Optimal Monetary Policy and Stock-Price Dynamics in a Non-Ricardian DSGE Model

Salvatore Nisticò
(Università degli Studi di Roma “La Sapienza” and LUISS “Guido Carli”, Rome)

XX International Tor Vergata Conference on Money, Banking and Finance: “Actors, Rules and Policies in the Global Financial Crisis”
December 7th, 2011
Motivation

- Standard small-scale DNK model: no role for stock-wealth effects but
  - small DNK model used to derive asset pricing implications
    (Sangiorgi-Santoro, ’05; Challe-Giannitsarou, ’07)

- Large and unsettled debate over the links between monetary policy and stock market fluctuations
  - What is the response of stock prices to monetary policy shocks?
    (Bernanke–Kuttner, ’05; Rigobon–Sack, ’04; Castelnuovo–Nisticò, ’10)
  - What is the appropriate MP response to stock market fluctuations?
    (Bernanke–Gertler, ’99, ’01; Cecchetti et al, ’00; Di Giorgio–Nisticò, ’07; Nisticò, ’05; Airaudo–Nisticò–Zanna, ’07)
  - What has been the response of major CB’s? (Bernanke–Gertler, ’99; Rigobon–Sack, ’03; Bjornland–Leitemo, ’08; Castelnuovo–Nisticò, ’10)

- Financial stability and monetary policy (Cùrdia–Woodford, ’09, ’10, ’11; Gertler–Karadi, ’09; Gertler–Kiyotaki, ’09)
Motivation

- Standard small-scale DNK model: no role for stock-wealth effects but
  - small DNK model used to derive asset pricing implications
    (Sangiorgi-Santoro, '05; Challe-Giannitsarou, '07)

- Large and unsettled debate over the links between monetary policy and stock market fluctuations
  - What is the response of stock prices to monetary policy shocks?
    (Bernanke–Kuttner, '05; Rigobon–Sack, '04; Castelnuovo–Nisticò, '10)
  - What is the appropriate MP response to stock market fluctuations?
    (Bernanke–Gertler, '99, '01; Cecchetti et al, '00; Di Giorgio–Nisticò, '07; Nisticò, '05; Airaudo–Nisticò–Zanna, '07)
  - What has been the response of major CB’s? (Bernanke–Gertler, '99; Rigobon–Sack, '03; Bjornland–Leitemo, '08; Castelnuovo–Nisticò, '10)
Motivation

- Standard small-scale DNK model: no role for stock-wealth effects but
  - small DNK model used to derive asset pricing implications
    (Sangiorgi-Santoro, '05; Challe-Giannitsarou, '07)

- Large and unsettled debate over the links between monetary policy and stock market fluctuations
  - What is the response of stock prices to monetary policy shocks?
    (Bernanke–Kuttner, '05; Rigobon–Sack, '04; Castelnuovo–Nisticò, '10)
  - What is the appropriate MP response to stock market fluctuations?
    (Bernanke–Gertler, '99, '01; Cecchetti et al, '00; Di Giorgio–Nisticò, '07; Nisticò, '05; Airaudo–Nisticò–Zanna, '07)
  - What has been the response of major CB’s? (Bernanke–Gertler, '99; Rigobon–Sack, '03; Bjornland–Leitemo, '08; Castelnuovo–Nisticò, '10)

- Financial stability and monetary policy (Cùrdia–Woodford, '09, '10, '11; Gertler–Karadi, '09; Gertler–Kiyotaki, '09)
This paper: contribution

- Derives a *small-scale* DSGE New-Keynesian model in which stock prices play an active role in shaping the business cycle (OLG-PY structure)
This paper: contribution

- Derives a *small-scale* DSGE New-Keynesian model in which stock prices play an active role in shaping the business cycle (OLG-PY structure)
  - Additional dynamic distortion wrt the Social Planner allocation
Derives a \textit{small-scale} DSGE New-Keynesian model in which stock prices play an active role in shaping the business cycle (OLG-PY structure)

✓ Additional dynamic distortion wrt the Social Planner allocation
⇒ The SPA is Efficient and Equitable, while the decentralized allocation implies \textit{cross-sectional consumption dispersion}
This paper: contribution

- Derives a *small-scale* DSGE New-Keynesian model in which stock prices play an active role in shaping the business cycle (OLG-PY structure)
  - ✓ Additional dynamic distortion wrt the Social Planner allocation
  - ⇒ The SPA is Efficient and Equitable, while the decentralized allocation implies *cross-sectional consumption dispersion*

- Derives the relevant Welfare Criterion (aggregation issues: indefinite number of heterogeneous agents)
This paper: contribution

- Derives a small-scale DSGE New-Keynesian model in which stock prices play an active role in shaping the business cycle (OLG-PY structure)
  - Additional dynamic distortion wrt the Social Planner allocation
  - The SPA is Efficient and Equitable, while the decentralized allocation implies *cross-sectional consumption dispersion*

- Derives the relevant Welfare Criterion (aggregation issues: indefinite number of heterogeneous agents)
  - Additional term in the Welfare Criterion: *Financial Stability*
This paper: contribution

- Derives a *small-scale* DSGE New-Keynesian model in which stock prices play an active role in shaping the business cycle (OLG-PY structure)
  - Additional dynamic distortion wrt the Social Planner allocation
  - The SPA is Efficient and Equitable, while the decentralized allocation implies *cross-sectional consumption dispersion*

- Derives the relevant Welfare Criterion (aggregation issues: indefinite number of heterogeneous agents)
  - Additional term in the Welfare Criterion: *Financial Stability*
  - Financial Stability is a Welfare-relevant Monetary Policy target, *additional and independent* wrt price and output stability
This paper: contribution

- Derives a *small-scale* DSGE New-Keynesian model in which stock prices play an active role in shaping the business cycle (OLG-PY structure)
  - Additional dynamic distortion wrt the Social Planner allocation
  - The SPA is Efficient and Equitable, while the decentralized allocation implies *cross-sectional consumption dispersion*

- Derives the relevant Welfare Criterion (aggregation issues: indefinite number of heterogeneous agents)
  - Additional term in the Welfare Criterion: *Financial Stability*
  - Financial Stability is a Welfare-relevant Monetary Policy target, *additional and independent* wrt price and output stability

- Studies Optimal Monetary Policy
This paper: contribution

- Derives a *small-scale* DSGE New-Keynesian model in which stock prices play an active role in shaping the business cycle (OLG-PY structure)
  - Additional dynamic distortion wrt the Social Planner allocation
  - The SPA is Efficient and Equitable, while the decentralized allocation implies *cross-sectional consumption dispersion*

- Derives the relevant Welfare Criterion (aggregation issues: indefinite number of heterogeneous agents)
  - Additional term in the Welfare Criterion: *Financial Stability*
  - Financial Stability is a Welfare-relevant Monetary Policy target, *additional and independent* wrt price and output stability

- Studies Optimal Monetary Policy
  - Endogenous trade-off between inflation/output and financial stabilization
This paper: contribution

- Derives a *small-scale* DSGE New-Keynesian model in which stock prices play an active role in shaping the business cycle (OLG-PY structure)
  - Additional dynamic distortion wrt the Social Planner allocation
  - ⇒ The SPA is Efficient and Equitable, while the decentralized allocation implies *cross-sectional consumption dispersion*

- Derives the relevant Welfare Criterion (aggregation issues: indefinite number of heterogeneous agents)
  - Additional term in the Welfare Criterion: *Financial Stability*
  - ⇒ Financial Stability is a Welfare-relevant Monetary Policy target, *additional and independent* wrt price and output stability

- Studies Optimal Monetary Policy
  - Endogenous trade-off between inflation/output and financial stabilization
  - ⇒ Strict Price Stability is no longer an optimal monetary policy regime
Asset Prices in DNK Models

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(C_t) - v(N_t) \right]
\]

s.t. \[ C_t + E_t \left\{ \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} B_{t+1} \right\} \leq W_t N_t - T_t + B_t + D_t \]
Asset Prices in DNK Models

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(C_t) - v(N_t) \right]
\]

s.t.
\[
C_t + E_t \left\{ F_{t,t+1} \frac{P_{t+1}}{P_t} B_{t+1} \right\} \leq W_t N_t - T_t + B_t + D_t
\]

- Equilibrium SDF = IMRS

\[
F_{t,t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}
\]
Asset Prices in DNK Models

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(C_t) - v(N_t) \right]
\]

s.t. \( C_t + E_t \left\{ \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} B_{t+1} \right\} \leq W_t N_t - T_t + B_t + D_t \)

- Equilibrium SDF = IMRS

\[
\mathcal{F}_{t,t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}
\]

- For any stochastic return \( R \)

\[
1 = E_t \left\{ \mathcal{F}_{t,t+1} R_{t+1} \right\}
\]
Asset Prices in DNK Models

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(C_t) - v(N_t) \right]
\]

s.t. \[ C_t + E_t \left\{ F_{t,t+1} \frac{P_{t+1}}{P_t} B_{t+1} \right\} \leq W_t N_t - T_t + B_t + D_t \]

- Equilibrium SDF = IMRS

\[ F_{t,t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)} \]

- For any stochastic return \( R \)

\[ 1 = E_t \left\{ F_{t,t+1} R_{t+1} \right\} \]

- Implied Stock-Price Dynamics \( R_{t+1} = \frac{P_{t+1}}{P_t} \frac{Q_{t+1} + D_{t+1}}{Q_t} \)

\[ Q_t = E_t \left\{ F_{t,t+1} \frac{P_{t+1}}{P_t} (Q_{t+1} + D_{t+1}) \right\} \]
Asset Prices in DNK Models

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(C_t) - v(N_t) \right]
\]

s.t. \( C_t + E_t \left\{ F_{t,t+1} \frac{P_{t+1}}{P_t} B_{t+1} \right\} \leq W_t N_t - T_t + B_t + D_t \)

- Equilibrium SDF = IMRS

\[
F_{t,t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}
\]

- For any stochastic return \( R \)

\[
1 = E_t \left\{ F_{t,t+1} R_{t+1} \right\}
\]

- Implied Stock-Price Dynamics

\[
R_{t+1} = \frac{P_{t+1}}{P_t} \frac{Q_{t+1} + D_{t+1}}{Q_t}
\]

\[
Q_t = E_t \left\{ F_{t,t+1} \frac{P_{t+1}}{P_t} (Q_{t+1} + D_{t+1}) \right\}
\]

- All distortions and shocks affect SP dynamics (through effects on \( D_t \))
However:

\[ y_t = E_t y_{t+1} - \gamma (r_t - E_t \pi_t + 1 - \rho) \]

...nor on inflation (dichotomy)

\[ \pi_t = \beta E_t \pi_{t+1} + 1 + \kappa x_t \]

no reason for welfare-maximizing CB to be concerned with FS

\[ L_t = E_t \sum_{k=0}^{\infty} \beta^t \left[ x_{t+k} + \epsilon \kappa \pi_{t+k} \right] . \]
Asset Prices in DNK Models

However:

1. No feedback effects on real activity

\[ y_t = E_t y_{t+1} - \frac{1}{\gamma} (r_t - E_t \pi_{t+1} - \rho) \]
However:

1. No feedback effects on real activity

\[ y_t = E_t y_{t+1} - \frac{1}{\gamma} \left( r_t - E_t \pi_{t+1} - \rho \right) \]

2. ...nor on inflation (dichotomy)

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]
Asset Prices in DNK Models

However:

1. No feedback effects on real activity

\[ y_t = E_t y_{t+1} - \frac{1}{\gamma} (r_t - E_t \pi_{t+1} - \rho) \]

2. ...nor on inflation (dichotomy)

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]

3. no reason for welfare-maximizing CB to be concerned with FS

\[ \mathcal{L}_t = E_t \sum_{k=0}^{\infty} \beta^{t+k} \left[ x_{t+k}^2 + \frac{\epsilon}{\kappa} \pi_{t+k}^2 \right]. \]
The Model with Stock-Wealth Effects

- Discrete-time stochastic version of the perpetual youth model by Blanchard (1985)-Yaari (1965). Economy consists of an indefinite number of cohorts, of size $\xi$: each period a fraction $\xi$ of the population is replaced by “newcomers” with no financial wealth.
The Model with Stock-Wealth Effects

- Discrete-time stochastic version of the perpetual youth model by Blanchard (1985)-Yaari (1965). Economy consists of an indefinite number of cohorts, of size $\xi$: each period a fraction $\xi$ of the population is replaced by “newcomers” with no financial wealth.


- Market for claims on monopolistic firms’ profits (stock market)

- Welfare-Maximizing Central Bank
The Model: the Households

✓ Hh’s in cohort $j$ solve

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta^t (1 - \xi)^t \left[ \delta \log C_t(j) + (1 - \delta) \log(1 - N_t(j)) \right]$$
The Model: the Households

✓ Hh’s in cohort $j$ solve

$$\max E_0 \sum_{t=0}^{\infty} \beta^t (1 - \xi)^t \left[ \delta \log C_t(j) + (1 - \delta) \log(1 - N_t(j)) \right]$$

s.t.  

$$C_t(j) + E_t \left\{ F_{t,t+1} \frac{P_{t+1}}{P_t} B_{t+1}(j) \right\} + \int_0^1 Q_t(i) Z_{t+1}(j, i) \, di$$

$$\leq W_t N_t(j) - T_t(j) + \frac{1}{1 - \xi} \left[ \int_0^1 \left( Q_t(i) + D_t(i) \right) Z_t(j, i) \, di + B_t(j) \right]$$

for all $j \leq t - 1$, the “old traders”,

✓ 1

$1 - \xi$: return on insurance contract à la Blanchard (1985)

✓ $T_t(j) = T_t(j) + \phi(j)$

$R_t$: participation fee, $R_t \equiv 1 - \xi - E_t \left\{ F_{t,t+1} \frac{P_{t+1}}{P_t} B_{t+1}(j) \right\}$
The Model: the Households

✓ Hh’s in cohort \( j \) solve

\[
\max \quad E_0 \sum_{t=0}^{\infty} \beta^t (1 - \xi)^t \left[ \delta \log C_t(j) + (1 - \delta) \log(1 - N_t(j)) \right]
\]

s.t.

\[
C_t(j) + E_t \left\{ \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} B_{t+1}(j) \right\} + \int_0^1 Q_t(i) Z_{t+1}(j, i) \, di \\
\leq W_t N_t(j) - T_t(j) + \frac{1}{1-\xi} \left[ \int_0^1 \left( Q_t(i) + D_t(i) \right) Z_t(j, i) \, di + B_t(j) \right]
\]

for all \( j \leq t - 1 \), the “old traders”, and

\[
C_t(j) + E_t \left\{ \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} B_{t+1}(j) \right\} + \int_0^1 Q_t(i) Z_{t+1}(j, i) \, di \leq W_t N_t(j) - T_t(j)
\]

for \( j = t \), the “newcomers”. 
The Model: the Households

✓ Hh’s in cohort \( j \) solve

\[
\max \quad E_0 \sum_{t=0}^{\infty} \beta^t (1 - \xi)^t \left[ \delta \log C_t(j) + (1 - \delta) \log(1 - N_t(j)) \right]
\]

s.t.
\[
C_t(j) + E_t \left\{ \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} B_{t+1}(j) \right\} + \int_0^1 Q_t(i) Z_{t+1}(j, i) \, di \\
\leq W_t N_t(j) - T_t(j) + \frac{1}{1-\xi} \left[ \int_0^1 \left( Q_t(i) + D_t(i) \right) Z_t(j, i) \, di + B_t(j) \right]
\]

for all \( j \leq t - 1 \), the “old traders”, and

\[
C_t(j) + E_t \left\{ \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} B_{t+1}(j) \right\} + \int_0^1 Q_t(i) Z_{t+1}(j, i) \, di \leq W_t N_t(j) - T_t(j)
\]

for \( j = t \), the “newcomers”.

✓ \( \frac{1}{1-\xi} \): return on insurance contract à la Blanchard (1985)
The Model: the Households

- Hh’s in cohort $j$ solve

$$\max \ E_0 \sum_{t=0}^{\infty} \beta^t (1 - \xi)^t \left[ \delta \log C_t(j) + (1 - \delta) \log(1 - N_t(j)) \right]$$

s.t. 

$$C_t(j) + E_t \left\{ F_{t+1} \frac{P_{t+1}}{P_t} B_{t+1}(j) \right\} + \int_0^1 \! Q_t(i) Z_{t+1}(j, i) \, di \leq W_t N_t(j) - T_t(j) + \frac{1}{1 - \xi} \left[ \int_0^1 \! \left( Q_t(i) + D_t(i) \right) Z_t(j, i) \, di + B_t(j) \right]$$

for all $j \leq t - 1$, the “old traders”, and

$$C_t(j) + E_t \left\{ F_{t+1} \frac{P_{t+1}}{P_t} B_{t+1}(j) \right\} + \int_0^1 \! Q_t(i) Z_{t+1}(j, i) \, di \leq W_t N_t(j) - T_t(j)$$

for $j = t$, the “newcomers”.

- $\frac{1}{1 - \xi}$: return on insurance contract à la Blanchard (1985)

- $T_t(j) = T_t + \phi(j) R_t$: participation fee, $R_t \equiv \frac{1}{1 - \xi} - E_t \left\{ F_{t+1} \frac{P_{t+1}}{P_t} \right\}$
The Model: the Households

→ Individual equilibrium conditions:

1. **Euler Equation:**
   \[
   \beta P_t C_t(j) = E_t\{F_t, t+1 P_{t+1} C_{t+1}(j)\}
   \]

2. Consumption of "old traders":
   \[
   C_t(j) = \sigma H_t + \sigma_{1-\xi} [\Omega_t(j) - \phi(j)]
   \]

3. Consumption of "newcomers":
   \[
   C_t(t) = \sigma(H_t + \phi_{nc})
   \]

where \(\Omega_t(j)\) denotes the stock of individual financial wealth:
\[
\Omega_t(j) = \int_0^1 \{Q_t(i) + D_t(i)\} \{Z_t(j,i) di + B_t(j)\}
\]
and \(H_t\) the stock of human wealth:
\[
H_t = P_t E_t\{\infty \sum_{k=0} F_{t+k} (1-\xi)^k (W^*_t+k - P_{t+k} T_t+k)\}
\]
The Model: the Households

→ **Individual** equilibrium conditions:

1. **Euler Equation:**

\[ \beta P_t C_t(j) = E_t \{ \mathcal{F}_{t,t+1} P_{t+1} C_{t+1}(j) \} \]
The Model: the Households

→ **Individual** equilibrium conditions:

1. **Euler Equation:**
   \[ \beta P_t C_t(j) = E_t \{ \mathcal{F}_{t,t+1} P_{t+1} C_{t+1}(j) \} \]

2. **consumption of “old traders”:**
   \[ C_t(j) = \sigma H_t + \frac{\sigma}{1-\xi} \left[ \Omega_t(j) - \phi(j) \right] . \]

3. **consumption of “newcomers”:**
   \[ C_t(t) = \sigma \left( H_t + \phi^{nc} \right) . \]
The Model: the Households

→ **Individual** equilibrium conditions:

1. **Euler Equation:**

\[
\beta P_t C_t(j) = E_t \{ \mathcal{F}_{t,t+1} P_{t+1} C_{t+1}(j) \}
\]

2. **Consumption of “old traders”:**

\[
C_t(j) = \sigma H_t + \frac{\sigma}{1-\xi} \left[ \Omega_t(j) - \phi(j) \right].
\]

3. **Consumption of “newcomers”:**

\[
C_t(t) = \sigma (H_t + \phi^{nc}).
\]

where \( \Omega_t(j) \) denotes the stock of individual financial wealth

\[
\Omega_t(j) \equiv \int_0^1 \left( Q_t(i) + D_t(i) \right) Z_t(j, i) \, di + B_t(j)
\]

and \( H_t \) the stock of human wealth:

\[
H_t \equiv \frac{1}{P_t} E_t \left\{ \sum_{k=0}^{\infty} \mathcal{F}_{t,t+k} (1 - \xi)^k \left( W_{t+k}^* - P_{t+k} T_{t+k} \right) \right\}
\]
The Model: the Households

→ **Aggregate** equilibrium relations

- Aggregator: 
  \[ X_t \equiv \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} X_t(j) \]

- Aggr. financial wealth: 
  \[ \Omega_t \equiv \sum_{j=-\infty}^{t-1} \xi (1 - \xi)^{t-1-j} \Omega_t(j) \]

- Redistribution scheme: 
  \[ 0 = \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} \phi(j) \]
The Model: the Households

→ **Aggregate** equilibrium relations

- Aggregator: \( X_t \equiv \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} X_t(j) \)
- Aggr. financial wealth: \( \Omega_t \equiv \sum_{j=-\infty}^{t-1} \xi (1 - \xi)^{t-1-j} \Omega_t(j) \)
- Redistribution scheme: \( 0 = \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} \phi(j) \)

↓ Stock-price dynamics: \( Q_t = E_t \left\{ \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} \left[ Q_{t+1} + D_{t+1} \right] \right\} \)
The Model: the Households

→ **Aggregate** equilibrium relations

- Aggregator: \[ X_t \equiv \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} X_t(j) \]
- Aggr. financial wealth: \[ \Omega_t \equiv \sum_{j=-\infty}^{t-1} \xi (1 - \xi)^{t-1-j} \Omega_t(j) \]
- Redistribution scheme: \[ 0 = \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} \phi(j) \]

1 Stock-price dynamics: \[ Q_t = E_t \left\{ \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} \left[ Q_{t+1} + D_{t+1} \right] \right\} \]

2 Consumption dynamics:

\[ \beta P_t C_t = E_t \left\{ \mathcal{F}_{t,t+1} P_{t+1} C_{t+1} \right\} \]
The Model: the Households

→ **Aggregate** equilibrium relations

- **Aggregator:** \(X_t \equiv \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} X_t(j)\)

- Aggr. financial wealth: \(\Omega_t \equiv \sum_{j=-\infty}^{t-1} \xi (1 - \xi)^{t-1-j} \Omega_t(j)\)

- **Redistribution scheme:** \(0 = \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} \phi(j)\)

1. **Stock-price dynamics:** \(Q_t = E_t \{ F_{t+1} \frac{P_{t+1}}{P_t} \left[ Q_{t+1} + D_{t+1} \right] \}\)

2. **Consumption dynamics:**

\[
\beta P_t C_t = E_t \{ F_{t+1} P_{t+1} C_{t+1} \} + \frac{\xi \sigma}{(1 - \xi)} E_t \{ F_{t+1} P_{t+1} (\Omega_{t+1} - \phi_{nc}) \}
\]

(*)

Claim: (*) captures a dynamic distortion wrt the Social Planner Allocation.
The Model: the Households

→ **Aggregate** equilibrium relations

- Aggregator: \[ X_t \equiv \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} X_t(j) \]
- Aggr. financial wealth: \[ \Omega_t \equiv \sum_{j=-\infty}^{t-1} \xi (1 - \xi)^{t-1-j} \Omega_t(j) \]
- Redistribution scheme: \[ 0 = \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} \phi(j) \]

1. Stock-price dynamics: \[ Q_t = E_t \left\{ \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} \left[ Q_{t+1} + D_{t+1} \right] \right\} \]

2. Consumption dynamics:

\[ \beta P_t C_t = E_t \left\{ \mathcal{F}_{t,t+1} P_{t+1} C_{t+1} \right\} + \frac{\xi \sigma}{1 - \xi} E_t \left\{ \mathcal{F}_{t,t+1} P_{t+1} \left( \Omega_{t+1} - \phi_{nc} \right) \right\} \]

(**) \[ (*) \]

Claim: \( (*) \) captures a dynamic distortion wrt the Social Planner Allocation.
Social Planner Allocation

The Social Planner solves

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} \left[ \delta \log C_t(j) + (1 - \delta) \log (1 - N_t(j)) \right]$$

subject to the resource constraint

$$Y_t \equiv \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} C_t(j)$$

and the aggregate production function

$$Y_t = A_t N_t \equiv \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} N_t(j)$$

The equilibrium Social Planner Allocation:

Efficient and Equitable

$$C_e_t(j) = C_e_t = Y_e_t = \delta A_t N_e_t(j) = N_e_t = \delta$$
The Social Planner solves

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=-\infty}^{t} \xi(1 - \xi)^{t-j} \left[ \delta \log C_t(j) + (1 - \delta) \log(1 - N_t(j)) \right]$$

subject to the resource constraint

$$Y_t = C_t$$
The Social Planner solves

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=\infty}^{t} \xi(1 - \xi)^{t-j} \left[ \delta \log C_t(j) + (1 - \delta) \log (1 - N_t(j)) \right]
\]

subject to the resource constraint

\[
Y_t = C_t \equiv \sum_{j=\infty}^{t} \xi(1 - \xi)^{t-j} C_t(j)
\]
Social Planner Allocation

The Social Planner solves

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} \left[ \delta \log C_t(j) + (1 - \delta) \log (1 - N_t(j)) \right]$$

subject to the resource constraint

$$Y_t = C_t \equiv \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} C_t(j)$$

and the aggregate production function

$$Y_t = A_t N_t$$
The Social Planner solves

$$\max_{t=0}^{\infty} \beta^t \sum_{j=-\infty}^{t} \xi (1-\xi)^{t-j} \left[ \delta \log C_t(j) + (1-\delta) \log (1 - N_t(j)) \right]$$

subject to the resource constraint

$$Y_t = C_t \equiv \sum_{j=-\infty}^{t} \xi (1-\xi)^{t-j} C_t(j)$$

and the aggregate production function

$$Y_t = A_t N_t = A_t \sum_{j=-\infty}^{t} \xi (1-\xi)^{t-j} N_t(j),$$
The Social Planner solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} \left[ \delta \log C_t(j) + (1-\delta) \log(1 - N_t(j)) \right]$$

subject to the resource constraint

$$Y_t = C_t \equiv \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} C_t(j)$$

and the aggregate production function

$$Y_t = A_t N_t = A_t \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} N_t(j),$$

The equilibrium Social Planner Allocation: **Efficient** and **Equitable**

$$C_t^e(j) = C_t^e = Y_t^e = \delta A_t$$

$$N_t^e(j) = N_t^e = \delta$$
Claim: (*) captures a dynamic distortion wrt the SP Alloc.

Decompose aggregate consumption in period $t + 1$

$$C_{t+1} = \sum_{j=-\infty}^{t+1} \xi (1 - \xi)^{t+1-j} C_{t+1}(j) = \xi C_{t+1}^{nc} + (1 - \xi) C_{t+1}^{ot}$$
Claim: (*) captures a dynamic distortion wrt the SP Alloc.

Decompose aggregate consumption in period \( t + 1 \)

\[
C_{t+1} = \sum_{j=-\infty}^{t+1} \xi(1 - \xi)^{t+1-j} C_{t+1}(j) = \xi C_{t+1}^{nc} + (1 - \xi) C_{t+1}^{ot}
\]

mass-weighted average of:

- consumption of “newcomers”:
  \[
  C_{t+1}^{nc} \equiv C_{t+1}(t + 1)
  \]

- consumption of “old traders”:
  \[
  C_{t+1}^{ot} \equiv \sum_{j=-\infty}^{t} \xi(1 - \xi)^{t-j} C_{t+1}(j)
  \]
Claim: (*) captures a dynamic distortion wrt the SP Alloc.

Decompose aggregate consumption in period $t + 1$

$$C_{t+1} = \sum_{j=-\infty}^{t+1} \xi(1 - \xi)^{t+1-j} C_{t+1}(j) = \xi C_{t+1}^{nc} + (1 - \xi) C_{t+1}^{ot}$$

mass-weighted average of:

- consumption of “newcomers”:
  $$C_{t+1}^{nc} \equiv C_{t+1}(t + 1)$$

- consumption of “old traders”:
  $$C_{t+1}^{ot} \equiv \sum_{j=-\infty}^{t} \xi(1 - \xi)^{t-j} C_{t+1}(j)$$

Apply aggregator to individual Euler Equation:

$$\beta P_t C_t = E_t \left\{ F_{t,t+1} P_{t+1} C_{t+1}^{ot} \right\}$$
Claim: (*) captures a dynamic distortion wrt the SP Alloc.

Decompose aggregate consumption in period $t + 1$

$$C_{t+1} = \sum_{j=-\infty}^{t+1} \xi (1 - \xi)^{t+1-j} C_{t+1}(j) = \xi C_{t+1}^{nc} + (1 - \xi) C_{t+1}^{ot}$$

mass-weighted average of:

- consumption of “newcomers”:
  $$C_{t+1}^{nc} \equiv C_{t+1}(t + 1)$$

- consumption of “old traders”:
  $$C_{t+1}^{ot} \equiv \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} C_{t+1}(j)$$

Apply aggregator to individual Euler Equation:

$$\beta P_t C_t = E_t \left\{ F_{t+1} P_{t+1} C_{t+1}^{ot} \right\}$$

To derive aggregate consumption dynamics we must account for the wedge

$$C_{t+1}^{ot} - C_{t+1}$$
Decompose aggregate consumption in period $t + 1$

$$C_{t+1} = \sum_{j=-\infty}^{t+1} \xi(1 - \xi)^{t+1-j} C_{t+1}(j) = \xi C_{t+1}^{nc} + (1 - \xi) C_{t+1}^{ot}$$

mass-weighted average of:

- consumption of “newcomers”:
  $$C_{t+1}^{nc} \equiv C_{t+1}(t + 1)$$

- consumption of “old traders”:
  $$C_{t+1}^{ot} \equiv \sum_{j=-\infty}^{t} \xi(1 - \xi)^{t-j} C_{t+1}(j)$$

Apply aggregator to individual Euler Equation:

$$\beta P_t C_t = E_t \{ F_{t,t+1} P_{t+1} C_{t+1}^{ot} \}$$

To derive aggregate consumption dynamics we must account for the wedge

$$C_{t+1}^{ot} - C_{t+1} = \xi \left( C_{t+1}^{ot} - C_{t+1}^{nc} \right) = \frac{\xi \sigma}{1 - \xi} \left( \Omega_{t+1} - \phi^{nc} \right) \rightarrow (*)$$
Claim: (*) captures a dynamic distortion wrt the SP Alloc.

Decompose aggregate consumption in period $t + 1$

$$C_{t+1} = \sum_{j=-\infty}^{t+1} \xi(1 - \xi)^{t+1-j} C_{t+1}(j) = \xi C_{t+1}^{nc} + (1 - \xi) C_{t+1}^{ot}$$

mass-weighted average of:

- consumption of “newcomers”:
  $$C_{t+1}^{nc} \equiv C_{t+1}(t + 1)$$

- consumption of “old traders”:
  $$C_{t+1}^{ot} \equiv \sum_{j=-\infty}^{t} \xi(1 - \xi)^{t-j} C_{t+1}(j)$$

Apply aggregator to individual Euler Equation:

$$\beta P_t C_t = E_t \left\{ F_{t,t+1} P_{t+1} C_{t+1}^{ot} \right\}$$

To derive aggregate consumption dynamics we must account for the wedge

$$C_{t+1}^{ot} - C_{t+1} = \xi \left( C_{t+1}^{ot} - C_{t+1}^{nc} \right) = \frac{\xi \sigma}{1 - \xi} (\Omega_{t+1} - \phi^{nc}) \rightarrow (*)$$

The wedge is closed if either

- $\xi = 0$: Representative Agent framework (standard DNK model)
Claim: (*) captures a dynamic distortion wrt the SP Alloc.

Decompose aggregate consumption in period $t + 1$

\[
C_{t+1} = \sum_{j=-\infty}^{t+1} \xi (1 - \xi)^{t+1-j} C_{t+1}(j) = \xi C_{t+1}^{nc} + (1 - \xi) C_{t+1}^{ot}
\]

mass-weighted average of:

- consumption of “newcomers”:
  \[
  C_{t+1}^{nc} \equiv C_{t+1}(t + 1)
  \]

- consumption of “old traders”:
  \[
  C_{t+1}^{ot} \equiv \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} C_{t+1}(j)
  \]

Apply aggregator to individual Euler Equation:

\[
\beta P_t C_t = E_t \{ F_{t+1} P_{t+1} C_{t+1}^{ot} \}
\]

To derive aggregate consumption dynamics we must account for the wedge

\[
C_{t+1}^{ot} - C_{t+1} = \xi \left( C_{t+1}^{ot} - C_{t+1}^{nc} \right) = \frac{\xi \sigma}{1 - \xi} \left( \Omega_{t+1} - \phi^{nc} \right) \rightarrow (*)
\]

The wedge is closed if either

- $\xi = 0$: Representative Agent framework (standard DNK model)
- $\Omega_{t+1} = \phi^{nc}$: Financial Stability
The Linear Model with optimal SS transfers

The Linear Model

\[ y_t = E_t y_{t+1} + \psi E_t \omega_{t+1} - (r_t - E_t \pi_{t+1} - \rho) \]

\[ \omega_t = \beta E_t \omega_{t+1} - (1 - \beta) \frac{1+\varphi-\mu}{\mu} x_t - \beta (r_t - E_t \pi_{t+1} - \rho) + (1 - \beta) \bar{y}_t \]

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]

in which

\[ \psi \equiv \xi \frac{1 - \beta (1 - \xi)}{1 - \xi} \frac{\delta}{1 - \beta} \frac{\mu}{1 + \mu} \quad x_t \equiv y_t - \bar{y}_t \quad \bar{y}_t = a_t \]
The Linear Model

\[ y_t = E_t y_{t+1} + \psi E_t \omega_{t+1} - (r_t - E_t \pi_{t+1} - \rho) \]

\[ \omega_t = \beta E_t \omega_{t+1} - (1 - \beta) \frac{1 + \varphi - \mu}{\mu} x_t - \beta (r_t - E_t \pi_{t+1} - \rho) + (1 - \beta) \bar{y}_t \]

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]

in which

\[ \psi \equiv \xi \frac{1 - \beta (1 - \xi)}{1 - \xi} \frac{\delta}{1 - \beta} \frac{\mu}{1 + \mu} \]

\[ x_t \equiv y_t - \bar{y}_t \quad \bar{y}_t = a_t \]

⇒ Stock-price dynamics no longer redundant
The Welfare Criterion: \( \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} u_{t+k} \right\} \)
The Welfare Criterion: \( \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} U_{t+k} \right\} \)

- Within the standard small-scale DNK model:
  \[
  U_t = u(C_t) - v(N_t)
  \]
  and a SOA implies
  \[
  \mathcal{L}_t \equiv -\mathcal{W}_t \sim E_t \sum_{k=0}^{\infty} \beta^{t+k} \left[ x_{t+k}^2 + \frac{\epsilon}{\kappa} \pi_{t+k}^2 \right].
  \]
The Welfare Criterion:  \( \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} \mathcal{U}_{t+k} \right\} \)

- Within the standard small-scale DNK model:

\[
\mathcal{U}_t = u(C_t) - v(N_t)
\]

and a SOA implies

\[
\mathcal{L}_t \equiv -\mathcal{W}_t \propto E_t \sum_{k=0}^{\infty} \beta^{t+k} \left[ x_{t+k} + \frac{\epsilon}{\kappa} \pi_{t+k} \right].
\]

- Within our PY-DNK model:

\[
\mathcal{U}_t = U_t - V_t = \sum_{j=-\infty}^{t} \xi(1 - \xi)^{t-j} \left[ u(C_t(j)) - v(N_t(j)) \right]
\]

where

\[
u(C_t(j)) \equiv \delta \log C_t(j) \quad v(N_t(j)) \equiv -(1 - \delta) \log(1 - N_t(j))\]
The Welfare Criterion: \( \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} U_{t+k} \right\} \)

Given the equitable steady state, for a given variable \( Z \):

\[
z_t \equiv \log(Z_t/Z) = \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} z_t(j) = E_j z_t(j).
\]
The Welfare Criterion: \( \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} U_{t+k} \right\} \)

- Given the equitable steady state, for a given variable \( Z \):

\[
\begin{align*}
z_t & \equiv \log(Z_t/Z) = \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} z_t(j) = E_j z_t(j).
\end{align*}
\]

- SOA of \( U_t \) implies

\[
\begin{align*}
U_t & \approx \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} \delta c_t(j) = \delta c_t
\end{align*}
\]
The Welfare Criterion: \( \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} \mathcal{U}_{t+k} \right\} \)

- Given the equitable steady state, for a given variable \( Z \):
  \[
  z_t \equiv \log(Z_t/Z) = \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} z_t(j) = E_j z_t(j).
  \]

- SOA of \( \mathcal{U}_t \) implies
  \[
  U_t \approx \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} \delta c_t(j) = \delta c_t
  \]
  \[
  V_t \approx \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} \delta \left( n_t(j) + \frac{1+\varphi}{2} n_t(j)^2 \right)
  \]
The Welfare Criterion: $\mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} \mathcal{U}_{t+k} \right\}$

- Given the equitable steady state, for a given variable $Z$:

$$z_t \equiv \log\left(\frac{Z_t}{Z}\right) = \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} z_t(j) = E_j z_t(j).$$

- SOA of $\mathcal{U}_t$ implies

$$U_t \approx \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} \delta c_t(j) = \delta c_t$$

$$V_t \approx \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} \delta \left( n_t(j) + \frac{1+\varphi}{2} n_t(j)^2 \right) = \delta \left( n_t + \frac{1+\varphi}{2} n_t^2 + \frac{1+\varphi}{2} \text{var}_j n_t(j) \right)$$
The Welfare Criterion: \( \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} \mathcal{U}_{t+k} \right\} \)

- Given the equitable steady state, for a given variable \( Z \):

\[
z_t \equiv \log(\frac{Z_t}{Z}) = \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} z_t(j) = E_j z_t(j).
\]

- SOA of \( \mathcal{U}_t \) implies

\[
U_t \approx \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} \delta c_t(j) = \delta c_t
\]

\[
V_t \approx \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} \delta \left( n_t(j) + \frac{1+\varphi}{2} n_t(j)^2 \right) = \delta \left( n_t + \frac{1+\varphi}{2} n_t^2 + \frac{1+\varphi}{2} \text{var}_j n_t(j) \right)
\]

- and therefore

\[
\mathcal{U}_t \approx -\frac{\delta (1 + \varphi)}{2} \left( x_t^2 + \frac{2}{1 + \varphi} \Delta_{p,t} + \frac{1}{\varphi^2} \text{var}_j c_t(j) \right)
\]
The Welfare Criterion: dealing with consumption dispersion

- set $\Delta_{c,t} \equiv \text{var}_j(c_t(j) | j \leq t)$
The Welfare Criterion: dealing with consumption dispersion

- set $\Delta_{c,t} \equiv \text{var}_j(c_t(j)| j \leq t)$

- notice:

$$c_t(j) = \begin{cases} \tilde{h}_t \\ \frac{\psi}{\xi} \frac{\Omega(j)}{\Omega} \omega_t(j) + \tilde{h}_t \end{cases}$$

for newcomers ($j = t$), whose mass is $\xi$

for old traders ($j \leq t - 1$), whose mass is $(1 - \xi)$
The Welfare Criterion: dealing with consumption dispersion

- set $\Delta_{c,t} \equiv \text{var}_j(c_t(j) | j \leq t)$

- notice:

$$c_t(j) = \begin{cases} \tilde{h}_t & \text{for newcomers } (j = t), \text{ whose mass is } \xi \\ \psi \frac{\Omega(j)}{\Omega} \omega_t(j) + \tilde{h}_t & \text{for old traders } (j \leq t - 1), \text{ whose mass is } (1 - \xi) \end{cases}$$

- law of total variance on above partition:

$$\Delta_{c,t} = E \left[ \text{var}_j(c_t(j) | j = t), \text{ var}_j(c_t(j) | j \leq t - 1) \right]$$
The Welfare Criterion: dealing with consumption dispersion

- set \( \Delta_{c,t} \equiv var_j(c_t(j) | j \leq t) \)

- notice:

\[
c_t(j) = \begin{cases} 
\tilde{h}_t & \text{for newcomers (} j = t \text{), whose mass is } \xi \\
\psi \frac{\Omega(j)}{\bar{\Omega}} \omega_t(j) + \tilde{h}_t & \text{for old traders (} j \leq t - 1 \text{), whose mass is } (1 - \xi) 
\end{cases}
\]

- law of total variance on above partition:

\[
\Delta_{c,t} = E \left[ var_j(c_t(j) | j = t), var_j(c_t(j) | j \leq t-1) \right] \\
+ var \left[ E_j(c_t(j) | j = t), E_j(c_t(j) | j \leq t-1) \right]
\]
The Welfare Criterion: dealing with consumption dispersion

- the mass-weighted mean of conditional variances:

\[ E \left[ \text{var}_j(c_t(j) | j = t), \text{var}_j(c_t(j) | j \leq t - 1) \right] = (1 - \xi) \text{var}_j(c_t(j) | j \leq t - 1) \]
The Welfare Criterion: dealing with consumption dispersion

- the mass-weighted mean of conditional variances:

\[
E \left[ \text{var}_j(c_t(j) | j = t), \text{var}_j(c_t(j) | j \leq t - 1) \right] = (1 - \xi) \text{var}_j(c_t(j) | j \leq t - 1)
\]

- using a FOA of the equilibrium stochastic discount factor

\[
\text{var}_j(c_t(j) | j \leq t - 1) = \text{var}_j(c_{t-1}(j) - f_{t-1}, t - \pi_t | j \leq t - 1)
\]
The Welfare Criterion: dealing with consumption dispersion

- the mass-weighted mean of conditional variances:
  \[ E\left[ var_j\left(c_t(j)\mid j = t\right), var_j\left(c_t(j)\mid j \leq t - 1\right)\right] = (1 - \xi) var_j\left(c_t(j)\mid j \leq t - 1\right) \]

- using a FOA of the equilibrium stochastic discount factor
  \[ var_j\left(c_t(j)\mid j \leq t - 1\right) = var_j\left(c_{t-1}(j) - f_{t-1, t-\pi t}\mid j \leq t - 1\right) \]
  \[ = var_j\left(c_{t-1}(j)\mid j \leq t - 1\right) = \Delta_{c, t-1} \]
The Welfare Criterion: dealing with consumption dispersion

- the mass-weighted mean of conditional variances:

\[ E \left[ \text{var}_j(c_t(j) | j = t), \text{var}_j(c_t(j) | j \leq t - 1) \right] = (1 - \xi) \text{var}_j(c_t(j) | j \leq t - 1) \]

- using a FOA of the equilibrium stochastic discount factor

\[ \text{var}_j(c_t(j) | j \leq t - 1) = \text{var}_j(c_{t-1}(j) - f_{t-1, t-\pi_t} | j \leq t - 1) \]

\[ = \text{var}_j(c_{t-1}(j) | j \leq t - 1) = \Delta_{c, t-1} \]

- therefore:

\[ \Delta_{c, t} = (1 - \xi) \Delta_{c, t-1} + \text{var} \left[ E_j(c_t(j) | j = t), E_j(c_t(j) | j \leq t - 1) \right] \]
The Welfare Criterion: dealing with consumption dispersion

- the mass-weighted variance of conditional means:

\[ \text{var} \left[ E_j(c_t(j) \mid j = t), E_j(c_t(j) \mid j \leq t-1) \right] \]
The Welfare Criterion: dealing with consumption dispersion

- the mass-weighted variance of conditional means:

\[
\text{var} \left[ E_j(c_t(j) \mid j = t), E_j(c_t(j) \mid j \leq t-1) \right] = \text{var} \left[ \tilde{h}_t, \frac{\psi}{\xi} \omega_t + \tilde{h}_t \right] = \psi \frac{\sigma}{1 - \beta} \frac{\mu}{1 + \mu} \omega_t^2
\]
The Welfare Criterion: dealing with consumption dispersion

- the mass-weighted variance of conditional means:

\[
\text{var} \left[ E_j(c_t(j) | j = t), E_j(c_t(j) | j \leq t-1) \right]
\]

\[
= \text{var} \left[ \tilde{h}_t, \frac{\psi}{\xi} \omega_t + \tilde{h}_t \right] = \psi \frac{\sigma}{1 - \beta} \frac{\mu}{1 + \mu} \omega_t^2
\]

- hence, consumption dispersion evolves as:

\[
\Delta_{c,t} = (1 - \xi) \Delta_{c,t-1} + \psi \frac{\sigma}{1 - \beta} \frac{\mu}{1 + \mu} \omega_t^2
\]

or else:

\[
\Delta_{c,t} = (1 - \xi)^{t+1} \Delta_{c,-1} + \psi \frac{\sigma}{1 - \beta} \frac{\mu}{1 + \mu} \sum_{s=0}^{t} (1 - \xi)^{t-s} \omega_s^2
\]
The Welfare Criterion: dealing with consumption dispersion

- the mass-weighted variance of conditional means:
  \[
  \text{var} \left[ E_j(c_t(j) | j = t), E_j(c_t(j) | j \leq t-1) \right] \\
  = \text{var} \left[ \tilde{h}_t, \frac{\psi}{\xi} \omega_t + \tilde{h}_t \right] = \psi \frac{\sigma}{1 - \beta} \frac{\mu}{1 + \mu} \omega^2_t
  \]

- hence, consumption dispersion evolves as:
  \[
  \Delta_{c,t} = (1 - \xi) \Delta_{c,t-1} + \psi \frac{\sigma}{1 - \beta} \frac{\mu}{1 + \mu} \omega^2_t
  \]
  or else:
  \[
  \Delta_{c,t} = (1 - \xi)^{t+1} \Delta_{c,-1} + \psi \frac{\sigma}{1 - \beta} \frac{\mu}{1 + \mu} \sum_{s=0}^{t} (1 - \xi)^{t-s} \omega^2_s,
  \]

- and the discounted sum over all periods \( t > 0 \):
  \[
  \sum_{t=0}^{\infty} \beta^t \Delta_{c,t} = \frac{\psi \delta \mu}{(1 - \beta)(1 + \mu)} \sum_{t=0}^{\infty} \beta^t \omega^2_t + t.i.p.
  \]
The Welfare Criterion: \( \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} u_{t+k} \right\} \)

Therefore:

\[ \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} u_{t+k} \right\} \]
The Welfare Criterion: \( \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} U_{t+k} \right\} \)

Therefore:

\[ \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} U_{t+k} \right\} \approx -\frac{\delta(1 + \varphi)}{2} E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} \left( x_{t+k}^2 + \frac{2}{1 + \varphi} \Delta_{p,t+k} + \frac{1}{\varphi^2} \Delta_{c,t+k} \right) \right\} \]
The Welfare Criterion: \( \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} U_{t+k} \right\} \)

Therefore:

\[
\mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} U_{t+k} \right\} \\
\approx -\frac{\delta(1 + \varphi)}{2} E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} \left( x_{t+k}^2 + \frac{2}{1 + \varphi} \Delta_{p,t+k} + \frac{1}{\varphi^2} \Delta_{c,t+k} \right) \right\} \\
= -\frac{\delta(1 + \varphi)}{2} E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} \left( x_{t+k}^2 + \alpha_\pi \pi_{t+k}^2 + \alpha_\omega \omega_{t+k}^2 \right) \right\}
\]

where

\[
\alpha_\pi \equiv \frac{\epsilon}{\kappa} = \frac{1 + \mu}{\mu \kappa} \quad \alpha_\omega \equiv \frac{\psi \delta \mu}{\varphi^2 (1 - \beta)(1 + \mu)}
\]
The Welfare Criterion: \( \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} U_{t+k} \right\} \)

Therefore:

\[ \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} U_{t+k} \right\} \]

\[ \approx - \frac{\delta(1 + \varphi)}{2} E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} \left( x_{t+k}^2 + \frac{2}{1 + \varphi} \Delta p_{t+k} + \frac{1}{\varphi^2} \Delta c_{t+k} \right) \right\} \]

\[ = - \frac{\delta(1 + \varphi)}{2} E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} \left( x_{t+k}^2 + \alpha_\pi \pi_{t+k}^2 + \alpha_\omega \omega_{t+k}^2 \right) \right\} \]

where

\[ \alpha_\pi \equiv \frac{\epsilon}{\kappa} = \frac{1 + \mu}{\mu \kappa} \quad \text{and} \quad \alpha_\omega \equiv \frac{\psi \delta \mu}{\varphi^2 (1 - \beta) (1 + \mu)} \]

- \( \alpha_\omega \) is increasing in \( \xi \) and \( \mu \) (notice: \( \alpha_\pi \) is decreasing in \( \mu \))
The Welfare Criterion:  \( \mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} U_{t+k} \right\} \)

Therefore:

\[
\mathcal{W}_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} U_{t+k} \right\} \\
\approx -\frac{\delta (1 + \varphi)}{2} E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} \left( x_{t+k}^2 + \frac{2}{1 + \varphi} \Delta_{p,t+k} + \frac{1}{\varphi^2} \Delta_{c,t+k} \right) \right\} \\
= -\frac{\delta (1 + \varphi)}{2} E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} \left( x_{t+k}^2 + \alpha_\pi \pi_{t+k}^2 + \alpha_\omega \omega_{t+k}^2 \right) \right\}
\]

where

\[
\alpha_\pi \equiv \frac{\epsilon}{\kappa} = \frac{1 + \mu}{\mu \kappa} \quad \alpha_\omega \equiv \frac{\psi \delta \mu}{\varphi^2 (1 - \beta)(1 + \mu)}
\]

\( \alpha_\omega \) is increasing in \( \xi \) and \( \mu \) (notice: \( \alpha_\pi \) is decreasing in \( \mu \))

Result:

Financial stability (\( \omega_t = 0 \)) explicit and additional target of welfare-maximizing monetary policy
Welfare-maximizing Central Bank solves

\[
\min \quad L_t = E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} \left( x_{t+k}^2 + \alpha \pi_{t+k}^2 + \alpha \omega_{t+k}^2 \right) \right\}
\]
Welfare-maximizing Central Bank solves

\[
\min L_t = E_t \left\{ \sum_{k=0}^{\infty} \beta^{t+k} \left( x_{t+k}^2 + \alpha \pi_{t+k}^2 + \alpha \omega_{t+k}^2 \right) \right\}
\]

subject to

\[
\omega_t = \beta (1 - \psi) E_t \omega_{t+1} + \eta x_t - \beta E_t x_{t+1} + (1 - \beta \rho_a) a_t
\]
\[
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t
\]

where

\[
\eta \equiv \frac{\mu - (1 - \beta)(1 + \varphi)}{\mu}
\]
The Central Bank solves

\[ \min_{\{x_t, \pi_t, \omega_t\}} x_t^2 + \alpha_\pi \pi_t^2 + \alpha_\omega \omega_t^2 \]

such that

\[ \omega_t = \eta x_t + K_{\omega,t} \]

and

\[ \pi_t = \kappa x_t + K_{\pi,t}, \]

where \( K_{\pi,t} \) and \( K_{\omega,t} \) collect the terms that the Central Bank cannot affect.
In the standard DNK model, the optimality condition:

\[ x_t + \alpha \pi \kappa \pi_t = 0. \]

implying, in the absence of cost-push shocks, the optimal

- rate of inflation \( \pi_t^d = 0 \)
- level of output gap \( x_t^d = 0 \)
- stock of financial wealth \( \omega_t^d = a_t \)

⇒ Strict Price-Stability \( (\pi_t = 0) \) is an optimal regime, as long as there are no cost-push shocks
Optimal Monetary Policy under Discretion

Optimality condition:

\[ x_t + \alpha \pi \kappa \pi_t + \alpha \omega \eta \omega_t = 0. \]
Optimal Monetary Policy under Discretion

Optimality condition:

\[ x_t + \alpha \pi \kappa \pi_t + \alpha \omega \eta \omega_t = 0. \]

implying the optimal

- rate of inflation \[ \pi_t^d = -\Phi_{\pi} a_t \]
- level of output gap \[ x_t^d = -\Phi_{x} a_t \]
- stock of financial wealth \[ \omega_t^d = \Phi_{\omega} a_t \]

where

\[ \Phi_{\pi}^d = \frac{\alpha \omega \kappa \eta (1 - \beta \rho_a)}{A + \alpha \omega B + \psi C} \]

\[ \Phi_{x}^d = \frac{\alpha \omega \eta (1 - \beta \rho_a)^2}{A + \alpha \omega B + \psi C} \]

\[ \Phi_{\omega}^d = \frac{A}{A + \alpha \omega B + \psi C} \]

and \( A, B, C > 0 \)

Result:
Strict price stability no longer optimal, even absent cost-push shocks: endogenous stabilization tradeoff (efficiency/equitability tradeoff).
Optimal Monetary Policy under Discretion

Optimality condition:

\[ x_t + \alpha \pi \kappa \pi_t + \alpha \omega \eta \omega_t = 0. \]

implying the optimal

- rate of inflation
  \[ \pi_t = -\Phi^d \pi a_t \]
- level of output gap
  \[ x_t = -\Phi^d x a_t \]
- stock of financial wealth
  \[ \omega_t = \Phi^d \omega a_t \]

where

\[ \Phi^d _\pi = \frac{\alpha \omega \kappa \eta (1 - \beta \rho_a)}{A + \alpha \omega B + \psi C} \]
\[ \Phi^d _x = \frac{\alpha \omega \eta (1 - \beta \rho_a)^2}{A + \alpha \omega B + \psi C} \]
\[ \Phi^d _\omega = \frac{A}{A + \alpha \omega B + \psi C} \]

and \( A, B, C > 0 \)

Result:

*Strict Price stability no longer optimal, even absent cost-push shocks: endogenous stabilization tradeoff (efficiency/equitability tradeoff).*
Conclusions

- Small-scale DNK model with real demand-side effects of stock-price dynamics: perpetual youth structure
Conclusions

- Small-scale DNK model with real demand-side effects of stock-price dynamics: perpetual youth structure

- Heterogeneity in households implies additional dynamic distortion wrt Social Planner Allocation: consumption dispersion
Conclusions

- Small-scale DNK model with real demand-side effects of stock-price dynamics: perpetual youth structure

- Heterogeneity in households implies additional dynamic distortion wrt Social Planner Allocation: consumption dispersion

- Micro-founded welfare-based loss function features an additional target: financial stability
Conclusions

- Small-scale DNK model with real demand-side effects of stock-price dynamics: perpetual youth structure

- Heterogeneity in households implies additional dynamic distortion wrt Social Planner Allocation: consumption dispersion

- Micro-founded welfare-based loss function features an additional target: financial stability

- Endogenous stabilization tradeoff: Strict Price Stability is no longer necessarily optimal even absent cost-push shocks
Conclusions

- Small-scale DNK model with real demand-side effects of stock-price dynamics: perpetual youth structure

- Heterogeneity in households implies additional dynamic distortion wrt Social Planner Allocation: consumption dispersion

- Micro-founded welfare-based loss function features an additional target: financial stability

- Endogenous stabilization tradeoff: Strict Price Stability is no longer necessarily optimal even absent cost-push shocks

- To appear soon(er or later):
  - gains from commitment
  - optimized simple rules: response to stock-price dynamics?