiency under financial repression and financial liberalization. Section 6 considers the anything-goes regime and characterizes a black-hole equilibrium. Section 7 concludes.

2 Outline and Related Literature.

We consider an endogenous growth model where the financially constrained input sector is the engine of growth. In a nutshell, the key determinant of aggregate growth and production efficiency is the share of intermediate goods production that is used for investment in the intermediate goods sector:

\text{Investment share: } \phi_t = \phi(\text{internal funds, financial regime, enforceability problems}) \tag{1}

The investment share $\phi_t$ determines the production of intermediate inputs and final goods through input-output linkages that in equilibrium take the following simple form

\begin{align*}
\text{Intermediate good: } q_{t+1} &= q(I_t), \\
\text{Final good: } y_t &= y(d_t), \\
I_t &= q_t \cdot \phi_t, \\
d_t &= q_t \cdot [1 - \phi_t] \tag{2}
\end{align*}

In equilibrium $\phi_t$ is determined by the interaction of bailout guarantees with contract enforceability problems. This interaction depends crucially on the regulatory regime. Under financial repression the $\phi_t$-sequence is smooth, but it can be inefficiently low and result in slow aggregate growth. Under financial liberalization the $\phi_t$-sequence has a higher mean, but it exhibits sharp and sudden contractions—associated with crises. The underlying mechanism is that when agents coordinate on systemic risk-taking—and by doing so exploit systemic bailout guarantees—they attain higher leverage, which increases investment and growth, but it also makes the economy vulnerable to crises.

\textsuperscript{8}Ranciere and Tornell (2011), and Levitin and Wachter (2011).
By emphasizing the link between borrowing constraints, sectoral misallocation and input-output linkages, this paper relates to Jones (2011a, 2011b) who emphasizes the consequence of resource misallocation in terms of intermediate inputs and its effects on aggregate productivity through input-output linkages.\textsuperscript{9} Analogous to the Jones’ steady-state input-output multiplier, in our setup higher production efficiency results from a dynamic input-output multiplier: an increase in today’s investment in the intermediate input sector ($\phi_i$) increases tomorrow’s production in the final good sector.

We show that, despite bailout costs and bankruptcy costs, a shift from a repressed to a liberalized regime—with regulatory limits—can increase aggregate growth, production efficiency and the present value of consumption. In contrast, a shift from financial repression to an anything-goes regime—without regulatory limits—can reduce production efficiency and create a financial black-hole, in which unproductive projects are funded. These results are linked to a vast empirical literature on the growth effects of financial liberalization. Henry (2007) and Bekaert, Harvey and Lundblad (2005) find that it is generally growth enhancing, but earlier literature obtains more mixed results (Edison et. al., 2004). A reason for this is that financial liberalization has typically lead both to higher growth and to more frequent crises. This dual effect is at the core of our theoretical mechanism. Ranciere, Tornell, Westermann (2006) and Bonfiglioli (2008) find robust evidence for this dual effect of financial liberalization. The average growth gains in tranquil times dominate the output costs associated with a higher propensity to crisis.\textsuperscript{10}

Bonfiglioli (2008), Kose, Prasad and Terrones (2009), Bekaert, Harvey and Lundblad (2011) find that the growth gains from financial liberalization come from an increase in aggregate TFP rather than from an increase in aggregate capital accumulation. Our model predicts that financial liberalization promotes a more efficient allocation of intermediate inputs across sectors and therefore increases aggregate TFP. Galindo, Schiantarelli and Weiss (2007) construct indexes of efficiency in the allocation of investment based on sales or profits per unit of capital for listed firms in 12 developing countries and find that financial liberalization improves allocative efficiency. Abiad, Oomes and Ueda (2008) provide similar evidence for such an allocative efficiency effect by comparing the dispersion of Tobin’s Q among listed firms in five emerging markets before and after financial liberalization.

\textsuperscript{9} A connected literature (e.g. Restuccia and Rogerston, 2007; Hsieh and Klenow, 2009) focuses on the aggregate TFP consequences of distortions causing resource misallocation between firms within sectors.

\textsuperscript{10} These results are related to Kaminsky and Schmukler (2008) who find that financial liberalization increase stock market volatility in the short run but reduce it in the long run or Loayza and Ranciere (2006) who show that financial development can reduce growth in the short run - through higher volatility and the incidence of crises - but increase it in the long run.
Levchenko, Thoenig and Ranciere (2008) find, using sector-level data, that sectors more dependent on external finance grow more and become more volatile after financial liberalization. In our model, the N-sector depends on external finance to fund investment, but the T-sector does not. In a liberalized regime the N-sector grows faster than the T-sector as long as a crisis does not occur. As a result, N-goods become cheaper and more abundant which, in turn, fosters more growth in the T-sector. However, liberalization also generates crisis risk. During crises, the N-sector suffers from severe financial distress costs and experiences a credit crunch that sharply reduces investment and output. Dell’Arricia, Detragiache, and Rajan (2008) and Kroszner, Laven and Klingebiel (2007) find indeed empirical evidence that sectors more dependent on external finance suffer disproportionately more during financial crises. Related, but based on a completely different setup, Buera, Kaboski and Shin (2009) show how a relaxation of financial constraints can result in more efficient allocation of capital and entrepreneurial talent across sectors.

Other theoretical papers emphasize the welfare gains from financial liberalization stemming from intertemporal consumption smoothing (Gourinchas and Jeanne, 2006), better international risk-sharing (Obstfeld, 1994) and better domestic risk-sharing (Ueda and Townsend, 2006). Gourinchas and Jeanne (2006) show that the welfare benefits associated with this mechanism are negligible in comparison to the increase in domestic productivity. The gains from risk-sharing can be much larger: Obstfeld (1994) demonstrates that international risk-sharing, by allowing a shift from safe to risky projects, increases strongly domestic productivity, production efficiency and welfare. In our framework the gains also stem from an increase in production efficiency but not from risk-sharing. The gains derive from a reduction of the contract enforceability problem not of the incomplete markets problem: efficiency gains are obtained by letting entrepreneurs take on more risk, not by having consumers face less risk. In Tirole and Patak (2007) currency mismatch also results in social welfare gains, but through a discipline effect on government policy, not through a better allocation of resources.

Systemic bailout guarantees play a crucial role in our framework. By affecting collective risk-taking and the set of fundable projects, they shape the growth and production efficiency effects of a regulatory regime. While there is ample evidence of ex-post systemic bailouts, evidence on bailout expectations is more difficult to obtain. By comparing the pricing of out-of-the money put options on a financial sector index with option on individual banks forming the index, Kelly, Lustig and Niewerburgh (2011) show that systemic bailouts, but not idiosyncratic bailouts are

Looking at the finance-growth nexus at the sector-level, Samaniego and Ilyina (2010, 2011) show that what really matters is the interaction between the ability to raise external finance and the need for such financing in order to fund growth-enhancing investment.
expected. Using firm-level data on loan pricing for a large sample of firms in Eastern Europe, Ranciere, Tornell and Vamvakidis (2010), find that some form of bailout expectations is necessary to rationalize the differences in the pricing of foreign and domestic currency debt across firms. Farhi and Tirole (2011) demonstrate how time-consistent bailout policies, designed by optimizing governments, generate a collective moral hazard problem that explains the wide-scale maturity mismatch and high leverage observed in the US financial sector before the 2007-2008 crisis.

Finally, the cycles generated by our model are very different from Schumpeter’s (1934) cycles in which the adoption of new technologies plays a key role. Our credit cycles are more similar to Juglar’s credit cycles (Juglar, 1863).

3 Model

We consider a sufficiently rich model so as to reproduce the empirical facts described above, but tractable enough so that the equilibria can be solved in closed-form and we can characterize analytically the relationships between regulation, systemic risk-taking, production efficiency and growth. We embed into a two-sector endogenous growth model the credit market game of Schneider and Tornell (2004), in which systemic-risk derives from the interaction of contract enforceability and bailout guarantees. A simpler one-sector framework would not be able to capture the empirical link between the regulatory regime and sectoral misallocation nor to explain systemic risk-taking and the resulting aggregate boom-bust cycles as an endogenous response to liberalization policies.

There are two goods: a final consumption good (T), and an intermediate good (N), which is used as an input in the production of both goods. We let the T-good be the numeraire and we denote the relative price of N-goods by $p_t = p_t^N / p_t^T$.  

Agents. There are competitive risk neutral international investors whose cost of funds equals the world interest rate $r$. These investors lend any amount as long as they are promised an expected

\footnote{Relatedly, by looking at differences in stock returns between large and small banks, Ghandi and Lustig (2009) provide evidence of an implicit guarantees on large banks in the US economy but not on small banks.}

\footnote{Bailout expectation are necessary to explain why: (i) firms in the non-tradable sector with currency mismatch on their book borrow at a cheapest rate than similar firms in the non-tradable sector but with no currency mismatch. (ii) the spread in interest rate between foreign and domestic currency debt is not significantly different for firms in the non-tradables and firms in the tradables sector.}

\footnote{Juglar characterizes asymmetric credit cycles along with the periodic occurrence of crises in France, England, and United States between 1794 and 1859. He concludes that: “The regular development of wealth does not occur without pain and resistance. In crises everything stops for a while but it is only a temporary halt, prelude to the most beautiful destinies.” Juglar (1863), page 13 (our translation).}

\footnote{In an international setup $p_e$ is the inverse of the real exchange rate.}
payoff of $1 + r$. They also issue a default-free T-bond that pays $1 + r$ next period.

There are overlapping generations of consumers that live for two periods and have linear preferences over consumption of T-goods: $c_t + \frac{1}{1 + r}c_{t+1}$. Consumers are divided into two groups of measure one: workers and entrepreneurs.

Workers are endowed with one unit of standard labor. In the first period of their life, a worker supplies inelastically his unit of labor ($l_t^T = 1$) and receives a wage income $v_t^T$. At the end of the first period, he retires and invests his wage income in the risk-free bonds.

Entrepreneurs are endowed with one unit of entrepreneurial labor. In the first period of her life, a young entrepreneur supplies inelastically one unit of entrepreneurial labor ($l_t = 1$) and receives a wage $v_t$. At the end of the first period, she starts running an N-firm and makes investment decisions. In the second period of her life, she receives the firm’s profits, if any.

Production Technologies. There is a continuum, of measure one, of firms run by entrepreneurs that produce N-goods using entrepreneurial labor ($l_t$) and capital ($k_t$). Capital consists of N-goods invested during the previous period ($I_{t-1}$), which fully depreciates after one period. The production function is

$$q_t = \Theta_t k_t^\beta l_t^{1-\beta}, \quad \Theta_t =: \Theta k_t^{1-\beta}, \quad k_t = I_{t-1}, \quad \beta \in (0, 1)$$

The technological parameter $\Theta_t$ embodies an external effect, where $\overline{k_t}$ is the average N-sector capital, that each firm takes as given.

There is a continuum, of measure one, of competitive firms that produce the T-good combining standard labor ($l_t^T$) and the N-good ($d_t$) using a Cobb-Douglas technology: $y_t = ad_t^\alpha (l_t^T)^{1-\alpha}$. The representative T-firm maximizes profits taking as given the price of N-goods ($p_t$) and standard labor wage ($v_t^T$).

$$\max_{d_t, l_t} \left[ y_t - p_t d_t - v_t^T l_t^T \right], \quad y_t = ad_t^\alpha (l_t^T)^{1-\alpha}, \quad \alpha \in (0, 1).$$

There is an alternative–inferior–technology to produce T-goods that will only be active in the financial black-hole equilibrium considered in Section 6. This technology uses only T-goods as inputs according to:

$$y_{t+1} = \varepsilon_{t+1} I_t^T, \quad \varepsilon_{t+1} = \begin{cases} \varepsilon & \text{with probability } \lambda, \\ 0 & \text{with probability } 1 - \lambda \end{cases}, \quad \varepsilon \leq 1 + r,$$

where $I_t^T$ denotes the input of T-goods.

Firm Financing. The investable funds of a firm consist of its internal funds $w_t$ plus the liabilities $B_t$ it issues. These investable funds can be used to buy default-free bonds $s_t$ or buy N-goods ($p_t I_t$)
to produce next period. It follows that the time $t$ budget constraint and time $t+1$ profits of an N-firm are, respectively:

$$p_t I_t + s_t = w_t + B_t$$

$$\pi(p_{t+1}) = p_{t+1} q_{t+1} + (1 + r) s_t - v_{t+1} l_{t+1} - L_{t+1},$$

where the cash flow of the firm equals the entrepreneur’s wage ($w_t = v_t$), and $L_{t+1}$ is the next period’s promised debt repayment, which we describe below. Since T-firms produce by combining instantaneously labor and intermediate inputs, they do not require financing.

There are two types of one-period bonds: standard bonds and catastrophe bonds. Under standard bonds a firm must promise to repay the same nominal amount in all states. In contrast, with catastrophe bonds a debtor can promise to repay an arbitrarily large amount in bad states and zero in good states.

Standard bonds can promise to repay in either $N$-goods or $T$-goods, with respective interest rates $\rho_t$ and $\rho_t^N$. It follows that if the firm issues $b_t$ T-bonds and $b_t^N$ N-bonds, the promised debt repayment is

$$L_{t+1} = (1 + \rho_t) b_t + p_{t+1} (1 + \rho_t^N) b_t^N.$$  

If at $t+1$ the firm does not repay, then it must default.

**Credit market imperfections.** Firm financing is subject to three credit market imperfections. First, firms cannot commit to repay their liabilities. This imperfection might give rise to borrowing constraints in equilibrium

**Contract Enforceability Problems.** Entrepreneurs cannot commit to repay their liabilities: if at time $t$ the entrepreneur incurs a non-pecuniary cost $h[w_{i,t} + B_{i,t}]$, then at $t+1$ she will be able to divert all the returns provided the firm is solvent (i.e., $\pi_i(p_{t+1}) \geq 0$).

Second, there are systemic bailout guarantees that cover lenders against systemic crises, but not against idiosyncratic default. This imperfection might induce N-firms to undertake insolvency risk by denominating their debt in T-goods rather that in N-goods.

**Systemic Bailout Guarantees.** If a majority of firms become insolvent, a bailout agency pays lenders the outstanding liabilities of each defaulting firm.

Lastly, there are bankruptcy costs. When a firm defaults, a share $1 - \mu - \mu_w$ of the insolvent firms’ revenues is lost in bankruptcy procedures. In this case, the bailout agency can recoup only $\mu p_t q_t$, and the workers receive a wage of only $\mu_w p_t q_t$. The parameters $\mu$ and $\mu_w$ satisfy

$$\mu \in [0, \beta] \quad \text{and} \quad \mu_w \in (0, 1 - \beta).$$  

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**Fiscal Solvency.** We impose the condition that bailout guarantees are domestically financed via taxation. We assume that the bailout agency is run by a government that has access to perfect capital markets and can levy lump-sum taxes \((T_t)\) on \(T\)-production. It follows that the intertemporal government budget constraint is

\[
E_t \sum_{j=0}^\infty \delta^j \{[1 - \xi_{t+j}][L_{t+j} - \mu p_{t+j} q_{t+j}] - T_{t+j}\} = 0, \quad \delta \equiv \frac{1}{1 + \tau}, \tag{10}
\]

where \(\xi_{t+j} = 1\) if no bailout is granted and zero otherwise.

**Regulatory Regimes.** The regulatory regime determines the set of liabilities that firms can issue. There are three regulatory regimes. First, a *financially repressed regime* under which a firm can only issue one-period standard bonds and must denominate debt in the good which it produces (i.e., cannot take on insolvency risk). Second, a *financially liberalized regime* under which a firm can only issue one-period standard bonds, but is free to take on insolvency risk. Finally, there is an *anything-goes regime* under which firms can issue both standard and catastrophe bonds, and can take on insolvency risk.

**Equilibrium Concept.** In this economy there is endogenous price risk: in an equilibrium \(p_{t+1}\) may equal \(\bar{p}_{t+1}\) with probability \(u_{t+1}\) or \(p_{t+1}\) with probability \(1 - u_{t+1}\). The probability \(u_{t+1}\) may equal either 1 or \(u\), and this is known at \(t\).

Since the only source of uncertainty is relative price risk, N-bonds constitute hedged debt. Meanwhile T-bonds generate insolvency risk because there is a mismatch between the denomination of liabilities and the price that will determine future revenues. Thus, an N-firm’s solvency will depend on the price of N-goods tomorrow.

A key feature of the mechanism is the existence of correlated risks across agents: since guarantees are systemic, the decisions of agents are interdependent. They are determined in the following credit market game, which is similar to that considered by Schneider and Tornell (2004). During each period \(t\), taking prices as given, every young entrepreneur proposes a plan \(P_t = (I_t, s_t, b_t, b^I_t, \rho_t, p^I_t)\) that satisfies budget constraint (6). Lenders then decide whether to fund these plans. Finally, funded young entrepreneurs make investment and diversion decisions.

Payoffs are determined at \(t + 1\). Consider first plans that do not lead to diversion. If the firm is solvent \((\pi(p_{t+1}) \geq 0)\), the old entrepreneur pays the equilibrium wage \(v_{t+1} = [1 - \beta]p_{t+1} q_{t+1}\) to the young entrepreneur and \(L_{t+1}\) to lenders. She then collects the profit \(\pi(p_{t+1})\). In contrast, if the firm is insolvent \((\pi(p_{t+1}) < 0)\), young entrepreneurs receive \(\mu_w p_{t+1} q_{t+1}\) \((\mu_w < 1 - \beta)\), lenders receive the bailout if any is granted, and old entrepreneurs get nothing. Consider next plans that entail diversion. If the firm is solvent, the young entrepreneur gets her wage \(\beta p_{t+1} q_{t+1}\), the old entrepreneur gets the remainder \([1 - \beta]p_{t+1} q_{t+1}\), and lenders receive the bailout if any is granted.
Under insolvency entrepreneurs get nothing and lenders receive the bailout if any is granted. The
problem of a young entrepreneur is then to choose an investment plan \( P_t \) and diversion strategy \( \eta_t \)
that solves

\[
\max_{P_t, \eta_t} E_t \left[ \zeta_{t+1}(p_{t+1}q_{t+1} + (1 + r)s_t - v_{t+1}l_{t+1} - (1 - \eta_t)L_{t+1}) - \eta_t h \cdot [w_t + B_t] \right]
\]

subject to (6), where \( \eta_t = 1 \) if the entrepreneur has set up a diversion scheme, and zero otherwise;
and \( \zeta_{t+1} = 1 \) if \( \pi(p_{t+1}) \geq 0 \), and zero otherwise.

**Definition.** A symmetric equilibrium is a collection of stochastic processes
\[ \{I_t, s_t, b_t, b^n_t, \rho_t, \rho^n_t, d_t, y_t, q_t, u_t, p_t, w_t, v^T_t, v_t \} \]

such that, (i) given current prices and the distribution of future prices, the plan \( (I_t, s_t, b_t, b^n_t, \rho_t, \rho^n_t) \) is determined in a symmetric subgame perfect equilibrium of the credit market game, and \( d_t \) maximizes T-firms’ profits; (ii) factor markets clear;
and (iii) the market for non-tradables clears:

\[
d_t(p_t) + I_t(p_t, p^n_{t+1}, \bar{L}_{t+1}, u_{t+1}) = q_t(I_{t-1})
\]

To close the model we assume that date zero young entrepreneurs are endowed with \( w_0 = (1 - \beta)p_oq_o \)
units of T-goods, while old entrepreneurs are endowed with \( q_o \) units of N-goods and have no debt
in the books.

### 3.1 Discussion of the Setup

Our framework is similar to a Rebelo-type two-sector AK model. The source of endogenous growth
is a production externality in the intermediate goods sector, which is also the investment sector.
This N-sector uses its own goods as capital, and as a result, the share of N-output commanded by
the N-sector for investment (\( \phi \)) is the key determinant of aggregate growth. Because the N-sector is
subject to borrowing constraints, \( \phi \) might be too small in equilibrium and so the economy as a whole
might experience a bottleneck to growth. Our result about the gains from financial liberalization
will derive from the fact that the undertaking of credit risk—by increasing the mean value of \( \phi \)—may
increase production efficiency and aggregate growth via linkages to the T-sector.\(^{16}\) This modeling
choice is consistent with the evidence provided by Harrison (2003) of robust positive externalities
in the investment sector but not in the consumption good sector. As shown by Feibelmayr and
Licandro (2005), the two-sector AK model is consistent with the time series evidence of a fall in
the price of the equipment sector relative to the final good sector (Whelan (2003)). The fall in the

\(^{16}\)In contrast, the assumptions that N-goods are not consumed and T-goods are not intermediate inputs are
convenient but not essential. If N-goods were consumed, there would a deeper fall in the demand of N-goods when
N-firms become insolvent, accentuating the self-fulfilling depreciation that generates crisis.
price of investment is the consequence of the production externality in the investment good sector and enables sustained growth in the aggregate economy.

The agency problem and the two-period lived entrepreneur set-up is considered by Schneider and Tornell (2004). The advantage of this set-up is that one can analyze financial decisions period-by-period. This will allow us to explicitly characterize the stochastic processes of prices and investment. These closed-form solutions are essential to derive the limit distribution of growth rates and establish our efficiency results.

Empirical evidence shows that the higher growth associated with financial liberalization comes together with more crisis-volatility. To capture this growth-volatility link, we consider a set-up with no exogenous source of shocks. In equilibrium endogenous insolvency risk arises from a self-reinforcing mechanism: N-firms find it profitable to issue T-debt in the presence of systemic guarantees and sufficient expected price variability. This variability, in turn, arises when N-firms issue enough T-debt: since N-goods are inputs in N-production, enough T-debt in the balance sheet of N-firms gives rise to the possibility of a crisis state characterized by the collapse of the N-good price and generalized bankruptcies.

To capture the dynamic and the static effects of crises we have allowed for two types of crisis costs: financial distress costs–indexed by $t^d \equiv 1 - \mu_w/(1 - \beta)$, and bankruptcy costs–indexed by $l \equiv 1 - \mu/\beta$. All the equilibria we characterize exist for any $t^d \in (0, 1)$ and any $l \in (0, 1)$.

Financing constraints affect sectors asymmetrically. Contract enforceability problems give rise to financing constraints, which affect mainly the N-sector as it needs external financing to invest. In contrast, T-firms that use N-inputs do not require financing because they transform instantaneously inputs into final output. This simplification provides the same insight than a more complex structure in which the N-sector would be more financially constrained than the T-sector.

The assumption that bailouts are granted only during a systemic crisis is essential. If instead, guarantees were granted whenever a single borrower defaulted, then the guarantees would neutralize the contract enforceability problems and borrowing constraints would not arise in equilibrium.

The three regulatory regimes we consider—repression, liberalization, and anything-goes—are meant to capture in a simple way three regulatory environments. One in which there is over-regulation, credit policies are restrictive and so leverage is low. Another situation in which agents are free to take on risk but there is financial discipline that ensures lenders impose strict repayment criteria on their loans. Finally, a situation where agents have the ability to implement scams that exploit bailout guarantees, like the ones that were used by AIG. As we shall see, standard bonds

\[AIG\] sold a large amount of CDSs prior to the crisis, but did not put aside the collateral necessary to meet the large promised payments during the crisis when the CDS were triggered. AIG liabilities from CDS were ultimately
induce more financial discipline than catastrophe bonds because if at \( t + 1 \) the firm does not repay, then it must default.

Finally, throughout the paper we will impose the following restrictions on the degree of contract enforceability \( h \), the crisis probability \( 1 - u \), and the entrepreneurs’ profit share \( \beta \)

\[
h < 1 + r \equiv \delta^{-1}, \quad u > h\delta, \quad \beta > h\delta/u
\]  

(12)

The restrictions \( h < \delta^{-1} \) and \( u > h\delta \) are necessary for borrowing constraints to arise in a safe equilibrium and a risky equilibrium, respectively. If \( h \), the index of contract enforceability, were greater than the cost of capital, it would always be cheaper to repay debt rather than to divert. Restriction \( \beta > h\delta/u \) is necessary and sufficient for prices to be finite (it implies that the share of N-output commanded by the N-sector \( \phi_t \) is always less than one).

3.2 Symmetric Equilibria (SE)

We will construct two types of symmetric equilibria: safe and risky. The former exists in the repressed as well as the liberalized regimes, while the latter exists only in the liberalized regime.

Consider first the final goods sector. The representative firm maximizes profits, taking goods and factor prices as given. It thus sets \( p_t d_t = \alpha y_t \) and \( v_t^T I_t^T = (1 - \alpha)y_t \). Since consumers supply inelastically one unit of labor, equilibrium T-output, consumer’s income and the T-sector demand for N-goods are, respectively

\[
y_t = d_t^\alpha, \quad v_t^T = [1 - \alpha] y_t, \quad d(p_t) = \left[ \frac{\alpha}{p_t} \right]^{\frac{1}{1-\alpha}}
\]  

(13)

Since \( t + 1 \) consumption is discounted using the riskless interest rate, consumers born in period \( t \) are indifferent between \( t \) and \( t + 1 \) consumption. Thus, we set:

\[
c_{t+1} = [1 - \alpha] y_t
\]  

(14)

Consider next the input-sector. Given prices \( (p_t, \bar{p}_{t+1}, \underline{p}_{t+1}) \) and the likelihood of crisis \( (1 - u_{t+1}) \) every entrepreneur chooses how much to borrow, how to denominate debt, and whether to setup a diversion scheme. Prices and the likelihood of crisis are, in turn, endogenously determined by the entrepreneurs’ choices. In a symmetric equilibrium, entrepreneurs’ choices and resulting prices validate each other. Propositions 3.1 and 3.2 characterize two such self-validating processes. The former characterizes a symmetric safe equilibrium in which all debt is hedged and crises never occur, while the latter characterizes a risky equilibrium where all debt is unhedged, and firms are solvent( insolvent) in the high(low) price state.

covered in full by a government bailout.
We start by characterizing the transition equations, which are common to both symmetric equilibria. We then endogeneize \((p_t, \bar{P}_{t+1}, \bar{P}_{t+1})\) and \(u_{t+1}\). Thus, suppose for a moment that expected returns are high enough so that an entrepreneur finds it optimal to borrow up to the limit and invest all her funds in intermediate goods production. As Propositions 3.1 and 3.2 show, if crises are not frequent, only plans that do not involve diversion are funded. Therefore, the borrowing limit is set by lenders so as to make diversion not profitable:

\[
E(L_{t+1}) \leq h(w_t + B_t), \quad \text{where} \quad B_t = b_t^N \quad \text{(N-debt)}
\]

in a safe equilibrium and \(B_t = b_t^T \quad \text{(T-debt)}\) in a risky one. Combining the binding no-diversion condition with the budget constraint \((p_t I_t = w_t + B_t)\) generates the following borrowing constraint and investment equation

\[
B_t = [m_t - 1]w_t, \quad I_t = m_t \frac{w_t}{p_t}
\]  

The key to our results will be value of the investment multiplier, which will depend on whether the equilibrium is risky or safe.

In any symmetric equilibrium the representative N-firm’s capital \((k_t)\) is equal to average N-sector capital \((\bar{k}_t)\). Thus, (3) implies that equilibrium N-output is linear in investment:

\[
q_t = \theta I_{t-1}
\]

If a firm is solvent, the young entrepreneur’s wage equals the marginal product of her labor, while under insolvency she just obtains a share \(\mu_w\) of revenues. Thus, in any SE the young entrepreneur’s internal funds are

\[
w_t = \begin{cases} 
[1 - \beta]p_t q_t & \text{if } \pi(p_t) \geq 0 \\
\mu_w p_t q_t & \text{if } \pi(p_t) < 0, 
\end{cases} \quad \mu_w \in (0, 1 - \beta)
\]  

Substituting (17) in (15), we have that under the assumption that expected returns are high enough so that it is optimal to invest all funds in the production of N-goods, N-sector investment is

\[
I_t = \phi_t q_t, \quad \phi_t = \begin{cases} 
[1 - \beta]m_t & \text{if } \pi(p_t) \geq 0 \\
\mu_w m_t & \text{if } \pi(p_t) < 0, 
\end{cases}
\]

Combining (13), (16) and (18) with market clearing condition (11), it follows that in a symmetric equilibrium N-output, prices and T-output evolve according to

\[
q_t = \theta \phi_{t-1} q_{t-1}
\]

\[
p_t = \alpha (q_t(1 - \phi_t))^{\alpha - 1}
\]

\[
y_t = [q_t(1 - \phi_t)]^\alpha = \frac{1}{\alpha} \frac{\phi_t}{p_t q_t}
\]

Equations (18)-(21) form an symmetric equilibrium provided that the implied returns validate the agents’ expectations. The next two propositions characterize two such equilibria. We assume

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throughout the rest of the paper that an entrepreneur denominates all debt in N-goods or T-goods, but not in both.\footnote{It is possible to have a small share of T-debt in a safe equilibrium, and a small share of N-debt in a risky equilibrium. Such debt mix does not alter the main properties of the equilibria.}

**Proposition 3.1 (Safe Symmetric Equilibria (SSE))** There exists an SSE if and only if (12) holds and the input-sector productivity $\theta$ is larger than a threshold $\tilde{\theta}^s$ given by (47). In an SSE

1. There is no currency mismatch ($b_t = 0$), and crises never occur ($u_{t+1} = 1$).

2. The interest rate is $1 + \rho_t^s = [1 + r]/p_{t+1}$.

3. The input-sector investment share and leverage are, respectively
   \begin{equation}
   \phi^s = \frac{1 - \beta}{1 - h\delta}, \quad \frac{w_t + b_t^s}{w_t} = \frac{1}{1 - h\delta} = m^s
   \end{equation}

4. Prices evolve according to $p_{t+1} = (\theta\phi^s)^{\alpha-1}$.

To see the intuition notice that, given that all other entrepreneurs choose the safe equilibrium strategy, an entrepreneur and her lenders expect that no bailout will be granted next period. Thus, lenders will not fund any plan that leads to diversion. Furthermore, since lenders must break-even, the interest rate that the entrepreneur has to offer is $1 + \rho_t^s = [1 + r]/p_{t+1}$. Since the expected debt repayment is $b_t^s[1 + r]$, the no-diversion condition is $b_t^s[1 + r] \leq h[w_t + b_t^s]$, which yields the borrowing constraint $b_t^s = [m^s - 1]w_t$, and investment $I_t = m^s w_t/p_t$. Notice that there are no incentives to denominate debt in T-goods because the expected interest payments are the same as those under N-debt.

An entrepreneur finds it profitable to borrow up to the limit and invest in the production of the intermediate input provided that her net return on equity is greater than the storage return. If the borrowing constraint is binding, so that $(1 + r)b_t = h(w_t + b_t)$, the marginal net return per unit of investment is $\theta\beta p_{t+1}/p_t - h$. Since the entrepreneur's leverage $\frac{w_t + b_t^s}{w_t}$ equals $m^s$, the return on equity is $[\theta\beta p_{t+1}/p_t - h] m^s w_t$, which is greater than $[1 + r]w_t$ when $\theta\beta p_{t+1}/p_t > 1 + r$.\footnote{Since in an SSE the wage is $\bar{v}_{t+1} = [1 - \beta]p_{t+1}\theta k_{t+1}$, and $l_{t+1} = 1$, the net return is $\pi^s_{t+1} = \theta p_{t+1} k_{t+1} - \bar{v}_{t+1} - b_t^s[1 + r] = \beta \frac{p_{t+1}}{p_t} [w_t + b_t^s] - b_t^s[1 + r]$. Replacing the borrowing limit, we have $[\beta \frac{p_{t+1}}{p_t} - h] m^s \geq 1 + r$, which is equivalent to $\beta \frac{p_{t+1}}{p_t} \geq 1 + r$.} Since prices evolve according to $p_{t+1} = (\theta\phi^s)^{\alpha-1}$, the net return on equity is greater than the storage return if and only if $\theta > \tilde{\theta}^s$. This parametric condition ensures the existence of an internally consistent mechanism whereby investment decisions generate the required expected price returns.

Next, we characterize risky symmetric equilibria (RSE) in which entrepreneurs choose unhedged T-debt. An entrepreneur finds it optimal to take on the implied insolvency risk only if: (i) $\bar{p}_{t+1}$
is high enough so that expected returns are greater than the storage return \(1 + r\), and (ii) \(p_{t+1}\) is low enough so that all firms with T-debt become insolvent next period and a bailout is triggered. The following proposition establishes the parameter conditions under which this self-validating mechanism arises: currency mismatch generates a large expected relative price variability, which in turn makes it optimal for entrepreneurs to denominate debt in T-goods.

**Proposition 3.2 (Risky Symmetric Equilibrium (RSE))** There exists an RSE for any crisis’ financial distress costs \(l^d \in (0, 1)\) and any bankruptcy costs \(l \in (0, 1)\) if (12) holds, the input-sector productivity satisfies \(\theta \in (\underline{\theta}, \overline{\theta})\), entrepreneurs’ profit share satisfies \(\beta < \overline{\beta}\), and crises are not frequent \((u > u)\). These bounds are defined by (48)-(50).

1. An RSE consists of lucky paths which are punctuated by crises. During a lucky period input-producing firms take on systemic-risk by denominating debt in final goods (i.e., currency mismatch). By doing so they attain high leverage, given by

\[
\frac{w_t + b_t}{w_t} = \frac{1}{1 - h\delta/u} \equiv m^r. \tag{23}
\]

2. Currency mismatch generates systemic risk: there can be a sharp fall in the input price that bankrupts all input-sector firms and generates a systemic crisis, during which creditors are bailed-out.

3. Crises cannot occur in consecutive periods. In the RSE where there is a reversion back to systemic-risk taking in the period immediately after the crisis, the probability of a crisis and the input-sector’s investment share satisfy:

\[
1 - u_{t+1} = \begin{cases} 
1 - u & \text{if } t \neq \tau_i \\
0 & \text{if } t = \tau_i
\end{cases}, \quad \phi_t = \begin{cases} 
\phi^l & \text{if } t \neq \tau_i \\
\phi^c & \text{if } t = \tau_i
\end{cases} \tag{24}
\]

where \(\tau_i\) denotes a crisis time.

4. Input and final goods production are \(q_t = \theta \phi_{t-1} q_{t-1}\) and \(y_t = q_t^\alpha [1 - \phi_t]^\alpha\), respectively. If \(t \neq \tau_i\), prices next period follow:

\[
p_{t+1} = \begin{cases} 
\bar{p}_{t+1} = (\theta \phi^l)^{\alpha-1} p_t & \text{probability } u \\
\underline{p}_{t+1} = (\theta \phi^c)^{\alpha-1} \left( \frac{1-\phi^c}{1-\phi^l} \right)^{1-\alpha} p_t & \text{probability } 1 - u
\end{cases} \tag{25}
\]

If \(t = \tau_i\), prices next period are \(p_{t+1} = (\theta \phi^l)^{\alpha-1} \left( \frac{1-\phi^c}{1-\phi^l} \right)^{1-\alpha} p_t\).

5. If parameters do not satisfy (12), an RSE does not exist.
The proposition says that, if \( \theta > \bar{\theta} \), the marginal gross return per unit of investment \( (u\theta \beta \bar{p}_{t+1}/p_t) \) is high enough so as to make it profitable to borrow up to the limit. Furthermore, because crises are infrequent, diversion schemes are not optimal in spite of the guarantees. Thus, borrowing constraints bind.\(^\text{20}\) Will the entrepreneur choose T-debt or N-debt? She knows that all other firms will go bust in the bad state (i.e., \( \pi(p_{t+1}) < 0 \)) provided there is insolvency risk – i.e., \( \frac{\beta \theta \bar{p}_{t+1}}{p_t} < \frac{h}{w} \). However, since there are systemic guarantees, lenders will get repaid in full. Thus, the interest rate on T-debt that allows lenders to break-even satisfies \( 1 + r = \frac{1}{1 + \theta} \). It follows that the benefits of a risky plan derive from the fact that choosing T-debt over N-debt reduces the cost of capital from \( 1 + r \) to \( 1 + r \). Lower expected debt repayments ease the borrowing constraint as lenders will lend up to an amount that equates \( u[1 + r]b_t \) to \( h[w_t + b_t] \). Thus, investment is higher relative to a plan financed with N-debt. The downside of a risky plan is that it entails a probability \( 1 - u \) of insolvency. Will the two benefits of issuing T-debt–more and cheaper funding–be large enough to compensate for the cost of bankruptcy in the bad state? If \( u\beta \theta \bar{p}_{t+1}/p_t \) is high enough, expected profits under a risky plan exceed those under a safe plan and under storage. High enough \( u\beta \theta \bar{p}_{t+1}/p_t \) is assured by setting the productivity parameter \( \theta > \bar{\theta} \).

To see why a crisis can happen consider a typical period \( t \) and suppose that all inherited debt is denominated in T-goods and agents expect a bailout at \( t + 1 \) in case a majority of firms goes bust. Since the debt repayment is independent of prices, there are two market clearing prices as in Figure 1. In the ‘solvent’ equilibrium (point A in Figure 1), the price is high enough to allow the N-sector to buy a large share of N-output. In contrast, in the ‘crisis’ equilibrium of point B, the price is so low that N-firms go bust:

\[
\frac{p_t q_t}{L_t} < 1 - \frac{1}{1 + r} = \frac{h}{w}.
\]

The key to having multiple equilibria is that part of the N-sector’s demand comes from the N-sector itself. Thus, if the price fell below a cutoff level and N-firms went bust, the investment share of the N-sector would fall (from \( \phi^k \) to \( \phi^c \)). This, in turn, would reduce the demand for N-goods, validating the fall in the prices. The upper bound on \( \theta \) ensures that the low price is low enough to bankrupt firms with T-debt, while the upper bound on \( \beta \) ensures that \( \frac{\theta}{\bar{\theta}} < \frac{1}{1 - \beta} \). A low enough \( \beta \) (high \( 1 - \beta \)) means that when a crisis hits, the fall in the young entrepreneurs’ cash flow (from \( [1 - \beta]p_{t-1}q_{t-1} \) to \( \mu_w p_t q_t \)) is translated into a large fall in input demand, validating the large fall in prices.

Three points are worth emphasizing. First, Proposition 3.2 holds for any \( l = 1 - \frac{\mu}{\beta} \in (0, 1) \) and any \( l' = 1 - \frac{\mu'}{\beta} \in (0, 1) \). That is, crisis costs are not necessary to trigger a crisis. A shift in expectations is sufficient: a crisis can occur whenever entrepreneurs expect that others will not

\(^{20}\) Diversion plans are not optimal when \( u \) is large because the interest rate that those plans imply becomes too large \( 1 + \rho^d = (1 + r)/(1 - u) \), and diversion requires the firm to be solvent.
undertake credit risk, so that there is a reversion to the SSE characterized in Proposition 3.1. Second, two crises cannot occur consecutively. Since investment in the crisis period falls, the supply of N-goods during the post-crisis period will also fall. This will drive post-crisis prices up, preventing the occurrence of insolvencies even if all debt were T-debt. That is, during the post-crisis period a drop in prices large enough to generate insolvencies is impossible. Third, we focus in the proposition above on a RSE where there is a reversion back to a risky path in the period immediately after the crisis. In the appendix, we relax this assumption and allow agents to choose safe strategies for multiple periods in the aftermath of crisis.

4 Growth

Here, we compare the long-run growth rates along the financially repressed and liberalized regimes—characterized in Propositions 3.1 and 3.2. In Section 6 we consider the anything-goes regime.

Since N-goods are intermediate inputs, while T-goods are final consumption goods, gross domestic product equals the value of N-sector investment plus T-output: $gdp_t = p_t I_t + y_t$. It then
follows from (18)-(21) that, in any symmetric equilibrium, GDP is given by
\[ \text{gdp}_t = p_t \phi_t q_t + y_t = q_t^\alpha Z(\phi_t) = y_t \frac{Z(\phi_t)}{1 - \phi_t} \text{ with } Z(\phi_t) = \frac{1 - (1 - \alpha)\phi_t}{[1 - \phi_t]^{1-\alpha}} \] (26)

As we can see, the key determinant of the evolution of GDP is the share of N-output commanded by the N-sector for investment: \( \phi_t \). This share is determined by the cash flow of young entrepreneurs and by the credit they can obtain. Importantly, (26) makes clear that because of output-input linkages, measured aggregate total factor productivity, \( Z(\phi_t) \), is a function of the share of investment in the N-sector. This result is linked with the literature on input misallocation (e.g., Jones 2010, 2011).\(^{21}\)

4.1 Growth in a Financially Repressed Economy

In an SSE the investment share \( \phi_t \) is constant and equal to \( \phi^s \). Thus, (26) implies that GDP and T-output grow at the same rate.
\[ 1 + \gamma^s := \frac{\text{gdp}_t}{\text{gdp}_{t-1}} = \frac{y_t}{y_{t-1}} = \left( \theta \frac{1 - \beta}{1 - h \delta} \right)^\alpha = (\theta \phi^s)^\alpha \] (27)

Absent exogenous technological progress in the T-sector, the endogenous growth of the N-sector is the force driving growth in both sectors. As the N-sector expands, N-goods become more abundant and cheaper allowing the T-sector to expand production. This expansion is possible if and only if N-sector productivity (\( \theta \)) and the N-investment share (\( \phi^s \)) are high enough, so that credit and N-output can grow over time: \( \frac{b_t}{b_{t-1}} = \frac{q_t}{q_{t-1}} = \theta \phi^s > 1 \). Notice that for any positive growth rate of N-output, \( \gamma^s \) increases with the intensity of the N-input in the production of T-goods (\( \alpha \)).

4.2 Growth in a Financially Liberalized Economy

Proposition 3.2 shows that any RSE is composed of a succession of lucky paths punctuated by crisis episodes. In the RSE characterized by Proposition 3.2 the economy is on a lucky path at time \( t \) if there has not been a crisis either at \( t - 1 \) or at \( t \). Since along a lucky path the investment share equals \( \phi^l \), (26) implies that the common growth rate of GDP and T-output is
\[ 1 + \gamma^l := \frac{\text{gdp}_t}{\text{gdp}_{t-1}} = \frac{y_t}{y_{t-1}} = \left( \theta \frac{1 - \beta}{1 - h \delta} \right)^\alpha = (\theta \phi^l)^\alpha \] (28)

\(^{21}\)Since at time \( t \) \( q_t^\alpha \) is predetermined by past investment, the contemporaneous effect of investment share changes on aggregate TFP at \( t \) can be decomposed as follows
\[ \frac{\partial \text{gdp}_t}{\partial \phi_t} = \frac{\partial Z(\phi_t)}{\partial \phi_t} = p_t q_t - \frac{\alpha y_t}{1 - \phi_t} + q_t \phi_t \frac{\partial p_t}{\partial \phi_t} = q_t \phi_t \frac{\partial p_t}{\partial \phi_t} > 0 \]

Market clearing in the N-goods market—i.e., \( (1 - \phi_t)p_t q_t = \alpha y_t \)—implies that the induced changes in investment and final output cancel out. Since an increase in investment raises contemporaneously the price of N-goods, \( q_t \phi_t \frac{\partial p_t}{\partial \phi_t} > 0 \), and thus measured aggregate TFP increases with an increase in \( \phi \).
A comparison of (27) and (28) reveals that as long as a crisis does not occur, growth in a risky economy is higher than in a safe economy. Along the lucky path the N-sector undertakes insolvency risk by issuing T-debt. Since there are systemic guarantees, financing costs fall and borrowing constraints are relaxed, relative to a safe economy. This increases the N-sector’s investment share \((\phi^l > \phi^s)\). Since there are sectorial linkages \((\alpha > 0)\), this increase in the N-sector’s investment share benefits both the T- and the N-sectors and fosters faster GDP growth.

However, in a risky economy a self-fulfilling crisis can occur with probability \(1 - u\), and during a crisis episode growth is lower than along a safe path. We have seen that any crisis episode consists of at least two periods: in the first period the financial position of the N-sector is severely weakened and the investment share falls from \(\phi^l\) to \(\phi^c < \phi^s\); then in the second period it jumps back to \(\phi^l\). Since these transitions occur with certainty, the mean crisis growth rate is given by:

\[
1 + \gamma^{cr} = \left( \frac{(\theta \phi^l)^\alpha Z(\phi^c)}{Z(\phi^l)} \right)^{1/2} \left( \frac{(\theta \phi^c)^\alpha Z(\phi^l)}{Z(\phi^c)} \right)^{1/2} = \left( \theta(\phi^l \phi^c)^{1/2} \right)\]

(29)

The second equality in (29) shows that the average loss in GDP growth stems only from the fall in the N-sector’s average investment share: \((\phi^l \phi^c)^{1/2}\). This reduction comes about through two channels: financial distress (indexed by \(l^d = 1 - \frac{\mu}{1-\tau}\)) and a reduction in leverage (indexed by \(1 - hdu - \tau\)). Notice that variations in GDP growth generated by relative price changes at \(\tau\) and \(\tau + 1\) cancel out (the derivation of this result is in Appendix A).

A crisis has long-run effects because N-investment is the source of endogenous growth, and so the level of GDP falls permanently. To determine under which conditions the mean long-run GDP growth in a liberalized economy is greater than in a repressed one despite the occurrence of crises, we derive the limit distribution of GDP’s compounded growth rate: \(\log(gdp_t) - \log(gdp_{t-1})\).

Recall that, because internal funds collapse in a crisis, it is not possible to generate enough leverage to make it possible for another crisis to occur next period (Proposition 3.2). Thus, in any RSE everyone must choose a safe plan during a crisis, and risky plans can be chosen in any period after the crisis period. In the RSE in which the undertaking of credit risk resumes the period immediately after the crisis, the growth process is characterized by the following three-state Markov chain:

\[
\Gamma = \begin{pmatrix}
\log \left( \frac{(\theta \phi^l)^\alpha}{Z(\phi^l)} \right) \\
\log \left( \frac{(\theta \phi^l)^\alpha Z(\phi^c)}{Z(\phi^l)} \right) \\
\log \left( \frac{(\theta \phi^c)^\alpha Z(\phi^l)}{Z(\phi^c)} \right)
\end{pmatrix}, \quad T = \begin{pmatrix}
u & 1 - u & 0 \\
0 & 0 & 1 \\
u & 1 - u & 0
\end{pmatrix}
\]

\[22\text{In Appendix B, we consider the alternative RSEs where a crisis is followed by a cool-off phase of arbitrary length, during which safe plans are undertaken.}\]
The three elements of $\Gamma$ are the growth rates in the lucky, crisis and post-crisis states, given by (24), (28) and (29). The element $T_{ij}$ of the transition matrix is the transition probability from state $i$ to state $j$. Since the transition matrix is irreducible, the growth process converges to a unique limit distribution over the three states that solves $T\Pi = \Pi$. The solution is
\[
\Pi = \left(\frac{u}{2-u}, \frac{1-u}{2-u}, \frac{1-u}{2-u}\right)^T,
\]
where the elements of $\Pi$ can be interpreted as the shares of time that an economy spends in each state over the long-run. It then follows that the mean long run GDP growth rate is $E(1 + \gamma^r) = \exp(\Pi^T \Gamma)$.

That is,
\[
E(1 + \gamma^r) = (1 + \gamma^l)^\omega (1 + \gamma^{cr})^{1-\omega} = \rho^\alpha (\phi^l)^\alpha (\phi^c)^{\omega \frac{1-u}{2-u}}, \quad \text{where} \quad \omega = \frac{u}{2-u} \quad (30)
\]
A comparison of long run GDP growth rates in (27) and (30) reveals the trade-offs involved in following safe and risky growth paths, and allows us to determine the conditions under which financial liberalization is growth enhancing.

**Proposition 4.1 (Long-Run GDP Growth)** In an RSE the mean long-run GDP growth rate is given by
\[
E(1 + \gamma^r) = (1 + \gamma^s)^\alpha \left(\frac{\phi^l}{\phi^s}\right)^{\frac{1}{1-u}} \left(\frac{\mu_w}{1-\beta}\right)^{\frac{1-u}{2-u}}
\]

- Mean Growth is greater in a financial liberalized than in a repressed economy if and only if financial distress costs $(l^d = 1 - \frac{\mu_w}{1-\beta})$ are not too severe:
\[
l^d < \overline{l}^d = 1 - \left(\frac{1 - h\delta u^{-1}}{1 - h\delta u}\right)^{1/(1-u)} \quad (31)
\]
Rewriting (31) as $(1 - u) \log (1 - \beta) - \log (\mu_w) < \log (\phi^l) - \log (\phi^s)$ makes clear what are the costs and benefits associated with a risky path. A liberalized economy outperforms a repressed one if the benefits of higher leverage and investment in no-crisis times ($\phi^l > \phi^s$) compensate for the shortfall in internal funds and investment in crisis times ($\mu_w < 1 - \beta$) weighted by the frequency of crisis $(1 - u)$. Notice that the larger $h$ within the admissible range $(0, \delta^{-1})$, the larger are the growth benefits of systemic-risk taking and therefore the less stringent is the condition on crisis costs $(l^d < \overline{l}^d)$.

\[\text{Table below gives the upper bound for the financial distress costs } \overline{l}^d : \text{ for different values of } h: \]

<table>
<thead>
<tr>
<th>$u = 0.95, \delta = 0.95$</th>
<th>$h = 0.4$</th>
<th>$\overline{l}^d = 48%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h = 0.6$</td>
<td>$\overline{l}^d = 76.7%$</td>
<td></td>
</tr>
<tr>
<td>$h = 0.8$</td>
<td>$\overline{l}^d = 96%$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 2: GDP Growth and Financial Distress Costs \((l^d = 1 - \frac{\mu_c}{1-\theta})\)

![Figure 2](image)

parameters \(\theta = 1.65 \quad \alpha = 0.35 \quad h = 0.76 \quad 1 - \beta = 0.2 \quad 1 - u = 5\%\)

Figure 2 illustrates Proposition 4.1 by plotting several risky growth paths associated with different degrees of crisis’ financial distress.\(^{25}\) As we can see, even if 90% of N-sector cash flow is lost during a crisis, a risky economy can outperform a safe economy over the long run. That is, a risky economy can exhibit greater mean growth than a safe economy despite large financial distress costs.

Figure 3 illustrates the limit distribution of GDP growth rates by plotting different GDP paths corresponding to different realizations of the sunspot process. Most of the risky paths outperform the safe path, except for a few unlucky risky paths. If we increased the number of paths, the cross section distribution would converge to the limit distribution.

\(^{25}\)See appendix C for details of the model calibration. The simulation plotted in Figure 4 include 4 crises in 80 period which corresponds to a probability of crisis of 5%.\)
Figure 3: Limit Distribution of GDP

parameters: $\theta = 1.65$, $\alpha = 0.35$, $h = 0.76$, $1 - \beta = 0.2$, $t^d = 70\%$, $1 - u = 5\%$
5 Production Efficiency and Consumption Possibilities

We have considered an endogenous growth model where the financially constrained N-sector is the engine of growth because it produces the intermediate input used throughout the economy. Thus, the share of N-output invested in the N-sector, $\phi_t$, is the key determinant of economic growth. When $\phi_t$ is too small T-output is high in the short-run, but long-run growth is slow. In contrast, when $\phi_t$ is too high, there is inefficient accumulation of N-goods. In this section we ask three questions. First, what is the Pareto optimal investment share sequence $\{\phi_t\}$? Second, can this Pareto optimal investment sequence be replicated in a financially repressed economy? If not, can the average investment share be higher in a financially liberalized economy where agents undertake credit risk and crises occur? Third, will the present value of consumption be greater in a liberalized economy after netting out the taxes necessary to finance the bailouts during crises? In Section 6 we consider the anything-goes regime.

5.1 Pareto Optimality

Consider a central planner who maximizes the present discounted value of the consumption of workers and entrepreneurs by allocating the supply of inputs to final goods production ($[1 - \phi_t]q_t$) and to input production ($\phi_t q_t$), as well as by assigning sequences of consumption goods to consumers and entrepreneurs for their consumption.

$$\max_{\{c_t, c_t', \phi_t\}} W^{PO} = \sum_{t=0}^{\infty} \delta^t [c_t' + c_t], \quad \text{s.t.} \quad \sum_{t=0}^{\infty} \delta^t [c_t + c_t' - y_t] \leq 0$$

$$y_t = [1 - \phi_t]^{\alpha} q_t^\alpha, \quad q_{t+1} = \theta \phi_t q_t$$

(32)

Pareto optimality implies efficient accumulation of N-inputs: the planner should choose the investment sequence $\{\phi_t\}$ to maximize the present value of T-production ($\sum_{t=0}^{\infty} \delta^t y_t$). We show in the Appendix that the Pareto optimal N-investment share is constant and equal to

$$\phi^{po} = (\theta^\alpha \delta)^{-1/\alpha}, \quad \text{if} \quad \alpha < \log(\delta^{-1})/\log(\theta)$$

(33)

The Pareto optimal share equalizes the discount rate $\delta^{-1}$ to the intertemporal rate of transformation. A marginal increase in the N-sector investment share ($\partial \phi$) reduces today’s T-output by $\alpha [(1 - \phi) q_t]^{\alpha - 1} \partial \phi$, but increases tomorrow’s N-output by $\theta \partial \phi$ and tomorrow’s T-output by $\alpha [(1 - \phi) \theta \phi q_t]^{\alpha - 1} \theta \partial \phi$. Thus, at an optimum $\theta^\alpha \phi^{\alpha - 1} = \delta^{-1}$.

Can a decentralized economy replicate the Pareto optimal allocation? The optimal investment share is determined by investment opportunities: $\theta^\alpha \delta$. In contrast, in a decentralized safe economy the N-investment share ($\phi^* = \frac{1 - \beta}{1 - \theta \phi}$) is determined by the credit market imperfections: the degree of contract enforceability ($h$) and the constrained sector’s cash flow ($1 - \beta$). Clearly, if either $h$ or $1 - \beta$
are low, the N-sector investment share will be lower than the Pareto optimal share: \( \phi^* < \phi^{po} \). That is, when the N-sector is severely credit constrained, low N-sector investment will keep the economy below production efficiency. For future reference we summarize with the following Proposition.

**Proposition 5.1 (Bottleneck)** Input production in a financially repressed economy is below the Pareto optimal level (i.e., there is a ‘bottleneck’) if contract enforceability problems are severe:

\[
\phi^* < \phi^{po} \quad \iff \quad h < (1 - (1 - \beta)\theta (\theta \delta)^{-\frac{1}{1-\alpha}})/\delta.
\]

When there is a bottleneck, the share of N-inputs allocated to T-production should be reduced and that allocated to N-production should be increased in order to bring the allocation nearer to the Pareto optimal level. This reallocation reduces the initial level of T-output, but increase its growth rate and the present value of cumulative T-production.

**Input-output Linkages and the Dynamic Multiplier Effect**

If there is a bottleneck, i.e., \( \phi^* < \phi^{po} \), an increase in the investment share \( \phi \) corresponds to a reduction of input misallocation. In the context of our two-sector endogenous growth model this increase in \( \phi \) leads to an increase in future final good production. This dynamic input-multiplier effect is analogous to the steady-state approach proposed by Jones (2010, 2011) in the context of a neoclassical growth model.

A marginal increase in the N-sector investment share \( (\partial \phi) \) reduces today’s T-output by \( \alpha [(1 - \phi)q_t]^{\alpha-1} \partial \phi \), but increases tomorrow’s N-output by \( \theta \partial \phi \) and tomorrow’s T-output by \( \alpha [(1 - \phi)\theta q_t]^{\alpha-1} \theta \partial \phi \). The intertemporal multiplier effect is therefore:

\[
M = \frac{\alpha [(1 - \phi)\theta q_t]^{\alpha-1} \theta}{\alpha [(1 - \phi)q_t]^{\alpha-1}} = \theta^\alpha \phi^{\alpha-1}.
\]

It follows that the long run dynamic gains in T-output resulting from a marginal increase in the investment rate in the N-sector are given by

\[
M + M^2 + ...M^j + ... = \sum_{j=1}^{\infty} M^j = \frac{1}{1 - M} - 1.
\]

These dynamic gains are maximized if \( M \) tends to 1, or equivalently if the investment share tends to \( \phi^g \equiv \theta^{\frac{\alpha}{1-\alpha}} \). Notice that the value of \( \phi^g \) is increasing in \( \alpha \), the strength of the input-output linkage.

To see the link between \( \phi^g \) and \( \phi^{po} \) note that \( \phi^g \) maximizes the sum of final goods production. If the objective were to maximize the discounted sum of final good production, we would obtain instead the Pareto optimal investment share: \( \phi^{po} = \delta^{\frac{1}{1-\alpha}} \theta^{\frac{\alpha}{1-\alpha}} \). Thus, \( \phi^{po} = \delta^{\frac{1}{1-\alpha}} \phi^g \).
5.2 Present Value of Consumption in a Decentralized Economy

Financial liberalization relaxes borrowing constraints and spurs growth. However, it also induces the adoption of insolvency risk that makes the economy vulnerable to crises, which entail deadweight losses as well as fiscal costs to cover bailouts. Here we determine conditions under which, the present value of consumption in a financially liberalized economy is greater than in a repressed economy, after netting out the crisis and bailout costs. We show that liberalization can improve consumption possibilities only if there is a bottleneck (i.e., $\phi^s < \phi^{ro}$), so that growth is inefficiently low under financial repression.

The expected discounted value of workers’ consumption and entrepreneurs’ consumption in our decentralized economy is equal to:

$$W^d = E_0 \left( \sum_{t=0}^{\infty} \delta^t (c_t + c^e_t) \right) = E_0 \left( \sum_{t=0}^{\infty} \delta^t ([1 - \alpha]y_t + \pi_t - T_t) \right)$$ (34)

To derive the second equation in (34) notice that in equilibrium workers’ income at $t$ is $[1 - \alpha]y_t$, entrepreneurs’ income is equal to their profits $\pi_t$, and the fiscal cost of bailouts is financed with lump-sum taxes $T_t$.

In order to obtain a closed-form solution notice that at any $t \geq 1$ profits equal the old entrepreneurs share in revenues minus debt repayments: $\pi_t = \beta p_t q_t - L_t = \frac{\alpha}{1 - \sigma} \beta y_t - \frac{\alpha}{1 - \sigma} \frac{b}{\phi^s} y_{t-1}$. Meanwhile, since at $t = 0$ there is no debt burden, $\pi_0 = \frac{\alpha}{1 - \sigma} \beta y_0$. In a financially repressed economy firms are always solvent and crises never occur. Thus, there are no bailouts and no taxes. It then follows from (34) that the present value of consumption equals the present value of T-output

$$W^s = \sum_{t=0}^{\infty} \delta^t y^s_t = \frac{1}{1 - \delta(\theta \phi^s)^\alpha} y^s_0 = \frac{(1 - \phi^s)^\alpha}{1 - \delta(\theta \phi^s)^\alpha} q^s_0 \quad \text{if } \delta(\theta \phi^s)^\alpha < 1$$ (35)

Consider a liberalized economy. Along the lucky path, the investment share is greater than in a safe economy. Thus, if there is a bottleneck and crises are rare events, the present value of T-output along the lucky path is greater than in a safe path. However, along a lucky path a crisis can occur with probability $1 - u$, and a crisis involves three costs.

First, there is a fiscal cost. Lenders receive a bailout payment equal to the debt repayment they were promised: $L_\tau = u^{-1} h \phi^s p_{r-1} q_{r-1}$. Since the bailout agency recuperates only a share $\mu \leq \beta$ of firms revenues $p_r q_r$, while the rest is dissipated in bankruptcy procedures, the fiscal cost of a crisis is $T(\tau) = L_\tau - \mu p_r q_r$. Second, there is a financial distress cost. In a crisis there is a fall in credit and investment, and so the investment share of the input sector is $\phi^c = \frac{\mu_w}{1 - h \delta}$ instead of $\phi^s$ in a safe economy, where $\mu_w$ can be arbitrarily small. During a crisis borrowing constraints are tighter than what they would be in a safe economy because (i) an N-firm’s net worth is $\mu_w p_r q_r$ instead of $[1 - \beta]p_r q_r$, and (ii) risk-taking is curtailed: only safe plans are financed. Third, since during a crisis all N-firms go bust, old entrepreneurs’ profits are zero.
The deadweight loss of a crisis for the economy as a whole is lower than the sum of these three costs. During a crisis there is a sharp redistribution from the N- to the T-sector generated by a large fall in the relative price of N-goods (a firesale). Thus, some of the costs incurred in the N-sector show up as greater T-output and consumers’ income. We show in the Appendix that after netting out the costs and redistributions, a crisis involves two deadweight losses: (i) the revenues dissipated in bankruptcy procedures: \[ [\beta - \mu]p_r q_r; \] and (ii) the fall in N-sector investment due to its weakened financial position: \[ [(1 - \beta) - \mu_w]p_r q_r. \] Note that (i) affects the fiscal burden of the bailouts but not future production. In contrast (ii) affects future production and future investment, so it is propagated through the dynamic multiplier described in subsection 5.1.

Using the market clearing condition \[ \alpha y_t = [1 - \phi_t]p_t q_t, \] we have that the sum of these two deadweight losses equals \[ \frac{\alpha}{1 - \phi} [1 - \mu - \mu_w]y_t \] in terms of T-goods. Thus, in an RSE the present value of consumption is given by

\[ W^r = E_0 \sum_{t=0}^{\infty} \delta^t k_t y_t; \quad k_t = \begin{cases} k^c := 1 - \frac{\alpha[1-\mu-\mu_w]}{1-\phi} & \text{if } t = \tau_i \\ 1 & \text{otherwise}, \end{cases} \]

where \( \tau_i \) is a crisis time. In order to compute this expectation we need to calculate the limit distribution of \( k_t y_t \). We do this in the Appendix and show that it is equal to

\[ W^r = \frac{1 + \delta(1 - u) \left[ \theta \phi^l \frac{1 - \phi^l}{1 - \phi} \right]^{\alpha} k^c}{1 - \left[ \theta \phi^l \right]^{\alpha} \delta u - \left[ \theta^2 \phi^l \phi^c \right]^{\alpha} \delta^2 (1 - u) [(1 - \phi^l)q_0]^{\alpha}} \]

By comparing (35) and (37) we can determine the conditions under which the ex-ante present value of consumption is greater in a financially liberalized economy.

**Proposition 5.2 (Present Value of Consumption)** In an economy where crisis are rare events and that satisfies parameter restrictions (66)-(71):

1. Financial liberalization increases the present value of consumption only if the investment share in a repressed regime (\( \phi \)) is less than the Pareto investment share (\( \phi^{po} \)).

2. When \( \phi < \phi^{po} \), financial liberalization increases the present value of consumption for any level of bankruptcy costs (i.e., any \( \mu \geq 0 \)) if financial distress costs are not too high (\( l^d < \tilde{l}^d \)) and the discount rate \( \delta \) is not too low.

This proposition establishes that the growth enhancing effect of systemic risk-taking in a liberalized regime translates into higher consumption possibilities for the economy as a whole only if it leads to an increase in production efficiency and if, in addition, dynamic crises costs are not too severe. Recall that systemic-risk taking is an equilibrium outcome only if crises are rate events.
Proposition 5.2 is proved in the Appendix by taking the derivative of $W^r$ with respect to $u$ and letting $u \rightarrow 1$. Since $W^r_{u=1} = W_s$, financial liberalization, which allows for systemic risk-taking, increases the present value of consumption if and only if $\frac{\partial W^r}{\partial u}|_{u=1}$ is negative. We have:

$$\left. \frac{\partial W^r}{\partial u} \right|_{u=1} = \left\{ \begin{array}{ll}
\alpha \phi' \left( \frac{D}{\phi} - 1 \right) & \text{Efficiency gains} \\
(1 - D)(1 - k_c \left( \frac{1 - \phi^c}{1 - \phi} \right)(1 - \phi)) & \text{Bankruptcy costs} \\
(1 - \phi)^\alpha \delta (\theta)^\alpha (\phi^\alpha - (\phi^c)^\alpha) & \text{Financial distress costs}
\end{array} \right\} K$$

where $D = \delta (\theta)^\alpha = (\phi^{po})^{1-\alpha} \phi^\alpha$ and $K$ is a strictly positive number.26 Since the derivative is evaluated at $u = 1$, we have $\phi \equiv \phi^l = \phi^s$.

The first term in (38) captures the efficiency gains from financial liberalization. It can be rewritten as $\alpha \phi' \left( \frac{\phi^{po}}{\phi} \right)^{1-\alpha} - 1$, which is negative if and only if $\phi < \phi^{po}$. The second term captures the bankruptcy costs associated with crises. The third term reflects the financial distress crisis costs, which are increasing in the difference between the tranquil times investment share ($\phi$) and the crisis investment share ($\phi^c$). When $\phi^c < \phi < \phi^{po}$, financial distress costs correspond to production efficiency losses since they bring the allocation of intermediate inputs farther away from the Pareto optimal level.

Since the second and third terms in (38) are positive, a necessary condition for $W^r > W^s$ is that the first term be negative, which occurs only if $\phi < \phi^{po}$. In other words, there are efficiency gains associated with financial liberalization only if there is a bottleneck. This is the first part of Proposition 5.2.

The second part of the proposition establishes conditions under which the crisis costs are outweighed by the efficiency gains. If the discount rate is high enough, the bankruptcy costs, which are static in nature, become vanishingly small. In this case, the gains (or losses) from financial liberalization depend on the relative sizes of efficiency gains and financial distress costs, both of which are dynamic. That is, they propagate to future periods through the investment channel, and affect future levels of T-production through input-output linkages. The efficiency gains depend on how much risk-taking reduces the distortion in the allocation of intermediate inputs in tranquil times, while financial distress costs measure how the allocation of intermediate inputs becomes more distorted in crises times. On net there are positive gains from financial liberalization when financial distress costs are below a threshold: $l^d < l^d$. Recall that these distress costs measure the severity of the credit crunch in the wake of crisis.

26 It is given by $K = \left[ \frac{\alpha \phi' (1 - \phi)^{n-1}}{1 - \phi \phi^{po} \phi^c (1 - u)} \right]^2 > 0$. 

29
6 Anything-Goes Regulatory Regime

Here, we analyze the consequences of unfettered liberalization that allows for the issuance, without collateral, of catastrophe bonds that promise a very big payoff in bad states and nothing otherwise. Recall that there are two technologies to produce final goods: the $\theta$-technology (4) that uses intermediate goods as inputs, and the inferior $\varepsilon$-technology (5) that uses only final goods as inputs and that yields less than the storage technology in all states. In the repressed and liberalized regimes, the $\varepsilon$-technology is never used in equilibrium. As we shall see, in the absence of bailout guarantees, this inferior technology is not funded in the anything-goes regime. The combination of bailout guarantees and catastrophe bonds with no collateral is the key for the funding of this inferior technology.

In the presence of bailout guarantees, the use of catastrophe bonds allows borrowers to shift all their liability repayments to the default state, where bailout payments are triggered. Therefore, the issuance of such securities implies that: (i) any positive return in the no-default state, even lower than the risk-free interest rate, is enough to ensure positive profits in that state; and (ii) the solution to the borrower-lender agency problem does not require equity investment: the borrowing limit is determined by the expected generosity of the bailout rather than by internal funds. As a consequence, the $\varepsilon$-technology is funded under the anything-goes regime.

In contrast, in the liberalized regime, the use standard debt contracts only restricts external finance to projects that return at least the risk-free rate in the no-default state, and imposes that borrowers can borrow at most a multiple of their own equity in order to eliminate incentives to divert.

A consequence of the lower bound on the project’s return and of having borrowers risk their own equity, is that the $\varepsilon$-technology is not funded and that borrowers only invest in projects that have a private return, net of debt repayments, greater than the storage return $(1 + r)$.

Here, we characterize a ‘financial black-hole’ equilibrium where the $\varepsilon$-technology is funded. To do so we add two ingredients to the setup of Section 3. First, we introduce a new set, of measure one, of entrepreneurs that have access to the $\varepsilon$-technology and that live for two periods. When young an $\varepsilon$-entrepreneur (who has zero internal funds) issues debt, and uses the proceeds to buy $T$-goods ($I^\varepsilon_t$), which he invests to produce $T$-goods using production function (5): $y_{t+1} = \varepsilon_{t+1}I^\varepsilon_t$. When old the $\varepsilon$-entrepreneur consumes his profits. Second, we add an upper bound on the bailout to ensure fiscal solvency.

Bailout Guarantees A bailout up to an amount $\Gamma_{i,t}$ is granted to lenders of a defaulting borrower if half of borrowers defaults.
We parametrize $\Gamma_{i,t}$ as a share $\gamma_i$ of final goods produced by the non-diverting part of the economy

$$\Gamma_{i,t} = \gamma_i [y_t^{\theta,nd} + y_t^{\varepsilon,nd}],$$

(39)

where $y_t^{\theta,nd}$ is the T-output produced using N-inputs from non-diverting N-firms, and $y_t^{\varepsilon,nd}$ is the T-output from non-diverting $\varepsilon$-firms. We set $\gamma_i = \gamma$ with $\gamma < \frac{1}{2}$ so that the total bailout granted $\Gamma_t$ is always lower than the value of the final good’s production.27 Bailouts are financed via lump-sum taxes on final output produced by the non-diverting part of the economy.28

The rest of the setup is the same as in Section 3. In particular, all entrepreneurs can issue both standard debt and catastrophe bonds with the following repayment schedule:

$$L_{t+1}^c = \begin{cases} 
0 & \text{if } \varepsilon_{t+1} = \varepsilon \\
1 + \rho_t^c & \text{if } \varepsilon_{t+1} = 0 
\end{cases}$$

(40)

Each lender observes whether the borrower is an $\varepsilon$- or $\theta$-entrepreneur, and decides whether to buy the bonds. At time $t + 1$, lenders receive the promised repayment from non-defaulting borrowers, or a bailout if one is granted.

### 6.1 A Black-Hole Equilibrium

In order for a black-hole equilibrium to exist it is necessary that bailout guarantees not be "too generous." If bailouts were too generous, input producing $\theta$-entrepreneurs would have incentives to issue catastrophe bonds and default in the bad state, so there would be no tax base to fund the bailouts. In the black-hole equilibrium characterized by the following proposition, the upper bound on the generosity of the bailout is tight enough so as to ensure that $\theta$-entrepreneurs choose the no-diversion safe plans characterized in Section 3, and so bailouts are fiscally viable.29

**Proposition 6.1 (Black-Hole Equilibrium)** Consider an anything-goes economy where (12) holds and N-sector productivity $\theta$ satisfies (47). Then a black-hole equilibrium exist if the generosity of bailout guarantees ($\gamma$) is below the threshold given by (51) in the Appendix. In this equilibrium:

---

27If both $\varepsilon$-entrepreneurs and $\theta$-entrepreneurs default, total bailout is $\Gamma_t = 2\gamma[y_t^{\theta,nd} + y_t^{\varepsilon,nd}] \leq y_t^{\theta,nd} + y_t^{\varepsilon,nd}$.

28The government cannot tax the diverting part of the economy—i.e., the black market. This is a realistic assumption, and it is also important for the working of the model. If output of the diverting sector were taxable, then one could construct equilibria where diversion is desirable because it would relax borrowing constraints as lenders would not impose the no-diversion condition.

29One could construct other black-hole equilibria where $\theta$-agents choose risky plans. We do not construct such more complicated equilibria because our objective is simply to show how financial discipline breaks down in an anything-goes regime.
1. $\theta$-agents issue standard bonds, hedge price risk, never default and input production is $q_t = \theta m^s w_t$.

2. $\varepsilon$-agents issue catastrophe bonds and default in the $\varepsilon = 0$ state with probability $1 - \lambda$.

3. Final goods’ production is 
   
   \[
   y_t^{nd} = y_t^{\theta, nd} + y_t^{\varepsilon, nd} = y_{t-1}^{nd} (\theta \phi^s)^\alpha (1 + \varepsilon_t \delta [1 - \lambda \gamma]).
   \]

To see why guarantees are necessary notice that if they were absent, the negative NPV $\varepsilon$-technology would not be funded. Since the $\varepsilon$-technology yields less than the riskless return in all states (because $\varepsilon < 1 + r$) any profitable strategy for an $\varepsilon$-entrepreneur involves the issuance of catastrophe bonds. Lenders, however, are unwilling to buy catastrophe bonds from an $\varepsilon$-entrepreneur as they will never be repaid: in the good state they are promised zero, and in the bad state the $\varepsilon$-entrepreneur will go bust. Only $\theta$-agents will be funded and, as in Section 3, they will choose a safe plan with no insolvency risk. Thus, in the absence of bailout guarantees, neither the catastrophe bonds nor the inferior $\varepsilon$-technology are used in equilibrium. They are irrelevant for the allocation of resources.

With bailout guarantees, the no-diversion condition does not generate a borrowing constraint for an $\varepsilon$-entrepreneur that issues catastrophe bonds. Since an $\varepsilon$-entrepreneur promises to repay zero in the good state and he will go bust in the bad state, the no-diversion constraint is $0 < hb_t^{\varepsilon, c}$. This constraint does not require equity investment (i.e., $w_t > 0$) and it is satisfied for any borrowing level. Rather, the borrowing limit to an $\varepsilon$-entrepreneur equals the expected present value of the bailout. A bailout next period will be granted to the creditors of an $\varepsilon$-entrepreneur if $\varepsilon_{t+1} = 0$. Since in the $\varepsilon_{t+1} = 0$ state, $y_{t+1}^{\varepsilon, nd} = 0$ and $y_{t+1}^{\varepsilon, nd} = y_{t+1}^{\theta} (\theta \phi^s)^\alpha$, total lending to the $\varepsilon$-sector in a black-hole equilibrium is $b_t^{\varepsilon, c} = \delta (1 - \lambda) \gamma y_t^{\theta} (\theta \phi^s)^\alpha$. Therefore, equilibrium final goods’ production using the $\varepsilon$-technology is 

\[
   y_{t+1}^{\varepsilon, nd} = \varepsilon_{t+1} I_t^c = \varepsilon_{t+1} \delta (1 - \lambda) \gamma y_t^{\theta} (\theta \phi^s)^\alpha.
\]

Since the $\varepsilon$-entrepreneur will make zero debt repayments in all states, his expected payoff is positive.

A $\theta$-entrepreneur believes that all other $\theta$-entrepreneurs will not default next period, and that $\varepsilon$-entrepreneurs will default if $\varepsilon_{t+1} = 0$. Thus, she expects a unique price $p_{t+1}$ next period, and that a bailout will be granted only if $\varepsilon_{t+1} = 0$. Given these expectations, she chooses whether to issue standard bonds or catastrophe bonds, and whether to implement a diversion scheme or not. The proof shows that if condition (56) holds and $\gamma < \gamma^*$, a $\theta$-entrepreneur finds it optimal to issue standard debt and not to implement a diversion scheme. To see the intuition notice that under a no-diversion plan, a $\theta$-agent is indifferent between both types of bonds. Under standard bonds, the best plan of a $\theta$-agent is the same as that in the safe equilibrium characterized in Proposition 3.1: all debt is indexed to $p_{t+1}$, and borrowing constraints bind with $b_t^s = [m^s - 1] w_t$. If the $\theta$-agent issues catastrophe bonds and will repay in all states, the lender require an interest rate no smaller
than \( \rho_t^c \), defined by
\[
1 + \rho^c = \frac{1 + r}{1 - \lambda}.
\]

The borrowing limit is then determined by the no-diversion condition
\[
[1 - \lambda][1 + \rho^c]b_{t}^{c,\theta} \leq h[w_t + b_{t}^{c,\theta}].
\]

Since \([1 - \lambda][1 + \rho^c] = 1 + r \) (by (41)), condition (42) implies that the borrowing constraint with catastrophe bonds is \( b_{t}^{c,\theta} \leq \frac{h\delta}{1 - h\delta} w_t \), which is the same as with standard debt \( b_{t}^{c} = [m^s - 1]w_t = \left[\frac{1}{1 - h\delta} - 1\right]w_t \). Furthermore, expected debt repayments are the same: \([1 - \lambda][1 + \rho^c]b_{t}^{c,\theta} = [1 + r]b_{t}^{c}\).

Thus, under no-default, a \( \theta \)-entrepreneur is indifferent between both types of bonds, a result similar to the Modigliani-Miller theorem.

Next, consider plans with catastrophe bonds that lead to default next period. Under such plans, lenders will get zero if \( \varepsilon_{t+1} = 0 \) and the bailout if \( \varepsilon_{t+1} = \varepsilon \). Thus, they lend up to the present value of the bailout: \( b_{t}^{c,\theta,\text{def}} = \delta[1 - \lambda]\gamma b_{t+1}^{\theta} \). Here is where the restriction on the bailout’s generosity kicks in: if \( \gamma < \frac{\varepsilon}{\varepsilon - \varepsilon} \), the borrowing limit under a default plan is smaller than under a no-default plan, and thus generate less expected profits than the equilibrium plans, as shown in the proof of Proposition 6.1.

Finally, we verify the fiscal solvency of the bailout agency. Since a bailout occurs with probability \( 1 - \lambda \), assuming that starting at \( t = 1 \) the entire final goods’ production of the non-diverting part of the economy can be taxed in a lump-sum way, fiscal solvency requires
\[
E \left( \sum_{t=0}^{\infty} \delta^{t+1} [y_{t+1}^{\theta, nd} + y_{t+1}^{\varepsilon, nd}] \right) \geq E \left( \sum_{t=0}^{\infty} \delta^{t+1} \Gamma_{t+1} \right).
\]

The proof shows that this inequality holds if and only if \( \gamma \leq \frac{\varepsilon}{\varepsilon} \).

**Present Value of Consumption.** From a growth perspective, the anything-goes regime outperforms the financially repressed regime since in the former regime average growth in the final output sector is \((1 + \varepsilon \lambda \delta[1 - \lambda] \gamma)(\theta \phi^s)^\alpha\), while it is \((\theta \phi^s)^\alpha\) in the later regime. However, this does not mean that the present value of consumption in an anything-goes-regime is greater than that in the safe equilibrium \((W^s)\). We need to net out the bailout costs of crises. In order to compute the expected present value of consumption in the the black-hole equilibrium characterized in Proposition 6.1, we add to equation (34) the terms corresponding to the consumption and profit of \( \varepsilon \)-entrepreneurs
\[
W^{agr} = E_0 \left( \sum_{t=0}^{\infty} \delta^t (c_t + c_t^c + c_t^\varepsilon) \right) = E_0 \left( \sum_{t=0}^{\infty} \delta^t [1 - \alpha]y_t^c + \pi_t + \pi_t^\varepsilon - T_t] \right),
\]
which can be simplified as follows:
\[
W^{agr} = \underbrace{W^s}_{\text{Safe Economy PVC}} + \underbrace{\sum_{t=1}^{\infty} \delta^t b_{t-1}^{c,\varepsilon} (\varepsilon - \frac{1 + r}{1 - \lambda})}_{\text{\( \varepsilon \)-expected PVC - Expected Bailout Costs}}.
\]
where $W^s$ is the present value of consumption in a financially repressed economy or in the safe
equilibrium of a financially liberalized economy.\textsuperscript{30} Since the $\varepsilon$ technology has negative net present value \((1 - \lambda)\varepsilon < 1 + r\), it follows that \(W^{agr} < W^s\).

This result implies that even if average growth is higher in an anything-goes economy than
in a repressed one, and production using $\varepsilon$-technology is optimal from the $\varepsilon$-agent’s individual
perspective, the losses it incurs during crises times more than offset private profits. Therefore a
financial black hole equilibrium generate net consumption losses for the economy as a whole.\textsuperscript{31}

7 Conclusions

We have shown that in an economy with credit market imperfections, financial liberalization can
help overcome obstacles to growth by improving allocative efficiency, i.e., easing financing con-
straints of bank-dependent sectors with no easy access to either stock markets or international
capital markets. As a result, constrained sectors can grow faster because they face less severe
bottlenecks–i.e., more abundant inputs produced by the constrained sector. However, as a side
effect financial fragility arises and thus crises occur from time to time.

We have seen that, despite crises, financial liberalization can increase long-run growth and con-
sumption possibilities. The key to this result is that even though the liberalized regime induces
risk-taking, it preserves financial discipline if regulation restricts liabilities to standard debt con-
tracts. In such liberalized regime, despite the presence of systemic bailout guarantees, lenders must
screen out unprofitable projects and incentivize borrowers not to divert by requiring them to risk
their own equity to cover a fraction of the investment. This discipline breaks down in an anything
goes regime. The possibility to issue catastrophe-like instruments that concentrate all repayments
in the default state allows borrowers without any profitable investment opportunities to invest
without putting equity down, and in this way exploit the bailout guarantees.

Our results have important implications for financial regulation in a world with capital market
imperfections. First, one should be cautious when interpreting the effects of financial liberaliza-
tion. From the finding that liberalization has lead to more crisis-induced volatility, one should not
conclude that liberalization per-se is bad for either growth or production efficiency. Furthermore,
policies intended to eliminate all risk taking and financial fragility might have the unintended effect
of blocking the forces that spur growth and allocative efficiency. Second, at the other extreme, the

\textsuperscript{30}The calculation assumes that $\varepsilon$ agents start to borrow in the first period \((t = 0)\) and therefore \(c_0 = 0\).

\textsuperscript{31}We have not discussed the issuance of stocks. Note however that stocks are different from catastrophe bonds.
Although stocks are liabilities that might promise very little in some states of the world, the issuance of stocks does
not bring with it the political pressure for systemic bailout guarantees.
gains from financial liberalization can be overturned in a regime with unfettered liberalization, in which option-like securities can be issued without collateral. In such an anything-goes regime, the presence of systemic bailout guarantees might lead to excessive leverage, a lack of discipline in lending decisions, and might result in misallocation of resources. Even though agents are optimizing and average growth might be higher under an anything-goes regime than under financial repression, the losses during crises more than offset private profits, generating net social losses.

Finally, while most of the current discussion regarding financial regulation concentrate on the asset side, our results demonstrate the importance of regulating the liability side—the types of liabilities that can be issued—to maintain financial discipline and reap the benefits of financial liberalization.

References


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Appendix

A. The Costs of Crises
During a crisis there are widespread bankruptcies, which generate deadweight losses as well as sectorial redistributions. Here, we net out these crises costs and show that the growth costs of crisis reduce to the fall in the N-sector’s investment share, as expressed in (29).32

If a crisis occurs at some date, say $\tau$, there is a firesale: there is a steep decline in the input price, and since there is currency mismatch, all N-firms default. As a result, the investment share falls from $\phi^d$ to $\phi^c$.33 The price of N-goods must fall to allow the T-sector to absorb a greater share of N-output, which is predetermined by $\tau - 1$ investment. At $\tau + 1$, N-output contracts due to the fall in investment at the time of the crisis. However, entrepreneurs adopt risky plans again, so the investment share increases from $\phi^s$ back to $\phi^d$. Thus, there is a real appreciation. At $\tau + 2$, the economy is back on a lucky path, but the level of cash flow and N-output are below their pre-crisis trend.

Although GDP fluctuations are affected by changes in the input price, T-output and N-investment, GDP growth during a crisis episode is solely determined by the mean investment share $[\phi^d, \phi^c]^T$ (by (29)). To understand why this is so note that GDP growth has two components: (i) relative price fluctuations (captured by $Z(\phi_t)/Z(\phi_{t-1})$) and (ii) output fluctuations (captured by $(\theta \phi_t)^\alpha$).34 In the crisis period, GDP growth falls below trend because there is a decline in the input price ($Z(\phi^d)/Z(\phi^c) < 1$). In the post crisis period, there are two effects: (i) since investment contracted during the previous period, N-output falls below trend and depresses growth; but (ii) there is a rebound of the input price as the investment share jumps from its crisis level $Z(\phi^c)/Z(\phi^d) > 1$. As we can see, variations in GDP growth generated by input price changes at $\tau$ and $\tau + 1$ cancel out. Thus, the average loss in GDP growth stems only from the fall in the N-sector’s average investment share.

In sum, a crisis has two distinct effects: sectorial redistribution and deadweight losses. At the

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32Although the main objective of the model is to address long-run issues, it is reassuring that it can account for key stylized facts of balance-sheet crises in emerging markets: a sharp real depreciation that coincides with a fall in credit growth, as well as the asymmetric sectoral response of N- and T-sectors.

33This is because young entrepreneurs income is only $w_p q_t$ instead of $[1 - \beta] p_t q_t$, and at $\tau$ entrepreneurs can only choose safe plans in which there is no currency mismatch (by Proposition 3.2).

34To interpret (29) note that variations in the investment share $\phi_t$ have lagged and contemporaneous effects on GDP. The lagged effect comes about because a change in $\phi_t$ affects next period’s GDP via its effect on N-output: $q_{t+1} = \theta I_t = \theta \phi_t q_t$. Using (26) and $y_t = (1 - \phi_t) q_t^\alpha$, the contemporaneous effect can be decomposed as:

$$\frac{\partial p_t}{\partial \phi_t} = -\frac{\alpha y_t}{1 - \phi_t} + p_t q_t + q_t \phi_t \frac{\partial p_t}{\partial \phi_t} = q_t \phi_t \frac{\partial p_t}{\partial \phi_t}$$

The first two terms capture variations in T-output and N-investment, while the third reflects input price fluctuations. Market clearing in the N-goods market i.e., $(1 - \phi_t) p_t q_t = \alpha y_t$ implies that the induced changes in N-sector investment and T-output cancel out. Therefore, the contemporaneous changes in the investment share affect GDP contemporaneously only through its effect on the real exchange rate. Since $GDP_t = Z(\phi_t) q_t^\alpha$, we can express $q_t \phi_t \frac{\partial p_t}{\partial \phi_t}$ as $q_t^\alpha \frac{\partial Z(\phi_t)}{\partial \phi_t}$. Thus, we can interpret $Z(\phi_t) q_t^\alpha$ as the effect of real exchange rate fluctuations on GDP.
time of the crisis the T-sector benefits from the financial collapse of the N-sector because it can buy N-output at firesale prices and expand production. This leads to a sharp fall in the N-to-T output ratio in the wake of crisis. The deadweight losses derive from the financial distress and the bankruptcy costs generated by crises. The former leads to a contraction in N-investment and thus has a long-run effect on output. In contrast, bankruptcy costs have only a static fiscal impact, which is the cost of the bailout.

B. Post-Crisis Cool-Off Phase and Growth. Here, we show that form the perspective of long-run growth, nothing is gained by delaying the onset of the new risky phase.

In Proposition 3.2, we characterized a RSE where there is a reversion back to a risky path in the period immediately after the crisis. We then compared growth in such a risky economy–where risk-taking occurs whenever it is possible–to growth in a safe economy where risk-taking never occurs. The comparison of these polar cases makes the argument transparent, but opens the question of whether the growth results presented in Proposition 4.1 are applicable to recent experiences in which systemic crises have been followed by protracted periods of low leverage, low investment and low growth. In order to address this issue, we construct an alternative RSE under which a crisis is followed by a cool-off phase during which all agents choose safe plans. The cool-off phase can be interpreted either as a period in which agents believe that others are following safe strategies or as a period during which agents are prevented from taking on risk.  

To keep the model tractable, we assume that in the aftermath of a crisis, all agents follow safe plans with probability $\zeta$. Hence, a crisis is followed by a cool-off phase of average length $1/(1-\zeta)$ before there is reversion to a risky path. In this case, the mean long-run GDP growth rate is

$$E(1 + \gamma) = \left(\theta \phi^s\right)^\alpha \left(\phi^l \right)^{\frac{1-\zeta}{(1-\zeta) + (1-u)}} \left(\frac{\mu_w}{1-\beta}\right)^{\frac{u(1-\zeta)}{(1-\zeta) + (1-u)}},$$

which generalizes the growth rate of proposition 4.1. Comparing (46) with (27) we can prove the following Lemma.

**Lemma .1**  Consider an RSE where a crisis is followed by a cool-off period of average length $1/(1-\zeta)$. Then:

35Or alternatively as a period where agents revise downwards their bailout expectations because they perceive that the surge in public debt associated with prior bailouts makes future bailout less likely.

36The average length of the cooling off period is computed as:

$$\lambda = (1-\xi) \sum_{k=0}^{\infty} \xi^{k-1}k = \frac{1}{1-\xi}$$

40
1. The conditions under which mean long-run GDP growth is greater in a risky than in a safe equilibrium are independent of $\zeta$, and are the same as those in Proposition 4.1.

2. The shorter the average cool-off period $1/(1 - \zeta)$, the higher the mean long-run GDP growth in a RSE.

The reason why the growth-enhancing properties of risk taking—stated in Proposition 4.1—are independent of $\zeta$ is that during the cool-off phase the economy grows at the same rate as in a safe equilibrium. Part 2 makes the important point that the faster risk-taking resumes in the wake of crisis, the higher will be mean long-run growth.

C. Model Simulations. The behavior of the model economy is determined by nine parameters: $u, \delta, \alpha, \theta, h, \beta, \mu_w, \mu$ and $\gamma$. We will set the probability of crisis $1 - u$, the world interest rate $r$ and the share of N-inputs in T-production $\alpha$ equal to some empirical estimates. Then, given the values of $u$, $\delta$ and $\alpha$, we determine the feasible set for the degree of contract enforceability $h$, the N-Sector’s cash flow-to-sales ratio $(1 - \beta)$ and total factor productivity in the N-sector $\theta$, such that both an RSE and an SSE exist. The values the crises’ costs $\mu_w$ and $\mu$ are irrelevant for the existence of equilibria. The generosity of the bailouts $\gamma$ is relevant only in Section 6, on the anything-goes regime. The admissible parameter set is determined by the following recursive conditions:

- $h < \delta^{-1}$: necessary condition for borrowing constraints in safe equilibrium
- $u > h\delta$: necessary condition for borrowing constraints to bind in the risky equilibrium
- $\beta > \beta = h\delta/u$: necessary and sufficient for positive prices: $\phi^* < 1$, $\phi^s < 1$.
- $\theta > \theta^s(\delta, h, \alpha, \beta)$: necessary and sufficient for safe plans to generate a rate on return on equity (ROE) larger than the risk-free rate
- $\theta < \theta(\delta, h, u, \alpha, \mu_w, \beta)$: necessary and sufficient for default in low price state
- $\theta > \theta(\delta, h, u, \alpha, \beta)$: necessary and sufficient for risky plans to generate a rate on return on equity larger than the risk-free rate.
- $\beta < \beta(\delta, h, u, \alpha, \mu_w)$: necessary and sufficient for $\bar{\theta} > \theta^r$
- $u > u$: sufficient for no-diversion in a risky equilibrium.
- $\gamma < \gamma$: necessary for a black-hole equilibrium. The bounds are:

$$\theta^s = \left[ \frac{1}{\beta\delta} \right]^{\frac{1}{\alpha}} \left[ \frac{1 - \beta}{1 - h\delta} \right]^{\frac{1-\alpha}{\alpha}}$$

(47)
\[
\theta \equiv \left[ \frac{1}{\delta} - \frac{h}{u} + h \right] \left[ \frac{1 - \beta}{1 - h\delta/u} \right]^{1-\alpha} \left[ \frac{1}{u\beta} \right]^{\frac{1}{\alpha}}, \quad \vartheta \equiv \left[ \frac{1}{\beta} \left[ \frac{1 - \beta}{1 - h\delta/u} \right] \right]^{1-\alpha} \left[ \frac{1}{1 - h\delta} \right]^{1-\alpha} \left( \frac{h}{u} \right) ^{\frac{1}{\alpha}} \tag{48}
\]
\[
\bar{\beta} = \frac{\beta}{2} + \left[ 1 - \frac{\mu_w}{1 - h\delta} \right] \left[ \frac{1}{h\delta} - \frac{1}{u} + 1 \right]^{\frac{1}{\alpha}} \left[ \frac{1}{1 - h\delta/u} \right]^{-1} \tag{49}
\]
\[
u \text{ s.t. } \max \left\{ \beta\theta^\alpha \left[ \frac{1 - h\delta/u}{1 - \beta} \right]^{1-\alpha} - \frac{h}{u[1-u]}, \quad \beta\theta^\alpha \left[ \frac{\beta - h\delta/u}{1 - \beta} \right]^{1-\alpha} - \frac{h}{u[1-u]} \right\} < 0 \quad \forall u > u \tag{50}
\]
\[
\tau \equiv \max \left\{ \frac{h\alpha}{1-\lambda} \frac{\phi}{1 - \phi} \left[ \theta^{\alpha} \phi^{\alpha - 1} - h \right] \frac{\theta^{\alpha} \phi^{\alpha + 1}}{1 - \phi}, \frac{1}{1 - \lambda [1 - \lambda \delta^u]} \right\}, \quad \phi \equiv \frac{1 - \beta}{1 - h\delta} \tag{51}
\]

In the simulations, the baseline crisis probability is set equal to 5%, slightly higher than 4.13%, the probability of a systemic crisis in emerging markets (Ranciere, Tornell and Westermann, 2008); \( \alpha \) is calibrated in reference to the use of non-tradable goods as inputs in tradables production, using Mexico’s input-output table. We choose \( h, \beta \) and \( \theta \) so that: (i) both an RSE and an SSE exist for the range \( u \in [0.91, 1] \), and (ii) we obtain plausible values for the growth rates along a safe economy and along a lucky path. In the baseline case: \( h = 0.76, \theta = 1.65, \beta = 0.8 \) and \( u = 0.95 \). These parameters imply a safe GDP growth rate of \( (1 + \gamma^s) = (1 - \beta)^\alpha \theta \frac{h}{1 - h\delta} = 3.8\% \) and a lucky GDP growth rate of \( (1 + \gamma^l) = (1 - \beta)^\alpha \left( \frac{\theta}{1 - h\delta} \right) ^\alpha = 8.7\% \). We choose the financial distress costs of crises \( l^d = 1 - \frac{\mu_w}{1 - h\delta} \) so that the cumulative decrease of GDP during a crisis episode is 13\%, which is the mean value in the sample considered by Tornell and Westermann (2002). In the model, the cumulative decrease in GDP growth during a crisis episode is \( (1 + \gamma^c)^2 = \left[ \frac{\mu_w}{1 - \beta} \right]^\alpha \left( \theta^2 \phi^\delta \phi^k \right)^\alpha \). Using the baseline case \( h = 0.76, \theta = 1.65 \), and \( \alpha = 0.35 \) we get that \( (1 + \gamma^c)^2 = (1 - 0.13) \) if \( \left[ \frac{\mu_w}{1 - \beta} \right] = 0.45 \). Thus, we set conservatively \( l^d = 0.7 \). In the baseline case, the level of bankruptcy costs is free. The discount rate \( \delta = 1/(1 + r) \) is set equal to 0.925.

Finally, in order for the welfare measures to be bounded, the expected discounted sum of tradable production has to be finite. In the safe economy this requires \( \delta(\theta\phi^s)^\alpha < 1 \). In the risky economy: \( [\theta \phi^l]^\alpha \delta u + [\theta^2 \phi^l \phi^k]^\alpha \delta^2 (1 - u) < 1 \). These two conditions impose an upper bound on \( \alpha \).\(^{37}\) In particular, they hold if \( \alpha < 0.6 \). Summing up:

\(^{37}\) Notice that the interior condition for the pareto optimal share, \( \phi^{\alpha_0} = [\theta \phi^s]^{\frac{1}{1-\alpha}} < 1 \) is sufficient for all boundness conditions if \( \phi^l < \phi^{\alpha_0} \). This condition is equivalent to an upper bound on \( \alpha : \frac{1}{\alpha} = \frac{\log(1 + \epsilon)}{\log(\epsilon)} \).
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ONLINE APPENDIX: PROOFS AND DERIVATIONS

Proof of Proposition 3.1. First, we determine the conditions on returns \( \frac{\partial \rho_{t+1}}{\partial \rho_t} \) that make the strategy of Proposition 3.1 optimal for an individual entrepreneur, given that all other entrepreneurs follow the equilibrium strategy: borrow up to the limit (i.e., \( b_t[p_{t+1}[1 + \rho_t] = h[w_t + b_t] \)), invest all funds in the production of N-goods (\( p_k = w_t + b_t, s_t = 0 \)), and never default. We then determine the parameter conditions under which the price sequences that result if all entrepreneurs follow the equilibrium strategy, generate a high enough return to validate the strategy of an individual entrepreneur.

Given that all other entrepreneurs follow the equilibrium strategy, crises never occur and prices are deterministic: \( u_{t+1} = 1 \) and \( E_t(p_{t+1}) = p_{t+1} \). Thus no bailout is expected. First, since no bailout is expected, lenders will get repaid zero with any plan that leads to diversion. Hence, lenders only fund plans where the no-diversion condition holds. Second, an entrepreneur has no incentives to deviate an issue T-debt. Since competitive risk-neutral lenders have to break even: the interest rate offered on N-debt is \( 1 + \frac{N}{1 + r} = \frac{h[w_t + b_t]}{E_t(p_{t+1})} \), while that on T-debt is \( r \). Thus, the expected interest costs are the same under both types of debt:

\[
1 + \frac{N}{1 + r} = 1 + r = 1 + r
\]

Hence, the borrowing limits are the same under both types of debt, and so there is no incentive to issue T-debt. It follows that if all other firms choose a safe plan, the payoff of a safe plan is the solution to the following problem

\[
\max_{b_{i,t}, k_{i,t+1}, l_{i,t+1}} Z_{i,t} = \left\{ E_t(p_{t+1}) \Theta_{t+1} k_{i,t+1}^{\beta-1} l_{i,t+1}^{1-\beta} - b_{i,t}[1 + r] - v_{t+1} l_{i,t+1} \right\}, \text{ subject to } \quad (52)
\]

\[
p_k k_{i,t} \leq w_{i,t} + b_{i,t} - s_{i,t}, \quad b_{i,t}[1 + r] \leq h[w_t + b_t], \quad \pi_{i,t+1}^s \geq 0,
\]

where prices and the wage are taken as given. Suppose for a moment that \( \frac{\partial E_t(p_{t+1})}{\partial \rho_t} \) is high enough so that it is optimal to borrow up to the limit allowed by the no-diversion condition (\( b_t[1 + r] = h[w_{i,t} + b_{i,t}] \)), and not store (so that \( p_t k_{i,t+1} = w_{i,t} + b_{i,t} \)). It follows that the first order conditions are

\[
\frac{\partial Z_{i,t}}{\partial k_{i,t+1}} = E_t(p_{t+1}) \Theta_{t+1} l_{i,t+1}^{1-\beta} k_{i,t+1}^{\beta-1} - h p_t \geq 0, \quad \frac{\partial Z_{i,t}}{\partial l_{i,t+1}} = p_{t+1} \Theta_{t+1} k_{i,t+1}^{\beta-1} l_{i,t+1}^{1-\beta} - v_{t+1} \geq 0 \quad (53)
\]

Notice that \( \pi_{t+1}^s \) is concave in \( k_{i,t+1} \) because \( \beta < 1 \). Since in a SSE all entrepreneurs choose the same investment level, \( \Theta_{t+1} k_{i,t+1}^{\beta-1} = \theta k_{i,t+1}^{\beta-1} l_{i,t+1}^{1-\beta} = \theta \). Furthermore, since labor is inelastically supplied (\( l^s = 1 \)), the equilibrium wage is

\[
\tilde{w}_{t+1} = p_{t+1} \theta k_{t+1}[1 - \beta] \quad (54)
\]

Substituting the equilibrium wage in (53) we have that in an SSE, the marginal return of capital...
Thus, if $\beta \theta E_t(p_{t+1}) > p_t h$ the solution to (52) entails borrowing and investing as much as allowed by the no-diversion condition: $b_{i,t} = \frac{1}{\lceil \frac{1}{\beta \delta} \rceil - 1} w_{i,t} \equiv [m^s - 1] w_{i,t}$. It follows that the payoff associated with the equilibrium strategy is

$$\pi_{t+1}^* = \beta \theta p_{t+1} k_{t+1} - h[w_t + b_t] = \left[ \beta \theta p_{t+1} - h \right] [w_t + b_t] = \left[ \beta \theta p_{t+1} - h \right] m^s w_t \quad (55)$$

In order for the above solution to be optimal, this return on equity must be greater than the storage return: $[\theta \beta p_{t+1}/p_t - h] m^s > 1 + r$, which is equivalent to $\theta \beta p_{t+1}/p_t > 1 + r$. To determine whether this condition is satisfied we need to endogeneize prices. To do so we use (19) and (20), and find that in a SSE $\frac{p_{t+1}}{p_t} = (\theta \phi^s)^{\alpha - 1}$. Therefore,

$$\frac{\beta \theta p_{t+1}}{p_t} > \frac{1}{\delta} \iff \beta \theta^s (\phi^s)^{\alpha - 1} > \frac{1}{\delta} \iff \theta > \theta^s \equiv \left[ \frac{1}{\beta \delta} \right] \frac{1 - \beta}{1 - h \delta}^{\frac{\alpha - 1}{\alpha}} \quad (56)$$

We conclude that the SSE characterized in Proposition 3.1 exists if and only if (56) holds, and $\beta < \beta$ so that prices are positive (i.e., $\phi^s < 1$). Finally, notice that, because there is a production externality, there exists also a degenerate equilibrium in which nobody invests because everyone believes others will not invest: $\theta k_{t+1} = 0$.

**Proof of Proposition 3.2.** The proof is in two parts. In part A we construct an RSE where two crises do not occur in consecutive periods. Then, in part B we show that two crises cannot occur in consecutive periods.

**Part A.** Consider an RSE where, if there is no crisis at $t$, prices next period can take two values as in (25). Meanwhile, if there is a crisis at $t$, there is a unique $p_{t+1}$. In a no crisis period a firm can choose three types of plans: a "risky plan" where a firm denominates debt in T-goods (i.e., with currency mismatch), will default if $p_{t+1} = p_{t+1}$, and does not divert; a "safe plan" where a firm denominates debt in N-goods, will never default and does not divert; finally a "diversion plan" where the firm will divert all funds. We will construct an RSE in which all entrepreneurs choose the risky plan during every period, except when a crisis erupts, in which case they choose the safe plan. In a first step we determine the conditions under which a risky plan is preferred to a safe plan and to storage. In a second step we determine the conditions under which diversion plans are not optimal.

**Step 1.** Suppose for a moment that $p_{t+1}$ is low enough so as to bankrupt firms with T-debt (we will determine below the parameter set under which this holds). Since in an RSE every firm issues T-debt, a bailout will be granted next period in the low price state. In this case lenders will be
repaid in all states (either by the borrowers or by the bailout agency) and so they break-even if the interest rate is \( \rho^T = r \). It follows that if all other firms choose a risky plan, the payoff of a risky plan is the solution to the following problem:

\[
\max_{b_{i,t}, k_{i,t+1}, \delta} u_{t+1}(\pi_{i,t+1}) = \left( \bar{p}_{t+1} \Theta_{t+1} k_{i,t+1}^{\beta} - b_{i,t} [1 + r] - v_{t+1} l_{i,t+1} \right), \quad \text{subject to} \quad p_{t+1} k_{i,t} \leq w_{i,t} + b_{i,t}, \quad \pi_{i,t+1}(\bar{p}_{t+1}) \geq 0, \quad \pi_{i,t+1}(p_{t+1})
\]

where \( (p_t, \bar{p}_{t+1}, v_{t+1}, \Theta_{t+1}) \) are taken as given. The first order conditions are

\[
\frac{\partial u_{t+1}(\pi_{i,t+1})}{\partial k_{i,t+1}} = u p_{t+1} \Theta_{t+1} l_{t+1}^{1-\beta} k_{i,t+1}^{\beta-1} - h p_t \geq 0, \quad \frac{\partial u_{t+1}(\pi_{i,t+1})}{\partial l_{i,t+1}} = p_{t+1} l_{t+1}^{1-\beta} k_{i,t+1}^{\beta} [1 - \beta] - v_{t+1} \geq 0
\]

Notice that \( \pi_{t+1} \) is concave in \( k_{i,t+1} \) because \( \beta < 1 \). Since in an RSE all entrepreneurs choose the same investment level, \( \Theta_{t+1} l_{t+1}^{1-\beta} k_{i,t+1}^{\beta-1} = \theta k_{i,t+1}^{\beta-1} = \theta \). Furthermore, the equilibrium wage is given by (54). Following the same steps as in the proof of Proposition 3.1, we have that if \( u \frac{\partial \pi_{t+1}}{\partial p_t} > h \), the solution to (57) entails borrowing up to the limit allowed by the no-diversion constraint: \( b_t = [m^r - 1] w_t = [m^r h \delta / u] w_t \). Thus, the payoff is

\[
E_t \pi_{t+1} = \left[ u \frac{\beta \theta p_{t+1}}{p_t} - h \right] m^r w_t
\]

In order for a firm to choose a risky plan the following conditions must be satisfied: (i) \( p_{t+1} \) must be low enough so as to bankrupt firms with T-debt, otherwise a bailout next period would not be expected and firms would not be able to take on systemic risk; (ii) \( \bar{p}_{t+1} \) must be high enough so as make the risky plan preferred both to storage and a to safe plan:

\[
\pi_{t+1}^r(p_{t+1}) < 0, \quad E_t \pi_{t+1}^r > w_t [1 + r], \quad E_t \pi_{t+1}^s > E_t \pi_{t+1}.
\]

Next we derive equilibrium returns and determine the parameter conditions under which (60) holds. Using the equations for internal funds (17), N-output (19) and prices (20), and noting that in an RSE the investment share \( \phi_{t+1} \) equals \( \phi^{\ell} \) if N-firms are solvent, while \( \phi_{t+1} = \phi^{c} \) if they are insolvent, it follows that equilibrium returns are

\[
R \equiv \beta \theta \frac{p_{t+1}}{p_t} = \beta \theta^{\alpha} \left( \frac{1}{\phi^{\ell}} \right)^{1-\alpha}, \quad R \equiv \beta \theta \frac{p_{t+1}}{p_t} = \beta \theta^{\alpha} \left( \frac{1}{\phi^{c}} \right)^{1-\alpha}
\]

First, substituting (61) in \( \pi^r(p_{t+1}) = \beta \frac{p_{t+1}}{p_t} q_{t+1} - L_{t+1} = \beta \frac{p_{t+1}}{p_t} m^r w_t - \frac{h}{u} m^r w_t \), it follows that the risky plan defaults in the low price state if and only if \( \left[ R - \frac{h}{u} \right] m^r w_t < 0 \):

\[
\pi^r(p_{t+1}) < 0 \iff \frac{h}{u} > R \iff \frac{h}{u} > \beta \theta^{\alpha} \left( \frac{1}{\phi^{\ell}} - 1 \right)^{1-\alpha} \left[ \frac{1}{1 - \phi^{c}} \right]^{1-\alpha}
\]

To simplify notation we will omit the subscripts that identify individual agents.
Second, the risky plan is preferred to storage if and only if \( [u\bar{R} - h]m^r w_t \geq w_t / \delta : \)

\[
E_t \pi^r \geq \frac{w_t}{\delta} \iff u\bar{R} - h \geq \frac{1}{\delta} - \frac{h}{u} \iff u\beta \theta^\alpha \left[ \frac{1}{\varphi^t} \right]^{1-\alpha} - h \geq \frac{1}{\delta} - \frac{h}{u} \tag{63}
\]

Third, to derive \( E_t \pi^r_{t+1} \), note that if an entrepreneur were to deviate and choose a safe plan, the interest rate it would have to offer is \( 1 + \rho^s = [1 + r]/p_{t+1}^c \), and her borrowing constraint would be \( b_t^u [1 + \rho^s]p_{t+1}^c \leq h[w_t + b_t^u] \). Following the same steps as in the proof of Proposition 3.1, we have that \( b_t^u = [m^s - 1]w_t \), and the payoff would be

\[
E_t \pi^s_{t+1} = \left[ \frac{u\beta \theta p_{t+1}^c}{p_t} + \frac{[1 - u]\beta \theta p_{t+1}^c}{p_t} - h \right] m^s w_t \tag{64}
\]

Thus, a risky plan is preferred to a safe one if and only if \( [u\bar{R} - h]m^r w_t \geq [u\bar{R} + (1-u)\bar{R} - h]m^s w_t \), which is equivalent to\(^{39}\)

\[
E_t \pi^r_{t+1} > E_t \pi^s_{t+1} \iff u\bar{R} - h \geq \bar{R}[1 - h\delta/u][h\delta/u]^{-1} \tag{65}
\]

Next, we verify that (62), (63) and (65) can hold simultaneously. Notice that the LHS of (63) and (65) are the same. Thus, (63) implies (65) if and only if \( \frac{1}{\delta} - \frac{h}{u} > \bar{R}[1 - (h\delta/u)](h\delta/u)^{-1} \). Rewriting \( \frac{1}{\delta} - \frac{h}{u} \) as \( [1 - (h\delta/u)]^{-1} \), the condition becomes \( [1 - (h\delta/u)]^{-1} > \bar{R}[1 - h\delta/u][h\delta/u]^{-1} \). Since an RSE exists only if \( u > h\delta \), we get \( \delta^{-1} > \bar{R}[\delta/(\delta/u)^{-1}] \) or equivalently \( h/u > \bar{R} \). Since \( h/u > \bar{R} \) is (62), we have that (63) is stronger than (65) if and only if (62) holds. Hence, we conclude that if (62) and (63) hold, then (65) must hold.

We next determine the parameter set such that (62) and (63) hold simultaneously. Condition (63) holds if and only if

\[
\theta \geq \bar{\theta}(\delta, h, u, \alpha, \beta) \equiv \left[ \frac{1}{\delta} - \frac{h}{u} + h \left[ \frac{1 - \beta}{1 - h\delta/u} \right]^{1-\alpha} \frac{1}{u\beta} \right]^{1/\alpha} \tag{66}
\]

Note that (61) implies that

\[
\bar{\theta}^{-\alpha} = \beta \left[ \frac{1}{\varphi^t} - 1 \right]^{1-\alpha} = \beta \left[ \frac{1 - h\delta u^{-1}}{1 - \beta} \right]^{1-\alpha} \left[ 1 - \frac{\mu_w}{1 - h\delta} \right]^{\alpha^{-1}}
\]

Thus, condition (62) holds if and only if

\[
\theta < \bar{\theta}(\delta, h, u, \alpha, \mu_w, \beta) \equiv \left[ \frac{1}{\beta} \left[ \frac{1 - \beta}{1 - h\delta/u} \right]^{1-\alpha} \left[ 1 - \frac{\mu_w}{1 - h\delta} \right]^{1-\alpha} \frac{h}{u} \right]^{1/2} \tag{67}
\]

\(^{39}\)Rewrite \( [u\bar{R} - h]m^r w_t \geq [u\bar{R} + (1-u)\bar{R} - h]m^s w_t \) as \( [u\bar{R} - h]m^r - m^s \geq (1-u)\bar{R}m^s \iff [u\bar{R} - h]((1-u)\bar{R} + m^s) \geq (1-u)\bar{R}m^s \iff [u\bar{R} - h][m^r - 1] \geq \bar{R} \).
In order for (66) and (67) to hold simultaneously it is necessary that \( \bar{\theta} > \theta \):

\[
\frac{1}{\beta} \left[ \frac{1 - \beta}{\beta - h\delta/u} \right]^{1-\alpha} \left[ 1 - \frac{\mu_w}{1 - h\delta} \right]^{1-\alpha} \frac{h}{u} > \left[ \frac{1}{\beta} - \frac{h}{u} + h \right] \left[ \frac{1 - \beta}{1 - h\delta/u} \right]^{1-\alpha} \frac{1}{u\beta} \left[ \frac{1}{\beta - h\delta/u} \right]^{1-\alpha} \left[ 1 - \frac{\mu_w}{1 - h\delta} \right]^{1-\alpha} \frac{1}{u}\beta.
\]

The LHS is decreasing in \( \beta \). It ranges from infinity, for \( \beta \rightarrow \beta \equiv h\delta/u \), to \( L(1) = \left[ \frac{1}{1-h\delta/u} \right]^{1-\alpha} \left[ 1 - \frac{\mu_w}{1-h\delta} \right]^{1-\alpha} \) for \( \beta = 1 \). Since \( L(1) \) is lower than the RHS (because \( u > h\delta \)), it follows that there is a unique threshold \( \bar{\beta} \) such that \( \bar{\theta} > \theta \) if and only if \( \beta < \bar{\beta} \). The above condition implies that this upper bound on \( \beta \) is

\[
\bar{\beta}(\delta, h, u, \alpha, \mu_w) = \beta + \left[ 1 - \frac{\mu_w}{1 - h\delta} \right] \left[ \frac{1}{h\delta} - \frac{1}{u} + 1 \right] \frac{1}{1-\alpha} \left[ \frac{1}{1 - h\delta/u} \right]^{-1} \quad (68)
\]

Summing up, given that parameters \((\delta, h, u, \beta)\) satisfy (12), condition (62) holds if and only if \( \theta < \bar{\theta} \), while condition (63) holds if and only if \( \theta > \bar{\theta} \). Furthermore, \( \bar{\theta} > \theta \) if and only if \( \beta < \bar{\beta} \). Thus, we conclude that during a no-crisis period, equilibrium expected returns are such that an entrepreneur prefers the equilibrium risky plan over both storage and a safe plan (i.e., the conditions in (61) hold) if and only if \( \theta \in (\bar{\theta}, \bar{\theta}) \) and \( \beta < \bar{\beta} \).

Consider next a crisis period. Given that all other entrepreneurs choose a safe plan, there can be no crisis and no bailout in the post-crisis period. Thus, an entrepreneur faces the same problem as that in a safe symmetric equilibrium. It follows from Proposition 3.1 that she will find it optimal to choose a safe investment plan if and only if \( \beta \theta p_{t+1}/p_t \geq \delta^{-1} \). This condition is equivalent to \( \beta \theta s \phi^s(\phi^s)_{\alpha-1} \geq \delta^{-1} \), which is implied by (63) because \( u > h\delta \).

Step 2. We show that the risky plan is preferred to a diversion plan if \( u \) is large. Consider first a "risky diversion plan" in which an entrepreneur deviates and chooses T-debt: she incurs a cost \( h[w_t + b_d^f] \) at \( t \), and will be able to divert all funds at \( t + 1 \) provided she will be solvent. This plan maximizes

\[
D_{t+1}^r = u \bar{\pi}_{t+1} \Theta_{t+1} l_{t+1}^{1-\beta} \kappa_{t+1}^{-\beta} - u w_{t+1} l_{t+1} - h[w_t + b_t], \quad \text{subject to}
\]

\[
1 + \rho^d = \frac{1 + r}{1 - u} \quad \pi_{t+1}^d(\bar{p}_{t+1}) \geq 0, \quad \pi_{t+1}^d(\bar{p}_{t+1}) < 0, \quad h[w_t + b_t] < b_t[1 + \rho^d]
\]

The interest rate is \( \frac{1 + r}{1 - u} \) because lenders must break-even by cashing the bailout next period, with probability \( 1 - u \). We will construct an upper bound on \( D^r \) and show that it is lower than \( E \pi_{t+1}^r \) if \( u \) is large enough. To do so we consider a fictitious auxiliary problem under which (i) only the first two conditions in the above problem are considered; and (ii) a diverting firm pays a wage \( \tilde{\nu}_{t+1}(b_t^d) = [1 - \beta] \bar{\nu}_{t+1} \Theta_{t+1} l_{t+1}^{1-\beta} \). Under this fictitious wage, the diverting firm will find it optimal to set its labor demand equal to one: \( l_{t+1} = 1 \), and so its payoff, denoted by \( \tilde{D} \), is

\[
\tilde{D}(b_t^d) = u \beta \bar{\nu}_{t+1} \Theta_{t+1} \kappa_{t+1}^{1-\beta} [b_t^d] - h[w_t + b_t^d].
\]
Next, we determine the values of \( u \) for which the solvency constraint \((\pi^{\text{div}}_{t+1}(\overline{p}_{t+1}) = \overline{p}_{t+1} \Theta_{t+1} l_{t+1}^{1-\beta} k_{t+1}^{\beta} - \overline{w}_{t+1}(b_t^d) l_{t+1} - b_t^{d (1+r)} / (1-u) \geq 0)\) is violated if \( b_t^d = b_t^r \).

\[
\pi^{\text{div}}_{t+1} = \frac{\beta \overline{p}_{t+1} \theta}{p_t} [w_t + b_t^r] - b_t^r \frac{1+r}{1-u} = \left[ \frac{\beta \overline{p}_{t+1} \theta}{p_t} m^r - \frac{m^r - 1}{1-u} \delta \right] w_t = \left[ \frac{\beta \overline{p}_{t+1} \theta}{p_t} - \frac{h}{u} \right] m^r w_t.
\]

Using (61), we know that in an RSE \( \beta \overline{p}_{t+1} / p_t = \beta \theta \left[ \frac{1}{\phi} \right]^{1-\alpha} \). Since \( 1/\phi^t = 1 - h \delta / u \),

\[
\pi^{\text{div}}_{t+1}(\overline{p}_{t+1}) < 0 \Leftrightarrow 0 > \beta \theta \left[ \frac{1 - h \delta / u}{1 - \beta} \right]^{1-\alpha} - \frac{h}{u} \left[ 1 - u \right] \equiv G^r(u).
\]

Recall that an RSE exists only if \( u \in (h \delta, 1) \). Since \( G^r(u) \) is continuous and \( \lim_{u \to 1} G^r(u) = -\infty \), there exist a lower bound \( u^r > h \delta \) so that \( G^r(u) < 0 \) for all \( u > u^r \). It follows that if \( u > u^r \) the solvency constraint of the auxiliary diversion problem \((\pi^{\text{div}}_{t+1}(\overline{p}_{t+1}) \geq 0)\) is violated at the equilibrium debt level \( b_t^r \). Since \( \pi^{\text{div}}_{t+1}(\overline{p}_{t+1}) = \frac{\beta \overline{p}_{t+1} \theta}{p_t} [w_t + b_t^r] - \frac{1+r}{1-u} b_t^d \) is decreasing in \( b_t^d \) (because \( \frac{\overline{p}_{t+1} \theta}{p_t} < \frac{h}{u} \left[ 1 - u \right] < \frac{1+r}{1-u} \) if \( u > u^r \),), diversion must entail \( b_t^d < b_t^r \). Lower debt in turn implies that the fictitious wage is lower than the actual equilibrium wage (i.e., the diverting firm gets a subsidy under the auxiliary problem), and so the payoff of the auxiliary problem \( D(b_t^r) \) is an upper bound on the diversion payoff. Finally, notice that if the diverting firm could borrow \( b_t^r \), its payoff would be \( D(b_t^r) = [u \overline{p}_{t+1} \theta / p_t - h] m^r w_t \), which equals the equilibrium payoff \( E_t \pi^{r,nd}_{t+1} \). Since \( D(b_t^r) > D(b_t^r) = E_t \pi^{r,nd}_{t+1} \), a firm has no incentives to choose a risky diversion plan if \( u > u^r \)

\[
u^r \text{ such that } \beta \theta \left[ \frac{1 - h \delta / u}{1 - \beta} \right]^{1-\alpha} - \frac{h}{u} \left[ 1 - u \right] < 0, \quad \forall u > u^r.
\]

Next, consider a "safe diversion plan" in which an entrepreneur chooses N-debt and will be solvent in both states next period. Under such plan the interest rate is \( \frac{1+r}{1-u} \). Using the same argument we can show that there is a \( u^s \), such that if \( u > u^s \), the solvency constraint associated with the auxiliary problem (i.e., \( 0 \leq \pi^{\text{div}}_{t+1}(\overline{p}_{t+1}) = \frac{\beta \overline{p}_{t+1} \theta}{p_t} [w_t + b_t^s] - \frac{1+r}{1-u} b_t^d \)) does not hold at the debt level of a safe plan \( b_t^s = \left[ m^s - 1 \right] w_t \). Thus, in an RSE, the payoff of a safe diversion plan is bounded above by the payoff of a safe no-diversion plan \( E_t \pi^{s,nd}_{t+1} \). Since we have shown in Part A that \( E_t \pi^{s,nd}_{t+1} < E_t \pi^{r,nd}_{t+1} \), it follows that a firm has no incentives to choose a safe diversion plan if

\[
0 > \pi^{\text{div}}_{t+1}(\overline{p}_{t+1}) = \left[ \frac{\beta \overline{p}_{t+1} \theta}{p_t} m^s - \frac{m^s - 1}{1-u} \delta \right] w_t = \left[ \frac{\beta \overline{p}_{t+1} \theta}{p_t} - \frac{h \delta}{1-u} \right] m^s w_t.
\]

Using (61) we have that there are no incentives to choose a safe diversion plan if \( u > u^s \):

\[
u^s \text{ such that } \beta \theta \left[ \frac{1 - \phi^s / \phi^r}{1 - \phi^s} \right]^{1-\alpha} \left[ \frac{1}{1 - \phi^s} \right]^{1-\alpha} - \frac{h}{1-u} < 0, \quad \forall u > u^s.
\]

We conclude that the risky equilibrium plan is preferred to a diversion plan if crisis are not frequent:
\[ u > \max\{u^r, u^s\} \]
\[
u \text{ s.t. } \max \left\{ \beta \theta^\alpha \left[ \frac{1 - \frac{h\delta}{u}}{1 - \beta} \right]^{1 - \alpha} - \frac{h}{u} \left[ \frac{1 - \frac{\mu_w}{1 - h\delta}}{1 - \beta} \right]^{1 - \alpha} \left[ \frac{1 - \frac{\mu_w}{1 - h\delta}}{1 - \beta} \right]^{\alpha - 1} - \frac{h}{1 - u} \right\} < 0 \ \forall u > u \]

(71)

**Part B.** We show that two crises cannot occur in consecutive periods. Suppose to the contrary that a crisis occurs at \( \tau \), firms choose risky plans at \( \tau \). We will show that it is not possible for firms to become insolvent in the low price state at \( \tau + 1 \) (i.e., \( \pi(p_{\tau+1}) < 0 \)), and so a bailout cannot be expected. It suffices to consider the case in which firms internal funds at \( \tau + 1 \) equal \( \mu_w \), and they undertake safe plans at \( \tau + 1 \), as this generates the lowest possible price \( p_{\tau+1} \).

We will show that even in these extreme case it is not possible to generate \( \pi(p_{\tau+1}) < 0 \): Along this path the \( \phi \)-investment share is \( \phi_\tau = \phi_c := \mu_w m^r \) and \( \phi_{\tau+1} = \phi_c := \mu_w m^s \). Thus,

\[
\hat{\pi}(p_{\tau+1}) = \beta p_{\tau+1} q_{\tau+1} - L_{\tau+1} = \left[ \frac{\beta \theta^\alpha [1 - \phi_c]^{\alpha - 1} [\theta \phi_c]^{\alpha - 1} - h}{u} \right] m^r \mu_w = \left[ \frac{\beta \theta^\alpha [1 - \phi_c]^{\alpha - 1} [\theta \phi_c]^{\alpha - 1} - h}{u} \right] \phi_c
\]

In order to get \( \hat{\pi}(p_{\tau+1}) < 0 \) it is necessary that

\[
u \beta \theta^\alpha \left[ \frac{1 - \phi_c^{\alpha - 1} [\phi_c]^{\alpha - 1}}{1 - \phi_c^{\alpha - 1}} \right] < h \iff u \beta \theta^\alpha \left[ \frac{1}{\phi_c} \right]^{1 - \alpha} < h \left[ \frac{1 - \phi_c^{\alpha - 1}}{1 - \phi_c} \right]^{1 - \alpha}
\]

(72)

Recall that a RSE requires that a risky plan be preferred to storage, i.e., condition (63): \( u \beta \theta^\alpha \left[ \frac{1}{\phi_c} \right]^{1 - \alpha} \geq h + \frac{1}{\phi} - \frac{h}{u} \). Since \( \frac{1}{\phi} - \frac{h}{u} > 0 \) (because a necessary condition for an RSE is \( u > h\delta \)), condition (63) implies \( u \beta \theta^\alpha \left[ \frac{1}{\phi_c} \right]^{1 - \alpha} \geq h \). Notice also that \( \phi_c^{\alpha - 1} [\phi_c]^{\alpha - 1} = \phi_c^{\alpha - 1} [\phi_c]^{\alpha - 1} \), because we have imposed financial distress costs of crisis: \( w_{\text{crisis}} = \mu_w < 1 - \beta \). Combining these facts we have

\[
u \beta \theta^\alpha \left[ \frac{1}{\phi_c} \right]^{1 - \alpha} > u \beta \theta^\alpha \left[ \frac{1}{\phi_c} \right]^{1 - \alpha} > h
\]

(73)

Finally, notice that (73) contradicts (72) because \( \left[ \frac{1 - \phi_c^{\alpha - 1}}{1 - \phi_c} \right] > 1 \). Hence, in an RSE it is not possible for agents to choose a risky plan during a crisis period. \( \square \)

**Proof of Proposition 4.1.** *Growth Limit Distribution.* Here, we derive the limit distribution of GDP’s compounded growth rate \( \log(gdp_t) - \log(gdp_{t-1}) \) along the RSE characterized in Proposition 3.2. In this RSE, firms choose safe plans in a crisis period and resume risk-taking the period immediately after the crisis. It follows from (24), (28) and (29) that the growth process follows a three-state Markov chain characterized by

\[
\Gamma = \begin{pmatrix}
\log ((\theta \phi^l)^\alpha) \\
\log ((\theta \phi^l)^\alpha Z(\phi^l)) \\
\log ((\theta \phi^c)^\alpha Z(\phi^c))
\end{pmatrix}, \quad T = \begin{pmatrix}
u & 1 - u & 0 \\
0 & 0 & 1 \\
u & 1 - u & 0\end{pmatrix}
\]

(74)
The three elements of $\Gamma$ are the growth rates in the lucky, crisis and post-crisis states, respectively. The element $T_{ij}$ of the transition matrix is the transition probability from state $i$ to state $j$. Since the transition matrix is irreducible, the growth process converges to a unique limit distribution over the three states that solves $T'\Pi = \Pi$. Thus, $\Pi = \left(\frac{u}{2-u}, \frac{1-u}{2-u}, \frac{1-u}{2-u}\right)^T$, where the elements of $\Pi$ are the shares of time that an economy spends in each state over the long-run. It then follows that the mean long run GDP growth rate is $E(1 + \gamma^r) = \exp(\Pi\Gamma)$.

We derive first the limit distribution of the growth rate process $\Delta \log(gdp_t) := \log(gdp_t) - \log(gdp_{t-1})$. Since in an RSE crises cannot occur in two consecutive periods, $\Delta \log(gdp_t)$ follows a three-state Markov chain characterized by the following growth vector and transition matrix

$$\Gamma = \begin{pmatrix} \log((\theta \phi^l)^\alpha) \\ \log((\theta \phi^l)^\alpha Z(\phi^c) Z(\phi^f)) \\ \log((\theta \phi^c)^\alpha Z(\phi^l) Z(\phi^c)) \end{pmatrix}, \quad T = \begin{pmatrix} u & 1-u & 0 \\ 0 & 0 & 1 \\ u & 1-u & 0 \end{pmatrix}$$

Since the transition matrix is irreducible, the growth process converges to a unique limit distribution over the three states that solves $T'\Pi = \Pi$. Thus, $\Pi' = \left(\frac{u}{2-u}, \frac{1-u}{2-u}, \frac{1-u}{2-u}\right)$ and the geometric mean long run GDP growth rate – equation (30) in the text– is $E(1 + \gamma^r) = \exp(\Pi\Gamma)$. It then follows from (27) and (30) that

$$\gamma^r > \gamma^s \iff \left(\frac{\mu_w}{1-\beta}\right) > \left(\frac{1 - h\delta u^{-1}}{1 - h\delta u}\right)^{1/(1-u)}$$

$$\iff l^d \equiv 1 - \frac{\mu_w}{1-\beta} < 1 - \left(\frac{1 - h\delta u^{-1}}{1 - h\delta u}\right)^{1/(1-u)}$$

**Derivation of (32).** Any solution to the Pareto problem is characterized by the optimal accumulation of N-goods that maximizes the discounted sum of T-production

$$\max_{\{d_t\} \in C^1} \sum_{t=0}^{\infty} \delta^t d_t^\alpha, \quad \text{s.t.} \quad k_{t+1} = \begin{cases} \theta k_t - d_t & \text{if } t \geq 1 \\ q_0 - d_0 & \text{if } t = 0 \end{cases}, \quad d_t \geq 0, \quad q_o \text{ given}$$

The Hamiltonian associated with this problem is $H_t = \delta^t[d_t]^\alpha + \lambda_t [\theta k_t - d_t]$. Since $\alpha \in (0, 1)$, the necessary and sufficient conditions for an optimum are

$$0 = H_d = \delta^t \alpha [d_t]^{\alpha-1} - \lambda_t, \quad \lambda_{t-1} = H_k = \theta \lambda_t, \quad \lim_{t \to \infty} \lambda_t k_t = 0 \quad (75)$$

Thus, the Euler equation is

$$d_{t+1} = \frac{[\delta \theta]}{\delta^t} d_t = \theta \phi d_t, \quad \phi := [\delta \theta^a] \frac{1}{1-\alpha} \quad t \geq 1 \quad (76)$$

$^40 E(1 + \gamma^r)$ is the geometric mean of $1 + \gamma^l, 1 + \gamma^c$ and $1 + \gamma^d$. 51
To get a closed form solution for $d_t$ we replace (76) in the accumulation equation:

$$ k_t = \theta^{t-1}k_1 - d_0 \sum_{s=0}^{t-2} \theta^{t-s-2} [\delta \theta]^{s+1} \frac{\phi}{1-\phi} = \theta^{t-1} \left[ k_1 - d_0 \phi \frac{1-\phi^{t-1}}{1-\phi} \right] = \theta^{t-1} \left[ k_1 - \frac{d_1 1 - \phi^{t-1}}{\theta} \right] $$  (77)

Replacing (76) and (77) in the transversality condition we get

$$ 0 = \lim_{t \to \infty} \delta^t k_t = \lim_{t \to \infty} \delta^t \left[ (\delta \theta)^t d_0 k_1 \right] \alpha \left[ \theta^{t-1} k_1 - d_0 \phi \frac{1-\phi^{t-1}}{1-\phi} \right] $$

$$ = \frac{\alpha d_0}{\theta} \left[ k_1 - d_0 \phi \frac{1}{1-\phi} \right] \quad \text{iff} \quad \phi < 1 $$

Since $k_1 = q_0 - d_0$, the bracketed term equals zero if and only if $d_0 = [1 - \phi] q_0$. The accumulation equation then implies that the unique optimal solution is $d_t = [1 - \phi] q_0$. □

**Derivation of (36).** To simplify notation we assume temporarily that there is only one crisis (at time $\tau$). It follows that profits and the bailout cost are:

$$ \pi_t = \frac{\alpha}{1-\phi^c} \beta y_t - \frac{\alpha \phi^c}{1-\phi^c} h y_{t-1}, \quad t \neq \{0, \tau, \tau + 1\} $$

$$ \pi_0 = \frac{\alpha}{1-\phi^c} \beta y_0, \quad \pi_{\tau+1} = \frac{\alpha}{1-\phi^c} \beta y_{\tau+1} - \frac{\alpha \phi^c}{1-\phi^c} h y_{\tau} $$

$$ T(\tau) = L_{\tau-1} - \mu p_t q_t = \frac{\alpha}{1-\phi^c} h \phi^c y_{\tau-1} - \mu p_t q_t = \frac{\alpha}{1-\phi^c} h \phi^c y_{\tau-1} - \mu \frac{\alpha}{1-\phi^c} h y_{\tau} $$  (79)

Replacing these expressions in (34) and using the market clearing condition $p_t q_t [1 - \phi_t] = \alpha y_t$, we get

$$ W(\tau) = (1-\alpha) y_\tau + \frac{\alpha \beta y_\tau}{1-\phi^c} + \sum_{t=1}^{\tau-1} \delta^t \left[ (1-\alpha) y_t + \frac{\alpha \beta y_t}{1-\phi^c} - \frac{\alpha \phi^c y_t}{1-\phi^c} \right] + \delta^\tau \left[ (1-\alpha) y_{\tau} + \frac{\alpha \beta y_{\tau}}{1-\phi^c} - \frac{\alpha \phi^c y_{\tau}}{1-\phi^c} \right] $$

$$ + \delta^{\tau+1} \left[ (1-\alpha) y_{\tau+1} + \frac{\alpha \phi^c y_{\tau+1}}{1-\phi^c} \right] + \sum_{t=\tau+2}^{\infty} \delta^t \left[ (1-\alpha) y_t + \frac{\alpha \beta y_t}{1-\phi^c} - \frac{\alpha \phi^c y_t}{1-\phi^c} \right] $$

$$ = \sum_{t \neq \tau} \delta^t y_t + K^c y_{\tau}, \quad K^c := 1 - \alpha + \mu \frac{\alpha}{1-\phi^c} - \frac{\alpha}{1-\phi^c} h \phi^c = 1 - \alpha \left[ 1 - (\mu + \mu_w) \right] $$

Notice that $K_c$ can be simplified as follows

$$ K_c = \alpha + \frac{\alpha}{1-\phi^c} (\mu - (1 - \mu_w) + (1 - \mu_w) - \delta h \phi^c) = \alpha + \frac{\alpha}{1-\phi^c} ((1 - \mu_w) - \delta h \phi^c) - \frac{\alpha}{1-\phi^c} [1 - (\mu + \mu_w)] $$

Notice that $\frac{1}{1-\phi^c} ((1 - \mu_w) - \delta h \phi^c) = \frac{(1-\mu_w)(1-h\delta)(-h\delta\mu_w)}{1-h\delta-\mu_w} = \frac{1-h\delta-M \mu_w}{1-h\delta-\mu_w} = 1$. Thus, $K_c = 1 - \frac{\alpha [1 - (\mu + \mu_w)]}{1-\phi^c}$.

The expression for the present value of consumption in (36) follows by allowing multiple crises to take place.
Derivation of (37). Consider T-output net of bankruptcy costs: \( \tilde{y}_t = K_t y_t \), where \( K_t \) is defined in (36). Notice that \( W^r = E_0 \sum_{t=0}^{\infty} \delta^t K_t y_t = E_0 \sum_{t=0}^{\infty} \delta^t \tilde{y}_t \), and \( \frac{\tilde{y}_t}{y_{t-1}} \) follows a three-state Markov chain defined by:

\[
\tilde{T} = \begin{pmatrix}
    u & 1-u & 0 \\
    0 & 0 & 1 \\
    u & 1-u & 0
\end{pmatrix}, \quad \tilde{G} = \begin{pmatrix}
    g_1 \\
    g_2 \\
    g_3
\end{pmatrix} = \begin{pmatrix}
    (\theta \phi^j)^\alpha \\
    \theta \phi^{1-\phi^c} \frac{\alpha}{K_c} \\
    \theta \phi^{1-\phi^c} \frac{1}{K_c}
\end{pmatrix} \tag{80}
\]

To derive \( W^r \) in closed form consider the following recursion

\[
V(\tilde{y}_0, g_0) = E_0 \sum_{t=0}^{\infty} \delta^t \tilde{y}_t = \tilde{y}_0 + \delta E_0 V(\tilde{y}_1, g_1) \\
V(\tilde{y}_t, g_t) = y_t + \beta E_t V(\tilde{y}_{t+1}, g_{t+1}) \tag{81}
\]

Suppose that the function \( V \) is linear: \( V(\tilde{y}_t, g_t) = \tilde{y}_t w(g_t) \), with \( w(g_t) \) an undetermined coefficient. Substituting this guess into (81), we get \( w(g_t) = 1 + \delta E_t g_{t+1} w(g_{t+1}) \). Combining this condition with (80), it follows that \( w(g_{t+1}) \) satisfies

\[
\begin{pmatrix}
    w_1 \\
    w_2 \\
    w_3
\end{pmatrix} = \left( \begin{array}{ccc}
    1 & 0 & 1 \\
    0 & 0 & 1 \\
    1 & 0 & 1
\end{array} \right) + \delta \begin{pmatrix}
    u & 1-u & 0 \\
    0 & 0 & 1 \\
    u & 1-u & 0
\end{pmatrix} \begin{pmatrix}
    g_1 w_1 \\
    g_2 w_2 \\
    g_3 w_3
\end{pmatrix}
\Rightarrow \begin{pmatrix}
    w_1 \\
    w_2 \\
    w_3
\end{pmatrix} = \begin{pmatrix}
    \frac{1+(1-u)\delta g_2}{1-(1-u)\delta^2 g_3 - u \delta g_1} \\
    \frac{1}{1-(1-u)\delta^2 g_3 - u \delta g_1} \\
    \frac{1+(1-u)\delta g_3}{1-(1-u)\delta^2 g_3 - u \delta g_1}
\end{pmatrix} \begin{pmatrix}
    g_1 \\
    g_2 \\
    g_3
\end{pmatrix}
\]

This solution exists and is unique provided \( g_1 \delta u + g_2 g_3 \delta^2 (1-u) < 1 \). Equation (37) follows by noting that at time 0 the economy is in the lucky state: \( V(y_0, g_0) = w_1 y_0 \), and by making the substitution \( g_2 g_3 = (\theta \phi^j)^\alpha (\theta \phi^c)^\alpha \).

Proof of Proposition 5.2

Consider the value functions \( W^s \) and \( W^r \) given by (35) and (36), respectively, and notice that if \( u = 1 \), both are equal. Since \( W^s \) does not depend on \( u \), we will prove the proposition by determining conditions under which \( W^r_u := \partial W^r / \partial u \) is negative. That is, an increase in crisis-risk improves the present value of consumption along a risky path. Let’s denote

\[
L = 1 - \left[ \theta \phi^j \right]^\alpha \delta u - \left[ \theta^2 \phi^j \phi^s \right]^\alpha \delta^2 (1-u), \quad T = \left( 1 + \delta (1-u) \left[ \theta \phi^j \frac{1-\phi^c}{1-\phi} \right]^\alpha k_c \right) (1-\phi)^j,
\]

so that

\[
W^r = \frac{T}{L} q_0^s, \quad \text{and} \quad W^r_u := \frac{\partial W^r}{\partial u} \bigg|_{u=1} = \frac{LT_u - L_u T}{L^2} q_0^s. \tag{82}
\]

The derivatives \( L_u := \partial L / \partial u \big|_{u=1} \) and \( T_u := \partial T / \partial u \big|_{u=1} \) are

\[
L_u = -\delta (\theta \phi)^\alpha - \alpha \phi \delta (\theta \phi)^{\alpha-1} + \left[ \theta \phi^j \phi^c \right]^\alpha \delta^2 \\
T_u = -\alpha \phi \left[ (1-\phi)^\alpha - \delta (\theta \phi)^\alpha (1-\phi)^{\alpha-1} \right] k_c (1-\phi) \left( \frac{1-\phi^c}{1-\phi} \right)^\alpha,
\]

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where \( \phi = \phi^s = \phi^l|_{u=1} \) and \( \phi' = \partial \phi^l/\partial u|_{u=1} \). It then follows from (82) that:

\[
\frac{L^2}{q_0} W_u^r = (D - 1) (1 - \phi)^{\alpha - 1} (\alpha \phi' + D (1 - \phi) k_c \left( \frac{1 - \phi^c}{1 - \phi} \right)^\alpha) + (1 - \phi)^\alpha (D + \alpha \phi' \frac{D}{\phi} - D \delta (\theta \phi^\alpha))
\]

\[
\frac{L^2}{q_0 (1 - \phi)^{\alpha - 1}} W_u = (D - 1) (1 - \phi)^{\alpha - 1} (\alpha \phi' + D (1 - \phi) k_c \left( \frac{1 - \phi^c}{1 - \phi} \right)^\alpha) + (1 - \phi)^\alpha (D + \alpha \phi' \frac{D}{\phi} - D \delta (\theta \phi^\alpha)),
\]

with \( D = \delta (\theta \phi)^\alpha \). Note that \( D < 1 \) because \( \delta < \delta_{\text{max}} : = (\theta \phi)^{-\alpha} \) is necessary for \( W^s \) in (35) to be well defined. After some algebraic manipulations, the expression above can be expressed as follows:

\[
\frac{L^2 W_u^r}{q_0 (1 - \phi)^{\alpha - 1}} = \frac{\alpha \phi' (D - 1)}{\phi} + (1 - D) (1 - k_c \left( \frac{1 - \phi^c}{1 - \phi} \right) (1 - \phi)) + \frac{(1 - \phi)^\alpha D \delta (\theta)^\alpha ((\phi)^\alpha - (\phi^c)^\alpha)}{\phi}
\]

Pareto gains
Bankruptcy costs
Financial distress costs

(83)

Since \( D = \delta (\theta \phi)^\alpha = (\phi^{po})^{1 - \alpha} \phi^\alpha \), the first term can be rewritten as \( \alpha \phi' ((\phi^{po})^{1 - \alpha} - 1) \), which is negative if and only if \( \phi < \phi^{po} \) because \( \phi' \) is negative (a reduction in \( u \) increases leverage). Since the two other terms are positive, a necessary condition for \( W^r > W^s \) is \( \phi < \phi^{po} \), where \( \phi^{po} \) is the Pareto optimal investment share. This establishes part (i) of the proposition. To prove part (ii), consider first the case in which financial distress costs are small \((\mu_w \to 1 - \beta)\). In this case the last term in (83) is zero as \( \phi^c = \phi \). Thus, \( W_u^r \) is negative if and only if:

\[
\frac{T^2 W_u^r}{q_0 (1 - \phi)^{\alpha - 1}} = \alpha \phi' \left( \left[ \frac{\phi^{po}}{\phi} \right]^{1 - \alpha} - 1 \right) + (1 - D) \left( \frac{\alpha [\beta - \mu]}{1 - \phi} \right) (1 - \phi) < 0
\]

\[
\Leftrightarrow \frac{\mu}{\beta} > 1 + \beta^{-1} \phi' \left( \left[ \frac{\phi^{po}}{\phi} \right]^{1 - \alpha} - 1 \right) (1 - D)^{-1}
\]

(84)

A necessary and sufficient condition for (84) is:

\[
\phi < \phi^{po} \text{ and } \mu > \mu^* := \max \left\{ 0, 1 + \frac{\phi'}{\beta (1 - D)} \left( \left[ \frac{\phi^{po}}{\phi} \right]^{1 - \alpha} - 1 \right) \right\} \beta.
\]

(85)

If in addition \( \delta \) is large enough, this condition holds for any \( \mu \geq 0 \). To see this observe that \( \lim_{\delta \to \delta_{\text{max}}} D = 1 \), and \( D \) is continuous and increasing in \( \delta \).

Second, consider the case \( \mu_w < 1 - \beta \), but let the discount factor \( \delta \to \delta_{\text{max}} \), so that \( D \to 1 \). In this case the second term in (83) converges to zero. Therefore, \( W_u^r < 0 \) is equivalent to

\[
(1 - \phi)^\alpha (1 - \left( \frac{\phi^c}{\phi} \right)^\alpha) < -\alpha \phi' \left( \left[ \frac{1}{\phi} \right]^{1 - \alpha} - 1 \right).
\]

(86)

A necessary and sufficient condition for (86) is:

\[
\phi < \phi^{po} \text{ and } l^d > \tilde{l}^d \equiv 1 - \left( 1 + \alpha \phi' ((\frac{1}{\phi})^{1 - \alpha} - 1)(1 - \phi)^{-\alpha} \right)^{1/\alpha}
\]

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To see this, develop (86):

\[
(1 - (\frac{\phi^c}{\phi})^\alpha) < -\alpha \phi'((\frac{1}{\phi})^{1-\alpha} - 1)(1 - \phi)^{-\alpha}
\]

\[
\left(\frac{\mu_w}{1 - \beta}\right)^\alpha < 1 + \alpha \phi'((\frac{1}{\phi})^{1-\alpha} - 1)(1 - \phi)^{-\alpha}
\]

\[
l^d > 1 - \left(1 + \alpha \phi'((\frac{1}{\phi})^{1-\alpha} - 1)(1 - \phi)^{-\alpha}\right)^{1/\alpha}
\]

**Proof of Proposition 6.1.** Throughout we assume that the returns condition (56) is satisfied.

**\(\theta\)-entrepreneurs.** In the black-hole equilibrium (FBE) we construct, all \(\theta\)-entrepreneurs issue standard bonds and never default. Meanwhile, \(\varepsilon\)-entrepreneurs default if \(\varepsilon_{t+1} = 0\). Thus, each \(\theta\)-entrepreneur expects next period a unique price \(p_{t+1}\) and that a bailout will be granted if and only if \(\varepsilon_{t+1} = 0\). Given these expectations, a \(\theta\)-entrepreneur’s problem is to choose whether to issue standard bonds or catastrophe bonds, and whether to implement a diversion scheme or not. We will show that if the bailout is not too generous, a \(\theta\)-entrepreneur has no incentives to deviate from the equilibrium.

First, if a \(\theta\)-entrepreneur issues standard bonds and will never default, her borrowing limit is \(b_s^t = [m^s - 1]w_t\) and and her expected profits are the same as those of the equilibrium safe plan of Proposition 3.1, given by (55). Second, consider plans with catastrophe bonds that will not default. Lenders require an interest rate no smaller than

\[
1 + \rho^c = 1/[1 - \lambda]\delta.
\]

To prevent diversion lenders lend up to an amount that satisfies the no-diversion condition

\[
[1 - \lambda][1 + \rho^c]b_t^{c,nd} \leq \delta[w_t + b_t^{c,nd}]
\]

Since \([1 - \lambda][1 + \rho^c] = 1 + r\) (by (87)), the no-diversion condition (88) implies that the borrowing constraint with catastrophe bonds is \(b_t^{c,nd} \leq \frac{\delta}{1 - \lambda}w_t\). Notice that this borrowing limit is the same as the one under the equilibrium strategy with standard debt: \(b_t^s = [m^s - 1]w_t\). Furthermore, the expected debt repayments are the same under both types of debt (i.e., \(b_t^s[1 + \rho^s] = b_t^{c,nd}[1 - \lambda][1 + \rho^c]\)). Thus, the expected profits are the same under both types of debt. Hence, conditional on no default, the \(\theta\)-entrepreneur has no incentives to deviate from the FBE.

Third, consider plans where the \(\theta\)-entrepreneur issues catastrophe bonds and will default next period (in the \(\varepsilon_{t+1} = 0\) state). Since catastrophe bonds promise to repay only in the \(\varepsilon_{t+1} = 0\) state, these plans include both diversion plans and no-diversion plans with an excessive promised repayment that will make the firm insolvent. Under both plans, lenders are willing to lend up to the present value of the bailout that they will receive in the \(\varepsilon_{t+1} = 0\) state

\[
b_t^{c,def} = \delta[1 - \lambda]r_{i,t+1}
\]
Since the bailout will be $\Gamma_{i,t+1} = \gamma y_{t+1}^\theta$, condition $\gamma < \overline{\gamma}$ in Proposition 6.1 implies that the borrowing limit for plans that lead to default is lower than the limit for non-defaulting plans

$$b_{i,t}^{c,def} = \delta [1 - \lambda] y_{t+1}^\theta < [m^s - 1] w_{i,t} = b_{i,t}^s$$

$$\iff \gamma < \overline{\gamma} = \frac{[m^s - 1]}{\delta [1 - \lambda]} \frac{w_{t}}{y_{t+1}} = \frac{[m^s - 1]}{\delta [1 - \lambda]} \frac{[1 - \beta] \alpha}{1 - \phi} \frac{1}{[\theta \phi]^\alpha}$$

This bound is time-invariant because along the equilibrium path $\frac{w_{t}}{y_{t+1}}$ is constant:

$$\frac{w_{t}}{y_{t+1}} = \frac{w_{t}}{y_{t}} \frac{y_{t+1}}{y_{t}} = \frac{(1 - \beta) p_{t} q_{t} (1 - \omega_{t} \mu_{t})}{\alpha} \cdot \frac{1}{[\theta \phi]^\alpha} = \frac{[1 - \beta] \alpha}{1 - \phi} \frac{1}{[\theta \phi]^\alpha}$$

Consider a no-diversion plan with catastrophe bonds that leads to default. Under such plan a $\theta$-entrepreneur borrows up to $b_{i,t}^{c,def}$, promises an interest rate $\frac{1}{[1 - \lambda] \delta}$, and will become insolvent when $\varepsilon_{t+1} = 0$. Under this deviation a $\theta$-entrepreneur avoids repaying debt altogether, but it sacrifices profits in the $\varepsilon_{t+1} = 0$ state. The requirement that the firm be insolvent in the $\varepsilon_{t+1} = 0$ state, implies that the maximum payoff under this deviation is $\lambda \Gamma_{i,t+1}$ (because the highest revenue consistent with insolvency in the $\varepsilon_{t+1} = 0$ state is $b_{i,t}^{c,def} [1 + \rho_{c}] = \Gamma_{i,t+1}$).

$$E_{i,t+1}^{\pi_{c,def}} \leq \lambda \gamma y_{t+1} = \lambda \gamma y_{t+1}^{\theta} y_{t}^{\theta} = \lambda \gamma [\theta \phi]^\alpha = \lambda \gamma [1 - \beta] \alpha \frac{1 - \phi}{[\theta \phi]^\alpha}$$

The last equality follows from (91). Comparing this upper bound with the equilibrium payoff in (55), we find that this deviation is not profitable provided the generosity of the guarantee is below $\overline{\gamma}$

$$E_{i,t+1}^{\pi_{c,def}} (b_{i,t}^{c,def}) < \pi_{i,t+1} (b_{i,t}^s) \iff \frac{\lambda \gamma}{[\theta \phi]^\alpha} \frac{1 - \phi}{[1 - \beta] \alpha} < \lambda \gamma \frac{1 - \phi}{[\theta \phi]^\alpha} \frac{1 - \phi}{[1 - \beta] \alpha} \frac{1}{1 - \beta}$$

Consider next diversion plans with catastrophe bonds. In a diversion plan the entrepreneur incurs a cost $h [w_{t} + b_{i,t}^s]$ at $t$, and is able to divert funds at $t + 1$ provided she is solvent. Under such plan her borrowing limit is $b_{i,t}^{c,def}$ in (89). Using the same argument as the one in the proof of Proposition 3.2, one can show that this deviation is not profitable because the debt ceiling under diversion is lower than under the equilibrium strategy ($b_{i,t}^{c,def} < b_{i,t}^{s}$), which is implied by $\gamma_i < \overline{\gamma}$. In sum, $\theta$-entrepreneurs issue standard debt, do not divert and invest according to Proposition 3.1 if the bailout is not too generous: $\gamma < \min\{\overline{\gamma}, \overline{\gamma}'\}$.

$\varepsilon$-entrepreneurs. Since the $\varepsilon$-technology has negative NPV, $\varepsilon$-agents find it profitable only to issue catastrophe bonds. In the presence of bailout guarantees, lenders are willing to buy these catastrophe bonds. Given the expected bailout $\Gamma_{i,t+1}$, lenders are willing to lend to each
\(\varepsilon\)-agent up to an amount \(b_{t+1}^\varepsilon = \delta[1 - \lambda]\Gamma_{t+1}\) at a rate \(\rho^\varepsilon\) (in (87)). At \(t + 1\), if the good state realizes \((\varepsilon_{t+1} = \bar{\varepsilon})\), lenders will get zero—as promised—while if \(\varepsilon_{t+1} = 0\) lenders will get the bailout \(\Gamma_{t+1} = b_{t+1}^\varepsilon[1 + \rho^\varepsilon]\). It follows that an \(\varepsilon\)-agent will de-facto repay zero in all states of the world, and so he does not gain anything by implementing a diversion scheme. His expected payoff is \(E\pi_{t+1}^\varepsilon = \lambda\bar{\varepsilon}b_{t+1}^\varepsilon = \lambda\bar{\varepsilon}^\frac{1}{2}\delta[1 - \lambda]\Gamma_{t+1}\). Since he does not need to risk his own capital, the \(\varepsilon\)-agent finds this project profitable.

**Fiscal Solvency.** To prove that bailouts are financeable via taxation of the final goods sector, we show that condition (43) holds iff \(\gamma \leq \gamma''\). Since bailouts are granted only in the \(\varepsilon_{t+1} = 0\) state, and in this state all \(\theta\)-firms are solvent while all \(\varepsilon\)-firms go bust \((y_{t+1}^{\varepsilon,nd} = \varepsilon_{t+1}^t I_t = 0)\), the bailout payment if \(\varepsilon_{t+1} = 0\) is \(\Gamma_{t+1} = \gamma y_{t+1}^{\varepsilon,nd}\).

\[
E \left( \sum_{t=0}^{\infty} \delta^{t+1} \left[ y_{t+1}^{\theta,nd} + y_{t+1}^{\varepsilon,nd} \right] \right) \geq E \left( \sum_{t=0}^{\infty} \delta^{t+1} \Gamma_{t+1} \right) \geq \sum_{t=0}^{\infty} \delta^{t+1} \gamma[1 - \lambda]y_{t+1}^{\theta,nd} 
\]

\[
\sum_{t=0}^{\infty} \delta^{t+1} y_{t+1}^{\theta,nd} [1 + \lambda\bar{\varepsilon}\delta[1 - \lambda] \gamma - \gamma[1 - \lambda]] \geq 0 
\]

\[
\sum_{t=0}^{\infty} \delta^{t+1} y_{t+1}^{\theta,nd} [1 + \gamma[1 - \lambda][\lambda\bar{\varepsilon}\delta - 1]] \geq 0 
\]

\[
\frac{y_0^\theta}{1 - \delta(\theta\phi^s)^\alpha} \left[ 1 + \gamma[1 - \lambda][\lambda\bar{\varepsilon}\delta - 1] \right] \geq 0 \quad \text{if } \delta(\theta\phi^s)^\alpha < 1 
\]

\[
\frac{(1 - \phi^s)^\alpha}{1 - \delta(\theta\phi^s)^\alpha} q_0^\theta \left[ 1 + \gamma[1 - \lambda][\lambda\bar{\varepsilon}\delta - 1] \right] \geq 0 \iff \gamma \leq \gamma'' := \frac{1}{[1 - \lambda][1 - \lambda\bar{\varepsilon}\delta]} 
\]  

(93)

Since \(\phi^s < 1\) and \(\delta(\theta\phi^s)^\alpha < 1\), the LHS is non-negative iff \(\gamma \leq \gamma''\). Putting together the three bounds in (90), (92) and (93) we conclude that a financial black-hole equilibrium exists if \(\gamma \leq \overline{\gamma}\), with

\[
\overline{\gamma} \equiv \max \left\{ \frac{h\alpha}{1 - \lambda} \frac{\phi}{1 - \phi} \left( \frac{1}{\theta^\alpha\phi^{\alpha-1} - h} \frac{\theta^\alpha\phi^{\alpha+1}}{1 - \phi} \frac{1}{[1 - \lambda][1 - \lambda\bar{\varepsilon}\delta]} \right) \right\}, \quad \phi = \phi^s = \frac{1 - \beta}{1 - h\delta} 
\]

(94)